

Compensation effect of growth of the lifetime of charged carriers in semiconductors at a magnetic field

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The condition of regulation of increasing of carrier lifetime by recombination processes in semiconductors at low temperatures (1–10) K and classical „strong“ magnetic fields ($3 \cdot 10^2 - 3 \cdot 10^4$) Gs are analyzed. The values of carrier concentrations ($10^{10} - 10^{14}$) cm^{-3} correspond to conditions of manifestation as cascade as resonant capture. It is indicated on necessarily to take into account of scattering of electrons on acoustic phonons along with electron-electron collisions, by it cascade capture on coulomb centers. As showed (on the basis of concrete estimates) namely scattering on acoustic phonons at cascade capture, stabilities of lifetime and controls of dynamics it increases in the presence of „strong“ magnetic field.

Keywords: effective coefficient of capture, cascade capture, recombination, classical „strong“ magnetic field, „shallow“ acoustic phonons, electron-electron collisions, lifetime.

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1. Introduction

At low temperatures (1–10) K in impurity semiconductors the recombination of charge carriers can occur through D^- (A^+)-centers [1,2]. In this case, recombination can occur due to the resonant capture of either an electron by a neutral donor (formation of D^- -center) with subsequent capture of a hole by attractive D^- -center, or hole by a neutral acceptor (formation A^+ -center) followed by electron capture [3]. In the temperature range (1–10) K at charge carrier concentrations ($10^{10} - 10^{14}$) cm^{-3} carriers capture by neutral and charged centers can have both resonant [3] and cascade nature [4]. The experimental and theoretical results of the papers [1–4] better correspond to the following scheme: the charge carrier is resonantly captured by neutral center with the formation of attractive center, on which further the cascade capture of the second carrier occurs, followed by recombination of the pair. The described scheme can be expressed by the equations: $e^- + D^0 \rightarrow D^- + h^+ \rightarrow D^0$, $h^+ + A^0 \rightarrow A^+ + e^- \rightarrow A^0$. The kinetic equation of the process has the form

$$\frac{d\Delta n}{dt} = G - \frac{\Delta n}{\tau}, \quad (1)$$

where Δn is change in photoelectron concentration, G is intensity of photoelectron generation under the action of external light, $\Delta n/\tau$ is photocarrier recombination rate, τ is lifetime of a photoelectron. Using germanium as an example, it was shown in the paper [4] that in the temperature range $T = (1-10)$ K the carrier concentrations $n_e = (10^{10} - 10^{14}) \text{cm}^{-3}$ and values of magnetic field induction $B = (3 \cdot 10^2 - 3 \cdot 10^4)$ Gs, the lifetime of secondary charge carriers under the prevalence of the electron-electron interaction can be increased by more than 100 times. The

induction values correspond to the area of „strong“ magnetic fields, in the specified temperature range. Then it follows from equation (1) that $\Delta n = G\tau [1 - \exp(-t/\tau)]$, and as a result — a significant increasing of the semiconductor photoconductivity. However, as a rule, this increasing is not observed in practice, except for the mechanisms of breakdown of $p-n$ junction. Note also the rather specific conditions for review (low-resistance germanium, low photoexcitation level) of the process of carriers recombination in the paper [4].

The results of the paper [4] are based on the Thomson model from low-temperature plasma physics [5] and are of a preliminary (evaluative) nature. At the same time, a disputable aspect of the paper [4] is the overestimation of the role of electron-electron collisions in cascade capture by Coulomb centers.

In semiconductors with strong electron-phonon interaction, a certain role in cascade capture is played by the carrier emitted „shallow“ acoustic phonons. The consideration of this circumstance ultimately leads to the stabilization of the recombination rate and the steady state process. In a nondegenerate electron gas $\sim 39\%$ of the number of electrons has energies above the average energy $3/2k_B T$, and the electron-phonon interaction acts as the main factor limiting the rapid growth of the lifetime, and thus stabilizes the growth dynamics of the electron-electron relaxation rate, changing it to the electron-phonon one.

