Spin-dependent tunneling in a double quantum dot in the „slow“ evolution regime

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The tunneling and spin dynamics is studied for the hole states in a GaAs-based double quantum dot in the presence of strong spin-orbit coupling and periodic electric field. The regimes of tunneling with the spin flip are considered for the „slow“ evolution when the field frequency is lower than the other energy parameters of the stationary part of the Hamiltonian. It is found that the under such conditions the spin flip tunneling may take place at both resonant and non-resonant regimes with respect to the Zeeman level splitting. In the latter case the driving frequency may be lower compared to the resonance one, and the system dynamics resembles the Landau−Zener−Stuckelberg−Majorana interference effects arising during the dynamic level passage in isolated quantum dots.

Keywords: double quantum dot, spin-orbit interaction, Zeeman splitting, tunneling, electric dipole spin resonance.

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1. Introduction

The problem of joint control of spatial motion and spin of charge carriers in heterostructures has been relevant for the past decades. One of the commonly used effects for controlling spin dynamics using an electric field is the Electric Dipole Spin Resonance (EDSR). During its course, the coherent dynamics of spin is induced by a periodic electric field with a frequency of ω equal to the Zeeman splitting of levels Δ_Z in quantum dots (QD) [1–3]:

$$\hbar\omega = \Delta_Z. \qquad (1)$$

A necessary condition for the EDSR is the connection between states with different spin projections that occurs when exposed to the electric field of the wave, which is possible if there is a significant spin-orbit interaction (SOI) in the system. As shown by recent experiments with double QD based on p-GaAs, between which tunneling occurs in the microwave field [4,5], in such a structure, non-trivial effects are possible for the dependence of the tunneling current on the system parameters, including the frequency and amplitude of the field, and both in the EDSR regime, and outside of it. In the latter case, the tunneling features resemble the interference effects of Landau–Zener–Stuckelberg–Majorana (LZSM), occurring at the dynamic intersection of levels related to isolated quantum dots [6–9]. In the simplest form, the main result of the Landau-Zener theory, which determines the probability of transition P in a two-level system with a single intersection of levels, i.e., on one cycle of changing the parameters of the system with the imposition of a periodic field, can be formulated as follows [6,7]:

$$P = \exp(-2\pi\delta). \qquad (2)$$

In the expression (2) the adiabaticity parameter δ is expressed in terms of the level splitting Δ and the rate of change of the distance between the levels $v = d(E_2 - E_1)/dt$ as follows (in the system of units with $\hbar = 1$):

$$\delta = \frac{\Delta^2}{4v}. \qquad (3)$$

If the adiabaticity parameter $\delta \gg 1$, i.e., according to (3) the rate of change of the distance between the levels is small, then the probability of transition (2) is also small. If the parameter is $\delta \ll 1$, then with multiple level crossings, the transition efficiency will be determined by the relationship between the amplitude and frequency of the perturbation, where interference amplifications and suppression of the transition probability are possible. In this paper, we will be interested in the probability of transition at a single intersection of levels in one period of the external field. In addition to the very probability of tunneling from one QD to another, the question of the effectiveness of the spin flip in such tunneling also remains open, if the EDSR condition (1) is not met. The presence of an effective spin flip at the LZSM-intersection of levels in neighboring QD for a small number of field periods would be of interest as another possibility of controlling spin using an electric field.

