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Electroacoustic waves in a PT-symmetric piezoelectric structure near the exceptional point

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Received June 24, 2022
Revised June 24, 2022
Accepted November 1, 2022

The spectral properties of gap electroacoustic waves in PT-symmetric piezoelectric structures of symmetry class 6 are investigated theoretically. It was found out that, at a certain level of loss and gain in piezoelectrics, the symmetric and antisymmetric modes intersect. The intersection point defines an exceptional point of the PT-symmetric structure. It was shown that the frequency dependence of the amplitude at the exceptional point has an extremely narrow resonance peak, which opens up the possibility of creating supersensitive sensors based on PT-symmetric physical structures.

Keywords: PT-symmetry, piezoelectrics, electroacoustic waves, gap structure.

DOI: 10.21883/TPL.2022.12.54954.19291

Energy of the collective modes of electroacoustic–waves can be transferred between coupled piezoelectric waveguides [1,2]. Thus, it is possible to affect the propagation of acoustic waves by varying the waveguide intrinsic loss. A particular case, when intrinsic loss in one waveguide is compensated by anti–damping in another one (balanced electroacoustic loss and gain), is just the parity–time–symmetric (PT) system. The idea of the PT-symmetry emerged in 1998 [3]. The PT-symmetry concept attracted a great interest and was developed for various physical systems in optics [4,5], electronics [6], acoustics [7] and magnetism [8,9].

Planar PT-symmetric piezoelectric waveguides have not been studied yet. They may have a form of simpler structures consisting of two (or more) piezoelectric dielectric films obtained from one and the same sample (and, thus, having identical parameters). At present, investigation of dispersive properties of electroacoustic gap waves in such structures ignoring the PT-symmetry are focused on revealing peculiar features associated with accounting for dielectric characteristics of the material of the layer located in the gap free of acoustic contact. Along with this, differences in material parameters and crystallographic symmetries of piezoelectrics were taken into account, influence of the transverse dimension of one of piezoelectrics forming the gap structure was considered, and the effect of electric fields delay was assessed [1,2]. Besides the above-listed aspects, paper [10] discussed, in connection with the requirements of rapidly developing mechatronics [11], the effect of relative longitudinal displacement of piezoelectrics of class 4 mm ($6mm$, ∞m) separated by an extremely thin gap on the behavior of electroacoustic gap waves. This paper considers for the first time the propagation of electroacoustic waves in a PT-symmetric structure with a gap formed by a pair of identical piezoelectrics of class 6 (4, $6mm$, $4mm$, ∞m).

In the geometry of the problem presented in Fig. 1 implies that both crystals belong to the symmetry class 6 and have identical orientations of crystallographic axes 6 perpendicular to the figure plane. In addition, in order to reveal whether this structure can exhibit the PT-symmetry properties [5], we took into account that electroacoustic waves get gained in one crystal and lost in another one. To account for the loss and gain, let us add complex parameter α to wave number k. The longitudinal (in the direction of propagation) wave number to be used in initial equations is

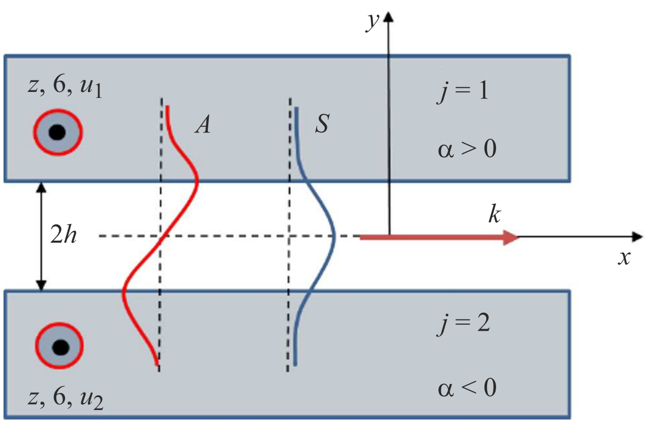

Figure 1. The geometry of the problem. Letters A, S designate the antisymmetric and symmetric modes.

