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Excitation of high cyclotron harmonics in a high-current relativistic gyrotron in the frequency multiplication regime

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Using the averaged equations supplemented with 3D particle-in-cells simulation methods, we have studied the frequency multiplication regime in a high-current relativistic gyrotron. The paper shows that the ratio between the output power of the higher (fifth or sixth) harmonics and that of the fundamental cyclotron resonance may be of about 0.1−0.3%. Accordingly, the nonlinear transformation coefficient is several orders of magnitude higher than the values achievable in gyrotrons with weakly-relativistic electron beams.

Keywords: gyrotron, harmonic excitation, high-current relativistic electron beams.

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At present, the most powerful sources of sub-terahertz radiation are gyrotrons that are capable of generating megawatt-power radiation at frequencies of up to 170 GHz in the mode of fundamental cyclotron resonance [1–3]. However, the necessity of creating sufficiently strong magnetic fields in large volumes is one of the key factors hindering extension of high-power gyrotrons to higher-frequency ranges. Therefore, investigation of radiation generation at cyclotron harmonics attracts interest in view of both increasing the generation frequency at a preset magnetic field and decreasing magnetic field at a preset frequency. Here one of attractive opportunities is utilizing the frequency multiplication effect [4–6]. Thereat, since the electron beam is a nonlinear medium, a low-frequency wave, either external or generated by the beam itself, induces in the beam current harmonics which then produce high-frequency radiation at the initial-wave frequency harmonics.

In the case of weakly-relativistic helical electron beams, the described mechanism possesses a substantial disadvantage that is a low nonlinear transformation coefficient (NTC) defined as a ratio between the high-frequency radiation power and the level of low-frequency generation at the fundamental cyclotron resonance. This is caused, on the one side, by an unequal spacing of the cylindrical waveguide spectrum modes and, on the other side, by a rapid decrease in the coupling coefficients with increasing harmonic number. For instance, presented in [7,8] power measurements for the gyrotron with operating frequency of 263 GHz showed that the NTC value at the second cyclotron harmonic was 10^{-4} (i. e. about 0.01% of the first-harmonic radiation power) while that at the third cyclotron harmonic was about 10^{-6} (0.0001%).

At the same time, it is well known that the extent of reduction of coupling coefficients at the harmonics decreases with increasing electron energy [9]. Actually, it follows from the properties of single-particle cyclotron radiation whose intensity at an s-th harmonic is defined by the Bessel function derivative $J'_s(\chi)$ [10] where argument $\chi = \kappa_\perp V_\perp/\omega_H = 2\pi a/l_\perp$ is the non-dipoleness factor, i. e. the ratio of the electron Larmor radius $a = V_\perp/\omega_H$ to the scale of transverse field non-uniformity $l_\perp = 2\pi/\kappa_\perp$. Here $V_\perp$ is the transverse electron velocity, m_e is the electron rest mass, $\omega_H = eH_0/m_e c\gamma$ is the gyrofrequency, H_0 is the guiding magnetic field, γ is the relativistic mass factor, $\kappa_\perp$ is the transverse wave number. Evidently, when the particle energy and Larmor radius increase, parameter $J'_s(\chi)$ decreases slower with increasing harmonic number s and, accordingly, there increases the efficiency of radiation generation by high-current harmonics arising in the process of azimuthal grouping of electrons. This paper shows that, when a relativistic high-current helical electron beam is used, the generation power even at the fifth and sixth gyrofrequency harmonics may be of tenths of percent of the first-harmonic radiation power.

Let us consider the gyrotron model in the form of a section of a weakly-irregular cylindrical waveguide with radius R_0, within which a helical electron beam excites a few TE modes with numbers $n = 1, 2, 3, \ldots$ and azimuthal and radial indices m_n and q_n, respectively. Assume that each mode interacts with the beam at the s_n-th cyclotron harmonic. Thereat, the radiation frequency at a specified mode is close to both the critical mode frequency in the resonator $\bar{\omega}_n^c$ and $s_n\omega_H$. The electric field of each work-space mode may be represented as $\mathbf{E}_n = \mathrm{Re}\big(A_n(z,t)\mathbf{E}_\perp^n(r)\exp(is_n\omega_H^0 t - im_n\varphi)\big)$, where $A_n(z,t)$ is the slowly varying complex amplitude of the n-th mode, function $\mathbf{E}_\perp^n(r)$ describes the mode radial structure, φ is the azimuthal angle, $\omega_H^0 = eH_0/m_e c\gamma_0$ is the unexcited gyrofrequency (gyrofrequency at the entrance to the interaction space). Using expansion $J'_s(\chi) \approx s\chi^{s-1}/2^s s!$, the electron−wave interaction may be described, with accounting for the velocity dispersion, by the following set of equations (compare with [11]):