This paper presents the results of a theoretical study of the evolution of hole states in a double QD with a strong SOI in the „mode of slow“ evolution, in which the Zeeman splitting is Δ_Z and, as a consequence, the frequency of the electric field is smaller than the other energy parameters of the stationary part of the Hamiltonian. The results of the evolution calculation are presented as in the EDSR mode, when the condition (1) is fulfilled, and at a lower frequency of the external field satisfying the condition $\hbar\omega = \Delta_Z/3$. We note that we performed calculations for a wider set of frequencies from $\hbar\omega = \Delta_Z/8$ to $\hbar\omega = \Delta_Z$, which showed the identity of the main conclusions that can be drawn from two cases presented in the paper: $\hbar\omega = \Delta_Z$ and $\hbar\omega = \Delta_Z/3$. The parameters of the investigated model were chosen close to the experimental conditions [4,5]. The total and spin-dependent probabilities of tunneling are calculated. A distinctive feature of the system is the strong spatial inhomogeneity of the periodic electric field in the QD region. Several basis states are also taken into account, taking into account the Zeeman splitting (up to ten), which is essential for the course of the EDSR, the nature of which in a multilevel system may differ significantly from that observed in a simple two-level approximation [10]. The results of calculations show that under such conditions, tunneling with a spin flip can occur both in the resonant mode (1) and in the non-resonant mode, for which we have chosen the ratio $\hbar\omega = \Delta_Z/3$. In the latter case, effective tunneling with a spin flip can be interpreted as a manifestation of the effects of LZSM interference [6–9] occurring during the dynamic interaction of levels in isolated quantum dots. The results obtained may be of interest in the creation of spintronics and nanoelectronics elements operating in the low frequencies.

2. Model

A model of quantum states is considered in the approximation of the effective mass in the lower zone of dimensional quantization for two-dimensional holes in GaAs, in which a one-dimensional structure with a profile of two close wells is created by the electrostatic potential of the gate, simulating a double QD. Tunneling, which is effectively one-dimensional, can occur between the potential minima corresponding to the isolated QD. The Hamiltonian of the system has the form

$$H = H_{2QD} + H_Z + H_{SO} + V(x,t). \tag{4}$$

The first term in (4) includes kinetic energy and the one-dimensional potential of the double QD [10]:

$$H_{2QD} = \frac{p^2}{2m} + U_0\left[\left(\frac{x}{d}\right)^4 - 2\left(\frac{x}{d}\right)^2\right]. \tag{5}$$

The potential energy in (5) corresponds to two minima spaced from each other at a distance of $2d$ and separated by a barrier of height U_0 relative to the bottom of the potential well. In the experiments [4,5], the typical value of d in a double QD structure was 58 nm, and the height of the potential barrier was estimated at $U_0 = 5$ meV. These values of the potential parameters (5) will be used in our model. The second term in (4) is the Zeeman interaction of a hole spin with a magnetic field B_z:

$$H_Z = \frac{1}{2} g\mu_B B_z \sigma_z, \tag{6}$$

that forms the Zeeman splitting of levels $\Delta_Z = g\mu_B B_z$ observed in experiments [4,5]. In a one-dimensional system,

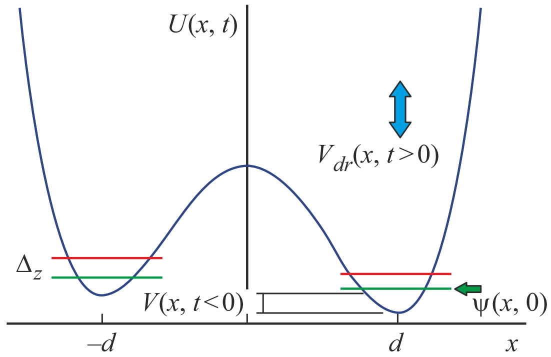

Figure 1. The shape of the potential and the structure of the lower levels in a double QD with a Hamiltonian (4), including the displacement of the minima of the potential (8) $V(x, t < 0)$ and Zeeman splitting of levels Δ_Z in each QD. The electric field with potential (10) $V_{dr}(x, t > 0)$ (blue double arrow) is applied mainly in the region of the right QD. The initial state of $\psi(x, 0)$ (green arrow) with spin down is in the right QD. (Colored version of the figure is presented in electronic version of the article).

the Zeeman term will be the only contribution from the magnetic field and there will be no contribution with a vector potential [10]. The third term in (4) describes the contribution to the spin-orbit interaction for hole states, linear in quasi-momentum, which is present in most low-dimensional structures with GaAs-based QD and has the following form in the one-dimensional model:

$$H_{SO} = \beta_D k_x \sigma_x, \tag{7}$$

where the constant β_D for hole states in QD based on GaAs is $\sim 3\,\text{meV}\cdot\text{nm}$ [11]. The last term $V(x,t) = U_d f(x) + V_{dr}(x,t)$ in the Hamiltonian (4) describes the scalar potential of the electric field forming both the static potential of the displacement of the minimum of the bottom of one well relative to another (detuning) and the non-stationary potential of the alternating field (driving). Up to the moment of time $t = 0$, the static field generates only a static displacement potential