The purpose of this paper is to obtain calculation formulas for the capture coefficient under conditions of cascade capture of carriers by emitting „shallow“ acoustic phonons ($k_B T \gg m^* s^2$) and the presence of „strong“ magnetic field, taking into account the results of the paper [4]. Here it is appropriate to note immediately that in the paper [4] for the

given capture coefficient, the relation was obtained

$$(C_B/C)_{e-e} = (r_B/r_T)^2 \propto B^{-2}, \quad (r_B \leq r_T), \quad (2)$$

where $r_B = \sqrt{2E/m^*\omega_B^2}$ is the Larmor radius of the electron, $m^* = m_\perp$ is electron effective mass ($m_\perp = 0.082m_e$), E is electron energy ($\sim 10^{-4} - 10^{-2}$ eV) [4], $\omega_B = eB/m^*c$ is cyclotron frequency, $r_T = e^2/2\kappa k_B T$ is radius of electron capture at A^+ -center, κ is shallow of semiconductor crystal, $c = 3 \cdot 10^{10}$ cm/s is speed of light in vacuum, s is speed of sound in a crystal ($\sim 10^5$ cm/s).

It is advisable to make preliminary estimates of the main parameters of the theory. For example, one can estimate the electron capture radius independently of the formula $r_T = e^2/2\kappa k_B T$. The average distance between Rydberg levels is $\Delta E \propto n^{-3}$, where n is the principal quantum number of the highly excited level.

According to the energy conservation law, the maximum energy of the emitted „small“ acoustic phonon is equal to

$$(\varepsilon_{ph})_{\max} = \sqrt{8m^*s^2\varepsilon} \approx \sqrt{8m^*s^2k_B T} \approx 6 \cdot 10^{-5} \text{ eV}. \quad (3)$$

The maximum energy of the emitted „small“ acoustic phonon is somewhat greater or comparable to the energy transferred to a free electron in electron-electron collisions. So, along with electron-electron collisions, as already noted, one must also take into account electron-phonon collisions. Since $(\varepsilon_{ph})_{\max} \leq 2I/n^3$ (I is ionization energy $\sim 10^{-2}$ eV), then the limitation $n \leq 7$ follows. Then $r_T \approx n^2 r_B \approx 10^{-5}$ cm. The mean free path of the electron in scattering on acoustic phonons is $\ell_{e-ph} = A/k_B T$ (A is constant, determined by thermal fluctuations of lattice) at a temperature 10 K is equal to $3 \cdot 10^{-5}$ cm, which is comparable to the value r_T , and the applicability condition for the cascade capture model with emission of „small“ acoustic phonons is well met [5]. Electron wavelength $\lambda = \hbar/\sqrt{2m^*k_B T} \approx 10^{-7}$ cm and $\lambda \ll r_T$, i.e., quasi-classical treatment is acceptable.

2. Calculation procedure

From a theoretical point of view, it is convenient to consider the cascade capture of the carrier to the charged center as diffusion in an abstract energy space. Unlike the usual coordinate space, the energy space, mathematically, is more convenient for calculations. The Fokker-Plank kinetic equation for the distribution function in the energy representation has the form

$$\rho_E \frac{\partial f}{\partial t} = -\frac{\partial J}{\partial E}, \quad (4)$$

where ρ_E is the density of electronic states in the total energy space, J is the charge carriers flow in the total energy space:

$$J = -B(E) \left(f + k_B T \frac{\partial f}{\partial E} \right). \quad (5)$$

Here $B(E)$ is coefficient of energy „friction“ factor equal to

$$B(E) = \int d\varepsilon \varepsilon \rho(\varepsilon) \frac{1}{\tau(\varepsilon)} \int_V \delta(E - \varepsilon - U(r)) dV, \quad (6)$$

where $\tau(\varepsilon)$ is the energy relaxation time for the emission of „velocity“ acoustic phonon in the model of the isotropic quadratic law of dispersion $\tau(\varepsilon) = \ell_o \sqrt{m^*/2\varepsilon}$, ℓ_o is characteristic mean free path for scattering by long-wavelength acoustic phonons (for germanium $\ell_o = 3.2 \cdot 10^{-3}$ cm), $\rho(\varepsilon) = \frac{(2m^*)^{3/2}}{2\pi^2 \hbar^3} \sqrt{\varepsilon}$ is the density of electronic states, $U(r) = -\frac{\alpha}{r}$ is Coulomb potential of a small impurity, $\alpha = Ze^2/\kappa$, Ze is charge $D^-(A^+)$ -center.