$$i\frac{\partial^2 a_n}{\partial Z^2}+s_n\frac{\partial a_n}{\partial \tau}+\big(i\Delta_n+i\delta_n(Z)+\sigma_n\big)a_n$$

$$=i\frac{I_n}{4\pi^2}\frac{\int\limits_0^{2\pi}e^{i(m_n-s_n)\varphi}\int\alpha(p_0)\langle p^s\rangle_{\theta_0}dpd\varphi}{\int\beta_{\|0}\alpha(p_0)/\bar{\beta}_{\|0}dp},$$

$$\frac{\bar{\beta}_{\|0}}{\beta_{\|0}}\frac{\partial p}{\partial Z}+\frac{\bar{g}_0^2}{4}\frac{\partial p}{\partial \tau}+ip\big(|p|^2-|p_0|^2\big)$$

$$=i\sum_n a_n\big(p^*\big)^{s_n-1}e^{-i(m_n-s_n)\varphi}, \tag{1}$$

where

$$a_n=\frac{eA_nJ_{m_n-s_n}\big(\nu_{m_n,q_n}R_b/R_0\big)}{mc\omega_H^0}\frac{s_n^{s_n}}{2^{s_n-1}s_n!}\frac{\bar{\beta}_{\perp0}^{s_n-4}}{\gamma_0},$$

$$Z=\frac{\bar{\beta}_{\perp0}^2}{2\beta_{\|0}}\frac{\omega_H^0}{c}z,\quad \tau=\frac{\bar{\beta}_{\perp0}^4}{8\beta_{\|0}^2}\omega_H^0 t, \tag{2}$$

$\bar{\beta}_{\perp0}=\bar{V}_{\perp0}/c$ and $\bar{\beta}_{\|0}=\bar{V}_{\|0}/c$ are the mean values of normalized transverse and longitudinal electron velocities at the entrance to the interaction area, $g=\bar{\beta}_{\perp0}/\bar{\beta}_{\|0}$ is the relevant pitch-factor, p is the complex transverse momentum normalized to the mean initial value, $\beta_{\|0}/\bar{\beta}_{\|0}=\sqrt{\bar{g}_0^2+1-\bar{g}_0^2|p_0|^2}$,

$$\Delta_n=\frac{8\bar{\beta}_{\|0}^2s_n^2}{\beta_{\perp0}^4}\frac{s_n\omega_H^0-\bar{\omega}_n^c}{\bar{\omega}_n^c},\quad \delta_n(Z)=\frac{8\bar{\beta}_{\|0}^2s_n^2}{\beta_{\perp0}^4}\frac{\bar{\omega}_n^c-\omega_n^c(Z)}{\bar{\omega}_n^c} \tag{3}$$

are the cyclotron and geometric (describing the resonator profile $R(z)$) mismatches for the n-th mode, $\omega_n^c(Z)=\nu_{m_n,q_n}c/R(z)$ is the function defining the dependence of the critical n-th mode frequency on the longitudinal coordinate,

$$G_n=64\frac{eI_b}{m_ec^3}\frac{\bar{\beta}_{\|0}\bar{\beta}_{\perp0}^{2(s_n-4)}}{\gamma_0}s_n^3\left(\frac{s_n^{s_n}}{2^{s_n}s_n!}\right)^2$$

$$\times\frac{J_{m_n-s_n}^2(\nu_{m_n,q_n}R_b/R_0)}{(\nu_n^2-m_n^2)J_{m_n}^2(\nu_{m_n,q_n})} \tag{4}$$

is the excitation parameter for a beam with injection radius R_b and current I_b, ν_{m_n,q_n} is the q_n-th root of equation $J'_{m_n}(\nu)=0$, $\sigma_n=4\bar{\beta}_{\|0}^2\bar{\beta}_{\perp0}^{-4}Q_n^{-1}s_n^2$ is the absorption factor, Q_n is the ohmic $Q-$factor of the respective mode. Function $\alpha(p_0)$ that is assumed to be Gaussian describes the initial transversevelocity dispersion of electrons. In the used normalizations, radiation power of each mode in the output cross-section $Z=L$ is calculated as $P_n[\mathrm{kW}]=511.765I[\mathrm{A}]\big(\gamma_0\beta_{\perp0}^2/G_n\big)\mathrm{Im}\big(a_n\partial a_n^*/\partial Z\big)\big|_{Z=L}$.