$$V(x, t < 0) = U_d f(x), \tag{8}$$

which has an amplitude of U_d and is described by a spatial profile

$$f(x) = \left(\frac{x}{d_1}\right)^3 - \frac{3}{2}\left(\frac{x}{d_1}\right)^2, \tag{9}$$

where the value $d_1 = 1.5d$ is chosen to ensure smooth crosslinking with a potential of (5) double QD. The sign of the amplitude $U_d < 0$ corresponds to the displacement of the bottom of the right QD downwards, as shown in Fig 1.

At time $t = 0$, an alternating electric field is added to the potential (8), the potential of which is

$$V_{dr}(x, t > 0) = U_d f(x)(-1 + \cos\omega t) \tag{10}$$

smoothly stitched with the detuning potential (8). Like the detuning potential, the variable field is mainly concentrated in the region of the right QD, which is due to the properties of the function (9), chosen by us to agree with the formulation of experiments [4,5].

A scheme for modeling dynamics with a Hamiltonian (4) consists in the following: first, its time-independent part is diagonalized $H_0 = H_{2QD} + H_Z + H_{SO} + U_d f(x)$ in the basis of the states of the double QD with the Hamiltonian H_{2QD} and a new basis is constructed, consisting of the eigenfunctions of the Hamiltonian H_0, which are two-component spinors $\psi_n(x)$, which correspond to the energy levels E_n. After that, the matrix elements V_{nk} of the periodic potential (10) are calculated in this basis and we solve the nonstationary Schrodinger equation for the function

$$\psi(x,t) = \sum_n c_n(t) e^{-iE_n t/\hbar} \psi_n(x) \quad (11)$$

with unknown coefficients $c_n(t)$ determining the E_n level occupancy. For the coefficients $c_n(t)$, a system of ordinary differential equations with periodic coefficients of the form is obtained

$$i\hbar \frac{dc_n}{dt} = \sum_k c_k V_{nk}\, e^{-i(E_k - E_n)t/\hbar}. \quad (12)$$

Even for a two-level model, the system (12) can be solved, generally speaking, only numerically. We include in the calculation the first ten levels in the double QD, taking into account the spin, which corresponds to five levels of spatial quantization in the potential (5). Such approximation provides a sufficient degree of completeness of the basis, since our modeling shows that more than 90% of the norm of the wave function (11) is provided by the contribution of the four lower levels corresponding to the main doublet in the double QD, taking into account the spin splitting in the magnetic field.

3. Evolution simulation results

The solution of the nonstationary Schrodinger equation in the representation of the system (12) was performed numerically by us using the Cayley scheme [12], which ensures the unitarity of the constructed evolution operator. In this paper, we focused on the EDSR regime in sufficiently weak magnetic fields or, equivalently, for sufficiently low frequencies of the periodic field. In other words, the Zeeman splitting $\Delta_Z = g\,\mu_B B_z$ or the energy $\hbar\omega$ equal to it in this mode are the smallest energy parameters of the problem, smaller in comparison with the energy level E_n, and with tunnel splitting $E_n - E_{n-1}$, $n = 2, 4, \ldots$. For the double QD parameters used in the Hamiltonian (4), the spin-conserving tunneling frequency corresponding to half of the tunneling splitting of the main doublet, which we denote by γ, is $\sim 1\,\mu$eV, and the spin-flip tunneling frequency corresponding to half of the matrix element from the spin-orbit interaction (7), which we denote by α, is $\sim 0.5\,\mu$eV [4,5]. To fulfill the above condition of the smallness of the Zeeman splitting and the frequency of the periodic field, we choose the value of the magnetic field induction $B_z = 0.0026$ T, which for the Zeeman term (6) with g-factor equal to 1.35 according to experimental data [4,5], leads to the value of the Zeeman splitting $\Delta_Z = 0.2\,\mu$eV, which ensures that the conditions are met