The stationary solution of the Fokker-Plank (4) equation corresponding to $J = \text{const}$, so that the desired solution passes into the equilibrium distribution at $E = 0$ (black wall), has the form

$$f(E = 0) = \frac{J}{k_B T} \int_{-\infty}^{E=0} \frac{\exp(\varepsilon/k_B T)}{B(\varepsilon)} d\varepsilon. \quad (7)$$

On the other hand, $f(E = 0) = n_e/N(T)$, $N(T) = (m^*k_B T/2\pi\hbar^2)^{3/2}$ — effective number of free states per unit volume with $E_k \leq k_B T$. Expressing the recombination flow in terms of the capture cross-section $J = n_e \sigma v_T$, where σ is the cascade capture cross-section, $v_T = \sqrt{8k_B T/\pi m^*}$ is the average thermal velocity of electrons, and using (7) one can receive the formula

$$\sigma = \frac{\pi^2 \hbar^3}{2k_B T m^*} \left[\int_{-\infty}^0 \frac{\exp(\varepsilon/k_B T)}{B(\varepsilon)} d\varepsilon \right]^{-1}. \quad (8)$$

In the presence of the magnetic field, the energy relaxation time decreases, which can be taken into account by replacing $\tau(\varepsilon) \rightarrow \tau(\varepsilon)[1 + \omega_B^2 \tau^2(\varepsilon)]^{-1}$.

Accordingly, the expression (6) can be rewritten as follows:

$$B_B(E) = \int d\varepsilon \varepsilon \rho(\varepsilon) \frac{1 + \omega_B^2 \tau^2(\varepsilon)}{\tau(\varepsilon)} \times \int_V \delta(E - \varepsilon - U(r)) dV. \quad (9)$$

The energy of interaction of a weakly bound electron with the magnetic field ($W_B = \mu_e B$) at $B = 5 \cdot 10^3$ G is $3 \cdot 10^{-5}$ eV, which is comparable to the maximum energy of the emitted „shallow“ acoustic phonon (3). Therefore, in expression (9) under the argument of δ -function we can neglect the energy of interaction of weakly bound electron with the magnetic field.

Calculating the integral (9) with the Coulomb potential and substituting the integration result into (8), we can obtain a quadrature:

$$\sigma_B = \frac{4\pi}{3\ell_o} \left(\frac{Ze^2}{\kappa} \right)^3 \frac{1}{(k_B T)^3} \left[\int_0^\infty \frac{x^2 \exp(-x)}{x + x_o} dx \right]^{-1}. \quad (10)$$

Here $x_0 = \bar{E}_k/2k_B T$, $\bar{E}_k = m^*(\omega_B \ell_0)^2/2$ is mean kinetic energy of the electron in the magnetic field at the mean average free path ℓ_0 . The integral in (10) can be expressed via the integral logarithm:

$$\int_0^{\infty} \frac{x^2 \exp(-x)}{x + x_0} dx = 1 - x_0 L(x_0), \quad (11)$$

where $L(x_0) = 1 - x_0 \exp(x_0)[-Ei(-x_0)]$.

Using the series expansion and the asymptotic expression for $-Ei(-x_0)$, one can show that

$$L(x_0) \approx 1/x_0 - 2/x_0^2 + \dots, \quad (x_0 \gg 1), \quad (12)$$

$$L(x_0) \approx 1 + x_0 \ln x_0 + \dots, \quad (x_0 \ll 1), \quad (13)$$

Substituting the asymptotic expansions (12) and (13) into (10) taking into account (11) gives

$$\sigma_B \approx \frac{4\pi}{3\ell_0} \left(\frac{Ze^2}{\kappa} \right)^3 \frac{1}{(k_B T)^3} \frac{\bar{E}_k}{4k_B T}, \quad (\bar{E}_k/2 \gg k_B T); \quad (14)$$

$$\sigma_{B \rightarrow 0} \approx \frac{4\pi}{3\ell_0} \left(\frac{Ze^2}{\kappa} \right)^3 \frac{1}{(k_B T)^3}, \quad (\bar{E}_k/2 \ll k_B T). \quad (15)$$