Based on equations (1), let us estimate the excitation level of high cyclotron harmonics for the described in [12] high-current gyrotron with a helical electron beam having the particle energy of 500 keV, current of 2 kA, pitch-factor $g=1$, and initial transverse−velocity dispersion of about 20%. In the given gyrotron, mode $\mathrm{TE}_{-3,2}$ with the operating frequency of 30 GHz gets excited at the fundamental cyclotron resonance; let us assign the mode number to 1 so that $s_1=1$, $m_1=-3$, $q_1=2$. In view of achieving the frequency range characteristic of the up-to-date megawatt gyrotrons, it is interesting to consider the generation of harmonics with $s_n\geqslant 5$. Efficient frequency multiplication takes place when there are fulfilled relation $m_n=s_nm_1$ and asymptotic condition of the critical frequency multiplicity $\bar{\omega}_n^c\approx s_n\bar{\omega}_1^c$ or, what is the same, $\nu_{m_n,q_n}\approx s_n\nu_{m_1,q_1}$, [4,5]. Analysis of the cylindrical waveguide mode spectrum shows that the most efficient multiplication at the fifth harmonic occurs to mode $\mathrm{TE}_{-15,7}$ ($s_2=5$, $m_2=-15$, $q_2=7$) in which the deviation from the frequency multiplicity condition is about 0.7%. At the sixth harmonic, the minimum deviation of $\sim 1\%$ is obtained for mode $\mathrm{TE}_{-18,8}$ ($s_3=6$, $m_3=-18$, $q_3=8$).

Fig. 1 demonstrates the magnetic field dependence of the radiation power calculated via equations (1). The maximum radiation power at the fundamental cyclotron resonance P_1 is about 200 MW at the magnetic field of 1.69 T, and gradually decreases with increasing magnetic field. The generation range is limited by the value of 1.92 T at which parasitic mode $\mathrm{TE}_{-4,2}$ is excited. In its turn, the radiation power at harmonics P_5 and P_6 gradually increases with increasing magnetic field and reaches the maximum value of ~ 1 MW (NTC is $1.5\cdot10^{-3}$) and ~ 0.3 MW (NTC is $1.5\cdot10^{-3}$) for $s_3=6$ near the parasitic mode excitation boundary. At the harmonic with number $s_4=7$, the radiation power in case of the mode $\mathrm{TE}_{-21,10}$ excitation does not exceed several tens of kilowatts.

For the sake of more comprehensive analysis of generation characteristics at high cyclotron harmonics, the gyrotron calculations were also performed based on the 3D PIC simulation by the particle-in-cells method using the CST Particle Studio code. Fig. 2, *a* illustrates the interaction space geometry, instantaneous arrangement of macroparticles, and their energy distribution. Fig. 2, *b* presents the magnetic field dependences of the radiation power at the fundamental operating mode $\mathrm{TE}_{-3,2}$ and modes $\mathrm{TE}_{-15,7}$, $\mathrm{TE}_{-18,8}$, to which generation at the fifth and sixth harmonics corresponds. The maximum $\mathrm{TE}_{-3,2}$-mode power gets reached at the magnetic field of 1.68 T and equals approximately 230 MW. At the magnetic field above 1.85 T, excitation of parasitic mode $\mathrm{TE}_{-4,2}$ takes place. As the magnetic field increases, the generation power at the gyrofrequency harmonics grows to 0.3 MW at the fifth harmonic and to 0.1 MW at the sixth one. The maximum power at the 150−GHz fifth harmonic amounts up to 0.3% of the first−harmonic generation power, while that at the 180−GHz sixth harmonic is about 0.1% (Fig. 3).

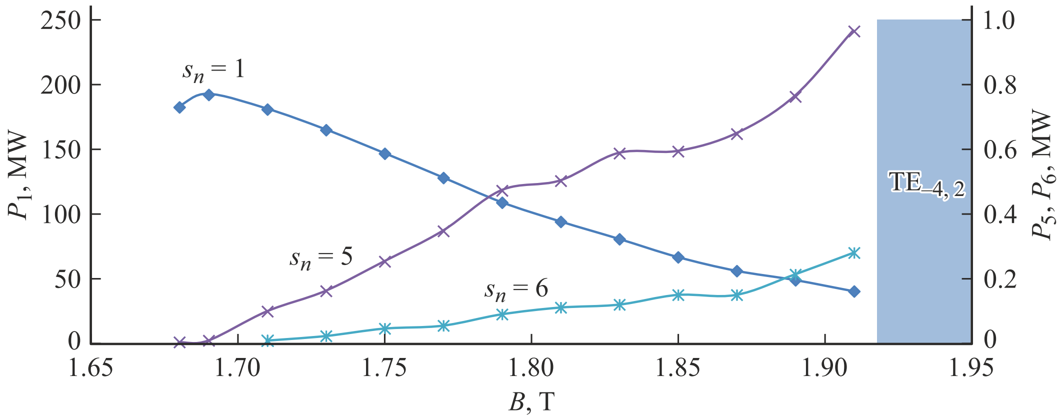

Figure 1. Simulation based on averaged equations. Magnetic field dependences of the generation power at the fundamental harmonic $P_1(s_n = 1)$ and harmonics P_5 and P_6 with numbers $s_n = 5$, 6. The shaded rectangular represents the excitation area of parasitic mode $TE_{-4,2}$.