$$\Delta_Z < (\alpha, \gamma); \quad \hbar\omega \le \Delta_Z. \quad (13)$$

We use the value of the amplitude detuning (8) and the periodic field (10) $U_d = -10\,\mu$eV, at which the ground state of the Hamiltonian H_0 corresponds to a hole localized in the right QD with spin down relative to the direction of the magnetic field. The EDSR frequency for the periodic field (10) is determined from the condition $\hbar\omega = \Delta_Z$ and corresponds to the linear frequency $f = 0.048$ GHz. For the non-resonant case at $\hbar\omega = \Delta_Z/3$ the linear frequency $f = 0.016$ GHz. Such frequencies are low compared to those used in the experiments [4,5], but technically their use seems feasible.

We are interested in the time evolution of the probability of a particle staying in the left QD, the time behavior of the spin projection σ_L^z on the direction of the magnetic field, also corresponding to the region of the left QD, and the dynamics of the full spin projection σ_{Full}^z, corresponding to the contribution from both QDs. The initial condition for the system of evolution equations (10) is a wave packet localized in the right QD and having the projection $\sigma_{\text{Full}}^z(0) = -1$, which meets the experimental conditions [4,5], where spin-polarized holes in the magnetic field were injected into the right QD in its ground state. After calculating the population levels of $c_n(t)$ when solving the system (12), we calculate using the wave function (11) the time-dependent probability of staying in the left QD,

$$P_L(t) = \int_{-\infty}^{0} |\psi(x,t)|^2 dx, \quad (14)$$

time-dependent average value of z-spin projection in the left and right QD regions

$$\sigma_L^z(t) = \int_{-\infty}^{0} \langle\psi|\sigma_z|\psi\rangle dx, \;\; \sigma_R^z(t) = \int_{0}^{\infty} \langle\psi|\sigma_z|\psi\rangle dx, \quad (15)$$

as well as the full z-spin projection

$$\sigma_{\text{Full}}^z(t) = \int_{-\infty}^{\infty} \langle\psi|\sigma_z|\psi\rangle dx. \quad (16)$$

When the conditions (13) are met, the electric field changes in time slowly enough so that all the processes of tunneling and spin flip occur only at a few of its periods. Fig. 2 shows the results of calculating the evolution of the mean values (14)−(16). For the potential of a periodic field (10), levels with different spin projections corresponding to the localization of the wave function in any one QD, in one period the fields are shifted relative to each other by a sufficiently large amount to overlap them, as can be imagined from Fig. 1, shifting the levels in the right QD upwards and down. This condition ensures efficient tunneling of a particle from one QD to another, which follows from the interaction of levels in the theory of LZSM interference [6–9]. A new result is that in the presence of a strong SOI, such interference with tunneling between QDs leads to pronounced spin dynamics at the same time intervals, which indicates the possibility of using the described mechanism to control spin.

4. Discussion of the results

From the results presented in Fig. 2, it can be concluded that in the „mode of slow“ evolution, when the conditions (13) are met, tunneling with a spin flip is observed already at 1−2 periods of the electric field, and as in the EDSR mode (red curves R), and for the non-resonant mode $\hbar\omega = \Delta_Z/3$ (blue curves N). The characteristic spin flip time in the resonant case for a resonant frequency of 0.048 GHz is ~ 40 ns, and for a non-resonant frequency of 0.016 GHz ~ 120 ns. The observed spin flip for both values of the field frequency deserves a discussion. If such a flip is expected for the resonant case [1–3,10], then its similar occurrence in the non-resonant mode, and at the same characteristic times in units of the field period, is a non-trivial result. With „slow“ evolution, when the conditions (13) are met, the main factor for effective tunneling with a spin flip is not the fulfillment of the resonant condition (1), but the intersection of levels corresponding to different QDs (see Fig. 1), on one period of the field. With such a coincidence, provided by the choice of the potential (10) of the periodic field, their effective interaction occurs, accompanied by both tunneling and spin flip. Such a mechanism has common features with the phenomena of LZSM interference, which are usually considered for the close passage of atomic levels [6], which are also mentioned in the context of the dynamics of levels in semiconductor structures [7–9]. The practical significance of the same presented in Fig. 2 of the result is as follows: in order to control spin through the SOI mechanism, it is not always necessary to accurately fulfill the EDSR condition (1). With a „slow“ evolution satisfying the conditions (13), the periodic field can have a frequency lower than the resonant one, while not limited by a strict condition of the form (1). As we have already mentioned, we also performed calculations for a wider set of frequencies from $\hbar\omega = \Delta_Z/8$ to $\hbar\omega = \Delta_Z$, which showed the identity of the main conclusions about the presence of tunneling with a spin flip on 1−2 periods of the external field, which can be made from the two cases described above: $\hbar\omega = \Delta_Z$ and $\hbar\omega = \Delta_Z/3$.