As expected, „weak“ magnetic field practically does not affect the rate of capture, while „strong“ magnetic field (14) changes the rate of the capture itself. The capture coefficient is expressed in terms of the capture cross-section: $C = \langle v \rangle \sigma = v_T \sigma$, $C_B = v_T \sigma_B = v_{\perp} \sigma_B$. Then, based on the limit formulas (14) and (15), we obtain the relation

$$(C_B/C)_{e-ph} = \bar{E}_k/4k_B T \propto B^2. \quad (16)$$

Direct calculation of integral (9) with effective energy relaxation time $\tau^* = \tau_{e-e} \tau_{ph}/(\tau_{e-e} + \tau_{ph})$ is quite difficult. However, it is clear that in one limit case the calculation of the integrals (9) and (10) must correspond to the result (2) of the paper [4], in the other limit case, as was shown, — to the result (16). Therefore, we can introduce the interpolation formula for the effective capture coefficient:

$$C_B^* = \left(\frac{1}{C_{B(e-e)}} + \frac{1}{C_{B(e-ph)}} \right)^{-1}, \quad (17)$$

which in limit cases corresponds to formulas (2) and (16). Formulas for coefficients $C_{B(e-e)}$ and $C_{B(e-ph)}$ [4,5] has the form

$$C_{B(e-e)} = C_{e-e} \left(\frac{r_B}{r_T} \right)^2 = C_{e-e} \frac{2E}{m^* \omega_B^2} \left(\frac{2\kappa k_B T}{e^2} \right)^2, \quad (18)$$

$$C_{e-e} = \frac{1}{3} (2\pi)^{3/2} n_e \left(\frac{e^2}{\kappa} \right)^5 \frac{Z^3 \Lambda}{\sqrt{m^*} (k_B T)^{9/2}}, \quad (19)$$

where Λ — Coulomb logarithm (approximately $\Lambda \approx 8$),

$$C_{B(e-ph)} = C_{e-ph} \frac{m^* (\omega_B \ell_0)^2}{8k_B T}, \quad (20)$$

Accounting for the relative contribution of electron-phonon relaxation to the overall balance of energy losses (for $n_e = 2 \cdot 10^{12} \text{ cm}^{-3}$, $E = 10^{-3} \text{ eV}$, $T = 10 \text{ K}$)

B , Gs	$C_{B(e-e, e-ph)}^*$, cm ³ /s	$C_{(e-e, e-ph)}^*$, cm ³ /s	$\frac{C_{B(e-e, e-ph)}^*}{C_{(e-e, e-ph)}^*}$	$\frac{C_{B(e-e)}}{C_{e-e}}$, from paper [4]
10 ²	0.32 · 10 ⁻³	8.2 · 10 ⁻⁶	0.39 · 10 ²	1
10 ³	1.46 · 10 ⁻³	8.2 · 10 ⁻⁶	1.78 · 10 ²	10 ⁻²
10 ⁴	1.52 · 10 ⁻⁵	8.2 · 10 ⁻⁶	1.85	10 ⁻⁴

$$C_{e-ph} = \langle v \rangle \sigma_{e-ph} = \sqrt{\frac{8k_B T}{\pi m^*}} \frac{4\pi}{3\ell_0} \left(\frac{Ze^2}{\kappa} \right)^3 \frac{1}{(k_B T)^3}. \quad (21)$$

Substituting formulas (18) and (20) into expression (17), and taking into account (19) and (21), leads to the relation ($Z = 1$) (see *Appendix*):

$$C_B^* = \frac{ab(k_B T)^{-5/2}}{4k_B T a B^{-2} + b B^2} [\text{cm}^3/\text{s}], \quad (22)$$

$$a = \frac{4}{3} \left(\frac{e^2}{\kappa} \right)^3 \frac{2E \Lambda c^2}{e^2} \sqrt{2\pi m^*} 2\pi n_e \approx 1.08 \cdot 10^{-34} [\text{SGSE}], \quad (23)$$

$$b = \frac{4}{3} \left(\frac{e^2}{\kappa} \right)^3 \sqrt{\frac{2\pi}{m^*}} \frac{\ell_0 e^2}{m^* c^2} \approx 1.27 \cdot 10^{-59} [\text{SGSE}] \quad (24)$$

(for estimates we assume $n_e = 2 \cdot 10^{12} \text{ cm}^{-3}$, $E = 10^{-3} \text{ eV}$, $B = 10^3 \text{ Gs}$, $\kappa = 16$). Numerical values (23) and (24) are given for germanium [4].