defined as

$$k^{(j)} = k \pm i\alpha, \tag{1}$$

where sign „minus“ is for the upper crystal (wave gain), $j = 1$, while sign „plus“ is for the lower crystal (wave loss), $j = 2$ (Fig. 1) at the given electroacoustic wave dependence on coordinate $x \sim \exp(ik^{(j)}x)$. The fact that representation of the wave number in the complex form (1) leads to the acoustic system PT-symmetry may be easily proved by comparing the structure of the Schrodingerś equation and Helmholtz equation for acoustic waves [5]. Assume that the crystallographic setting of a ferroelectric of class 6 (4, 6*mm*, 4*mm*, ∞m) is such that the sixth-order symmetry axis is parallel to axis z, where z is the axis of the laboratory reference system $x0yz$. Initial equations for shear waves at the given crystal symmetry type may be written as follows [12,13]:

$$\rho\frac{\partial^2 u_z}{\partial t^2} = c_{44}\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2}\right) + e_{1,5}\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right),$$

$$4\pi e_{1,5}\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2}\right) = \varepsilon\left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right), \tag{2}$$

where c_{44} is the element of the crystal elastic modulus tensor. Equation set (2) describes horizontally polarized waves: $\mathbf{u} \parallel z$. In the case of the class 6 crystals, adding of moduli $e_{1,4} = -e_{2,5}$ to the piezo–modulus matrix will not change the piezo–acoustic equations [14]. The presence of these piezo–moduli will manifest itself as introduction of extra terms to the piezo–effect equations [14] and will be finally exhibited as shear stresses and normal electric induction components involved in the boundary conditions. Equations (2) may be represented as follows:

$$\left[\frac{1}{c_{44}^{(j)*}}\frac{\partial^2}{\partial t^2} - \nabla^2\right]u_j = 0, \qquad \nabla^2\Phi_j = 0. \tag{3}$$

In equations (3), $c_{44}^{(j)*} = c_{44}^{(j)} + 4\pi e_{15}^{(j)2}/\varepsilon_j$, $\tilde{\nabla}^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, Φ_j is a part of total potential φ_j in the j-th crystal, which is an electric field induced from domain boundaries by piezo–polarization charges [15].

Let us search for solution of equations (3) in the form of waves propagating along the structure boundaries $y = \pm h$. In view of this, assume that u_j, Φ_j and $\Phi_0 \sim \exp[i(k^{(j)}x - \omega t)]$, where $k^{(j)}$ is the wave number defined by expressions (3) and (1), ω is the circular frequency of the gap electroacoustic wave in the laboratory reference system. Taking into account requirements of limited shear displacements and potentials of crystals electric fields, obtain based on (1) and (3)

$$u_1 = U_1\exp(i\varphi)\exp\big(-s_1(y-h)\big)\exp(\alpha, x),$$

$$\Phi_1 = F_1\exp(i\varphi)\exp\big(-k(y-h)\big)\exp(\alpha x)\exp\big(i\alpha(y-h)\big),$$

$$u_2 = U_2\exp(i\varphi)\exp\big(s_2(y+h)\big)\exp(-\alpha x),$$

$$\Phi_2 = F_2\exp(i\varphi)\exp\big(k(y+h)\big)\exp(-\alpha x)\exp\big(i\alpha(y+h)\big),$$

$$\Phi_0 = \exp(i\varphi)\big[A\exp(-\alpha x)\exp\big(-k(y+h)\big)$$

$$\times\exp\big(-i\alpha(y+h)\big) + B\exp(\alpha x)$$

$$\times\exp\big(k(y-h)\big)\exp\big(-i\alpha(y-h)\big)\big],$$

$$\varphi = kx - \omega t. \tag{4}$$

Parameters $s_{1,2}$ have the meaning of coefficients of decrease in the shear displacement magnitude in the relevant crystal with increasing distance from its boundary.