It is of interest to compare the numerically obtained results with an analytical estimate of (2), (3) the probability

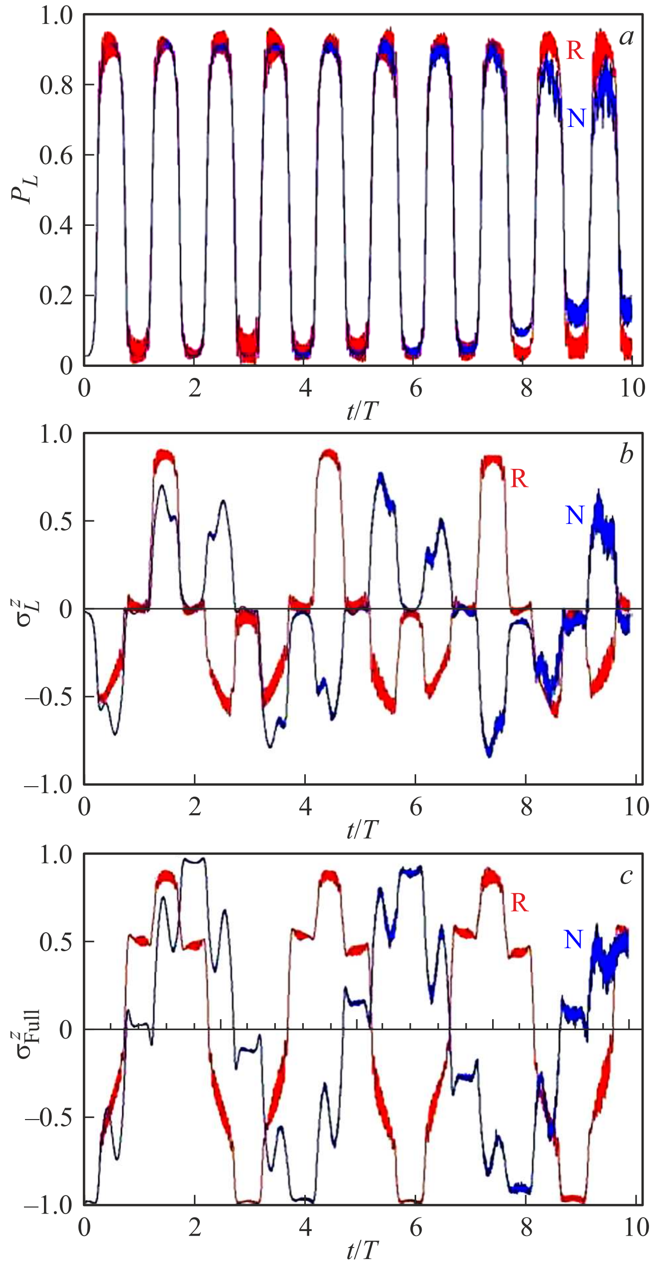

Figure 2. Time dependence of average values on 10 electric field periods with the initial condition $\sigma^z_{\rm Full}(0) = -1$ for (*a*) probability of (14) tunneling to the left QD; (*b*) z-projection of the spin (15) in the left QD and (*c*) full z-spin projections (16). The red curves R correspond to the EDSR mode (1), the blue curves N correspond to the non-resonant mode $\hbar\omega = \Delta_Z/3$. Tunneling with a spin flip is observed for both modes. (Colored version of the figure is presented in electronic version of the article).