In the absence of the magnetic field (or in very weak magnetic fields)

$$C_{e-e}/C_{e-ph} \approx 4.5 \cdot 10^{-5}/10^{-5} = 4.5. \quad (25)$$

In the presence of „strong“ magnetic field, the situation changes radically:

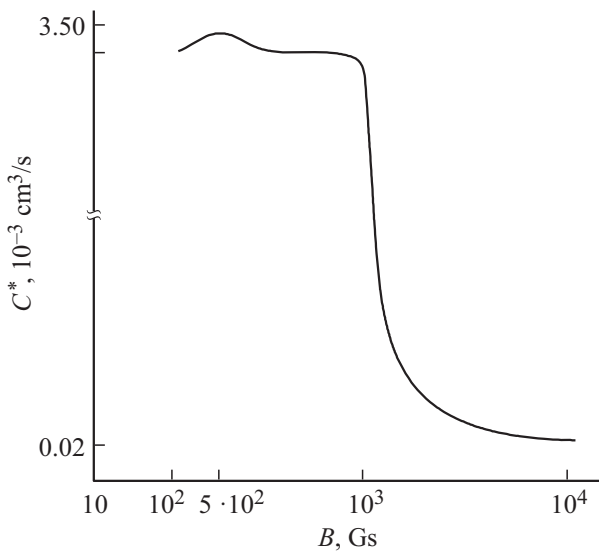
$$C_{B(e-e)}/C_{B(e-ph)} \approx 0.3 \cdot 10^{-2}. \quad (26)$$

The result (26) roughly corresponds to the result of the paper [4]:

$$C_{B(e-e)}/C_{e-e} \approx 10^{-2}, \quad (27)$$

which allows us to draw up an important conclusion: the energy loss rate in electron-electron collisions without the magnetic field approximately corresponds to the energy loss rate in electron-phonon collisions in the presence of „strong“ magnetic field.

To estimate C_B^* in the actual range of parameters, one should substitute into formula (22) the values $T = 10 \text{ K}$, $B = 10^3 \text{ Gs}$, the result is $C_B^* \approx 1.46 \cdot 10^{-3} \text{ cm}^3/\text{s}$. Estimated lifetime $\tau_B^* = 1/NC_B^*$ for concentration of capture centers $N = 10^{13} \text{ cm}^{-3}$ gives the value $\tau_B^* \approx 6.8 \cdot 10^{-11} \text{ s}$, which is almost the same as $5.7 \cdot 10^{-11} \text{ s}$ at $T = 10 \text{ K}$, for the same germanium [6]. This is quite understandable, since the electron-phonon relaxation, even in „strong“ magnetic field, manages to stabilize (compensate) the growth of the lifetime



An idealized plot of effective capture coefficient versus magnetic field induction. The presence of a characteristic plateau indicates the achievement of the lifetime stabilization effect [7], at which the energy loss rate due to electron-electron collisions is approximately equal to the energy loss rate in electron-phonon collisions.

until the prevalence of electron-electron relaxation: estimates (25)–(27). For germanium at $T = 10$ K, this starts approximately from concentrations $n_e = 3 \cdot 10^{12} \text{ cm}^{-3}$.

As can be seen from the Table, a directly opposite dynamics is observed, which is quite natural, since estimates were made for $n_e = 2 \cdot 10^{12} \text{ cm}^{-3}$, when the electron-electron interaction is not so effective as for capturing. In any case, we can conclude that the results [4] are valid, strictly speaking, for the concentrations $n_e \geq 3 \cdot 10^{12} \text{ cm}^{-3}$, i.e. for the region $(10^{12} - 10^{14}) \text{ cm}^{-3}$, however, in this case, the electron-phonon interaction should also be taken into account to refine the dynamics of lifetime growth.

Function (22) reaches its maximum value at

$$B_c = \sqrt[4]{4ak_B T/b} \approx 4.7 \cdot 10^2 \text{ Gs}. \quad (28)$$

The value (28) substitution into (22) gives

$$C_o^* = \frac{1}{4} \sqrt{ab} (k_B T)^{-3} \approx 3.5 \cdot 10^{-3} \text{ cm}^3/\text{s}. \quad (29)$$

Thus, the effective capture coefficient reaches its maximum value already at the initial values of the magnetic field induction and decreases sharply in the region $(10^3 - 10^4)$ Gs. In the region $(10^2 - 10^3)$ Gs it changes rather weakly (see Figure).