Assume that material parameters of the media are identical. This is one of the conditions for the media PT-symmetry. Since on non-metallized crystal boundaries $y = \pm h$ there should be met requirements of the continuity of potentials and normal components of the electric induction vectors D_y, as well as of the absence of shear stresses T_{yz}, substitution of relations (4) into the boundary conditions results in obtaining six homogeneous algebraic equations in amplitudes $U_{1,2}$, $F_{1,2}$, A and B. The requirement for solvability expressed as the equality to zero of the determinant of the set of obtained equations with accounting for the $\alpha = \alpha_{coeff}k$ substitution (where constant $\alpha_{coeff} = \alpha/k \ll 1$) provides the desired dispersion relationship for the gap electroacoustic waves in the layered structure of the class 6 piezoelectrics separated by a vacuum gap.

Fig. 2 presents numerical calculations via the dispersion equation at different values of coefficient α_{coeff}. The material whose spectrum is shown in Fig. 2 is lithium iodate ($LiIO_3$) of the symmetry class 6 (non-zero transverse piezo–activity) with parameters $K^2 = 0.25$, $K_\perp^2 = 0.005$,

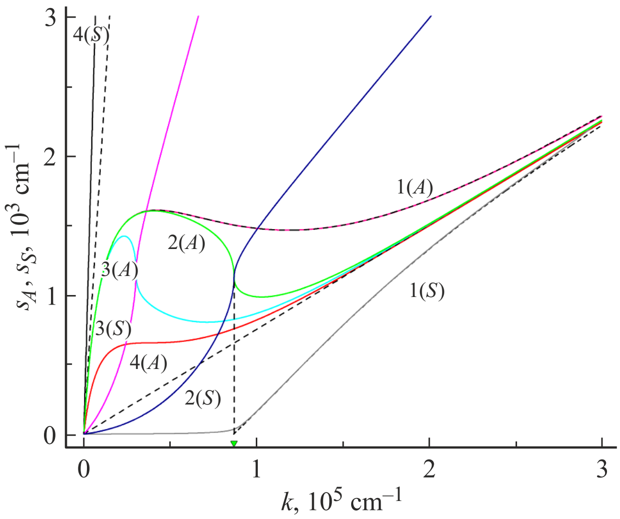

Figure 2. A spectrum of the gap electroacoustic waves (identical piezocrystals of class 6) with accounting for gain and loss. $h = 10^{-6}$ cm, $K^2 = 0.25$, $K_\perp^2 = 0.025$, $\varepsilon = 8$. Digits designate the spectra of the symmetric (S) and antisymmetric (A) modes at different gain and loss levels: $1(S,A) - \alpha_{coeff} = 10^{-6}$, $2(S,A) - \alpha_{coeff} = 9.62 \cdot 10^{-5}$, $3(S,A) - \alpha_{coeff} = 10^{-3}$, $4(S,A) - \alpha_{coeff} = 10^{-1}$. The wave number marked with a triangle is equal to that defining the exceptional point for $\alpha_{coeff} = 9.62 \cdot 10^{-5}$.