of a Landau−Zener transition at a single intersection of levels with a spin flip, i.e., on one period of the field. In our model, the parameter $\Delta = \alpha = 0.5\,\mu$eV, and the rate of change in the distance between levels can be estimated as $v = \omega U_d$. Substituting in (2) our perturbation frequencies $f_1 = 0.048$ GHz and $f_2 = 0.016$ GHz, as well as the amplitude of the field $10\,\mu$eV, we get that the adiabaticity parameter $\delta_1 \sim 0.032$ for $f = f_1$ and $\delta_2 \sim 0.096$ for $f = f_2$, i.e., the considered perturbations are still fast enough. According to (2), the probability of a transition in one period of the Landau–Zener field is $P_1 = 0.82$ and $P_2 = 0.55$ for resonant and non-resonant perturbations, respectively. Comparing these estimates with the graphs in Fig. 2, *a*, we are convinced that they agree quite well with the numerical calculation in the sense of a significant probability of transition even with a single intersection of levels, i.e., on one period of the field.

To assess the applicability of the results obtained at different time intervals, the role of the main spin relaxation mechanisms should be evaluated at least at low temperatures. The influence of relaxation mechanisms based on interaction with the phonon field and the mechanism of hyperfine interaction with nuclear spins should be noted.

The spin flip times obtained above, despite the „slow" evolution mode, are still significantly smaller than the typical spin relaxation times τ_s by the state mixing mechanism involving the phonon field [13,14]. Thus, the estimates obtained in [14] indicate the characteristic spin relaxation times of τ_s in QD based on GaAs having the order $\tau_s \sim 1$ ms at a magnetic field of 1 T, while with a decrease in the magnetic field of B, the relaxation time increases rapidly (as $(1/B)^5$). These times are several orders of magnitude higher than the characteristic spin flip times in our model.

Another spin relaxation mechanism important for hole states in GaAs-based QD is associated with hyperfine interaction with nuclear spins [15,16]. Spin relaxation times due to this mechanism can be quite short and amount to $\sim 10-20$ ns [15] for GaAs-based structures, however, the presence of a magnetic field in Faraday geometry when $B_z \parallel S_z(t = 0)$, leads to an increase in the proportion of spins $S_z(t \to \infty)/S_z(0)$ that do not experience relaxation, i.e., the magnetic field performs a stabilizing function. For the evolution examples we have considered with $B_z = 0.0026$ T is the ratio of the Larmor frequency Ω_B and the spin relaxation rate δ gives an estimate of $\Omega_B/\delta \sim 0.5-1$. This estimate leads to the value of the fraction $S_z(t \to \infty)/S_z(0)$, which is $\sim 0.40-0.55$ [15], i.e., the proportion of coherently evolving spins is significantly less, but not much less than one. We note that with additional consideration of the mechanisms of spin blockade and exchange interaction, the spin relaxation in the double QD at bólder times may not be exponential, but power-law character [16], which provides a sufficient degree of spin polarization at times $\sim 10-100$ ns. The estimates we mentioned suggest the possibility of observing spin dynamics at the time intervals under discussion in several periods of the electric field, when the effects of spin relaxation can be ignored in the first approximation.

One can hope that the discussed mechanism of spin flip under conditions (13) with „slow" evolution can be implemented for a wider range of semiconductor structures with a strong SOI and sources of alternating electric field compared to the classical EDSR, which will have a positive impact on the development and implementation of spintronics structures.

5. Conclusion

The evolution of hole states in a GaAs-based double quantum dot in the presence of a strong spin-orbit interaction and a periodic electric field is investigated. Spin-flip tunneling modes are considered under conditions of „slow" evolution when the frequency of the field is smaller than the other energy parameters of the stationary part of the Hamiltonian. It was found that spin-flip tunneling can occur in both resonant and non-resonant modes when the frequency of the field is smaller than the Zeeman splitting. The observed dynamics of tunneling and spin demonstrates effects resembling the effects of the interference of Landau–Zener–Stuckelberg–Majorana, which are manifested not only for spatial, but also for spin dynamics. The predicted spin flip modes implemented in a wide frequency range can contribute to the wider introduction of spin control mechanisms using an alternating electric field in semiconductor structures with strong spin-orbit interaction.

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Conflict of interest

The authors declare that they have no conflict of interest.

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