3. Conclusion

The main result of the paper can be summarized by the interpolation formula (22). The coincidence of the calculated value $\tau_B^* \approx 6.8 \cdot 10^{-11} \text{ s}$ with the results of theoretical calculations in other papers [4,6] can be

considered as confirmation of the conclusion about the essential role of energy losses due to the emission of „small“ acoustic phonons, along with electron-electron collisions, during cascade capture of carriers in „strong“ magnetic field. According to the numerical values (see Table) the effective lifetime of non-equilibrium electrons decreases by ~ 40 times ($B = 10^2$ Gs) compared to its value in the absence of the magnetic field, however, this effect can take place at sufficiently low concentrations of non-equilibrium electrons ($n_e \leq 2 \cdot 10^{12} \text{ cm}^{-3}$).

The origin of the limit field dependences (2) and (16) has a simple physical explanation: in the first case (with prevalence of electron-electron relaxation) „strong“ magnetic field „compresses“ the radius of the cyclotron orbit to the capture radius ($r_B \leq r_T$), in the second case it increases the probability of being captured by the center, since the electron moving along a cyclotron orbit with normal acceleration rapidly loses energy and can even emit „large“ acoustic phonon in the vicinity of the center.

Appendix

The average value of the effective capture coefficient (22) in the range $E = (10^{-4} - 10^{-2}) \text{ eV}$:

$$\begin{aligned} \langle C_B^* \rangle_E &= \frac{1}{E_2 - E_1} \int_{E_1}^{E_2} C_B^*(E) dE \\ &= \frac{bB^2}{4(k_B T)^{7/2}} \left(1 - \frac{E_o}{E_2 - E_1} \ln \frac{E_2 + E_o}{E_1 + E_o} \right). \end{aligned}$$

Here

$$E_o = \frac{\ell_o}{16\pi\Lambda} \frac{m^* \omega_B^4}{n_e k_B T}.$$

For the values $n_e = 2 \cdot 10^{12} \text{ cm}^{-3}$, $B = 10^3$ Gs, $T = 10$ K calculation gives $E_o = 0.021 \text{ eV}$. As a result $\langle C_B^* \rangle_E \approx 6 \cdot 10^{-3} \text{ cm}^3/\text{s}$, which is greater than the maximum value (29) by about 1.7 times. Consequently, a significant change in the energy parameter E has little effect on the dynamics of changes in the effective capture coefficient. This corresponds to the fact that photoelectrons are thermalized before they are captured, so that $E \propto k_B T$, and thus justifies the procedure for obtaining formula (22), which assumes free variation of the parameter E in the region of classical motion description [4].

Conflict of interest

The author declares that he has no conflict of interest.

References

- [1] E.M. Gershenson, G.N. Goltsman, A.P. Mel'nikov. Pis'ma ZhETF, **14** (5), 281 (1971). (in Russian).
- [2] E.M. Gershenson, Yu.P. Ladyzhinsky, A.P. Mel'nikov. Pis'ma ZhETF, **14** (9), 380 (1971). (in Russian).

- [3] T.T. Muratov. FTP, **53** (12), 1609 (2019) (in Russian).
- [4] B.B. Zelener, B.V. Zelener, E.A. Manykin. Pis'ma ZhETF, **95** (3), 164 (2012). (in Russian).
- [5] V.N. Abakumov, V.I. Perel', I.N. Yassievich. *Bezizluchatel'naya rekombinatsiya v poluprovodnikakh* (SPb, FTI, 1997). (in Russian)
- [6] A.G. Zaluzhny, A.Z. Varisov, V.I. Grafutin, O.V. Ilyukhina, G.G. Myasishcheva, E.P. Prokopiev, S.P. Timoshenkov, Yu.V. Funtikov. Poverknost. Rentgenovskie, sinkhrotronnye i neytronnye issledovaniya **2**, 15 (2008) (in Russian).
- [7] B.M. Ashkinadze, N.R. Tevs. FTP, **19** (6), 1122 (1985) (in Russian).