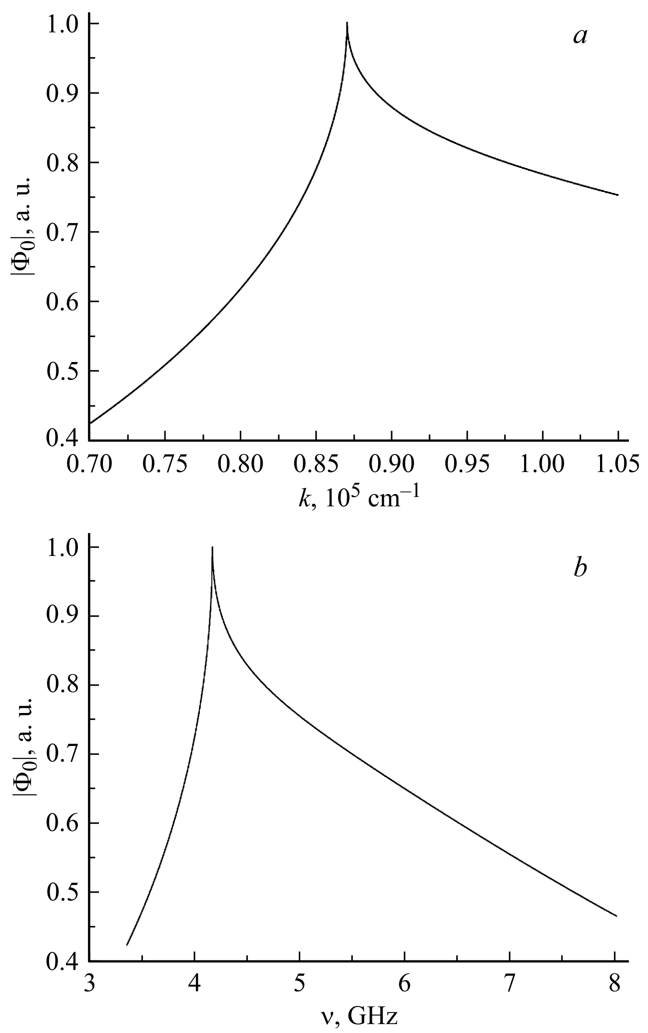

Figure 3. Dependences of amplitude of the symmetric−mode electric potential at $y = 0$ (gap center) on the wave number (a) and frequency (b) at $\alpha_{coeff} = 9.62 \cdot 10^{-5}$.

$\varepsilon = 8$ [16]. Here quantities $K_\perp^2 = 4\pi e_{1,4}^2/(c_{44}^*\varepsilon)$, $K^2 = 4\pi e_{1,5}^2/(c_{44}^*\varepsilon)$ are the squared coefficients of the crystal electromechanical coupling for the transverse and longitudinal piezoelectric effects, respectively. In calculating the spectrum for this material, we used, for the sake of clarity, parameter $K_\perp^2 = 0.025$ that is 5 times higher than the reference one. Generally, this substitution does not strongly change the spectrum shape but allows distinguishing the point of the symmetric mode origin from the zero on the wavenumber axis. The dashed straight lines represent linear spectra of the electroacoustic wave on the metallized ($s = kK^2$) and non-metallized ($s = k(K^2 - \varepsilon K_\perp^2)/(1+\varepsilon)$) boundaries of the piezoelectric crystal [14]. Thin dashed curves represent the spectrum of the gap electroacoustic wave modes in the absence of loss and gain. We can see that accounting for loss and gain in the neighboring piezoelectrics results in the fact that the higher is α_{coeff}, the larger is steepness of the symmetric mode curve. The curves of the symmetric and antisymmetric modes move opposite each other so that they intersect at certain values of α_{coeff} (at $\alpha_{coeff} > 10^{-6}$). When $\alpha_{coeff} > 10^{-3}$, the symmetric mode curve becomes straight and lies higher over the s_1 values of the antisymmetric mode curve. It is possible to assume that, similarly to optical and magnetic systems [14], violation of the purely symmetric (antisymmetric) field distribution over the structure thickness takes place behind the mode intersection point of the structure with a gap. The intersection point itself has been named in literature as an exceptional point which possesses in the PT-symmetric structures a number of interesting properties. An essential characteristic of the exceptional points is that not only eigenvalues but also relevant eigenvectors get degenerated at them [12]. In Hermitian systems, the eigenvalue space topology is of the double-cone type with degeneration points in the cone apexes. On the contrary, the eigenvalue space in non-Hermitian systems has the form of Riemann sheets with the centers near the exceptional points [17]. This unique characteristics make it possible to create supersensitive sensors based on PT-symmetric physical structures [18]. These structures indeed have an extremely narrow resonance curve. Let us demonstrate this by calculations of the amplitude dependence on frequency.

Figs. $3, a, b$ present the dependences of the symmetric−mode electric potential amplitude at $y = 0$ (gap center) on the wave number (a) and frequency (b) at $\alpha_{coeff} = 9.62 \cdot 10^{-5}$. Expectedly, $k = 83\,000\,\text{cm}^{-1}$ ($\nu_R = 4.15\,\text{GHz}$) at the exceptional point. This dependence has the form of a narrow resonance curve. The resonance line width of the PT-symmetric structure is about 0.35 of resonance frequency $\nu_R = 4.15\,\text{GHz}$. As mentioned above, this characteristic feature of exceptional points enables creating supersensitive sensors based on PT-symmetric physical structures [18].

Financial support

The study was supported by the Russian Scientific Foundation (project 19-19-00607).

Conflict of interests

The authors declare that they have no conflict of interests.

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