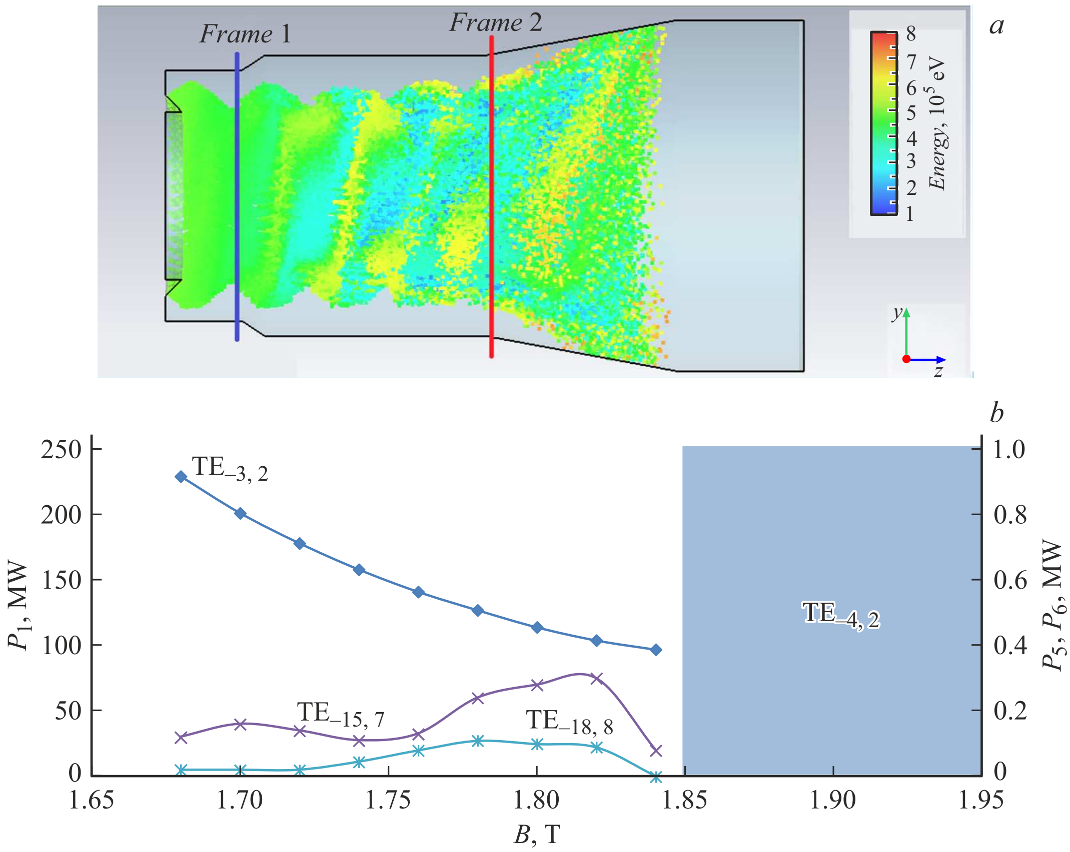

Figure 2. Results of the 3D PIC simulation. *a* — the gyrotron cavity geometry and instantaneous macroparticle arrangement; the energy distribution of particles is illustrated in colors (the colored figure is given in the electron version of the paper); *b* is the magnetic field dependence of the generation power at the first (mode $TE_{-3,2}$), fifth (mode $TE_{-15,7}$) and sixth (mode $TE_{-18,8}$) cyclotron harmonics. The shaded rectangular represents the excitation area of parasitic mode $TE_{-4,2}$.

Thus, the simulation results demonstrate the possibility of achieving in relativistic gyrotrons a sub-megawatt power of radiation in the 150–180 GHz frequency range at the generation at the fifth and sixth cyclotron harmonics with a multiple decrease in the magnetic field. Notice that at present relativistic gyrotrons with the output power of about 80 MW in the range of 300 GHz are being developed [13]. In the regime of frequency multiplication, such gyrotrons enable counting on obtaining radiation power of hundreds of kilowatts at frequencies > 1.5 THz. Such

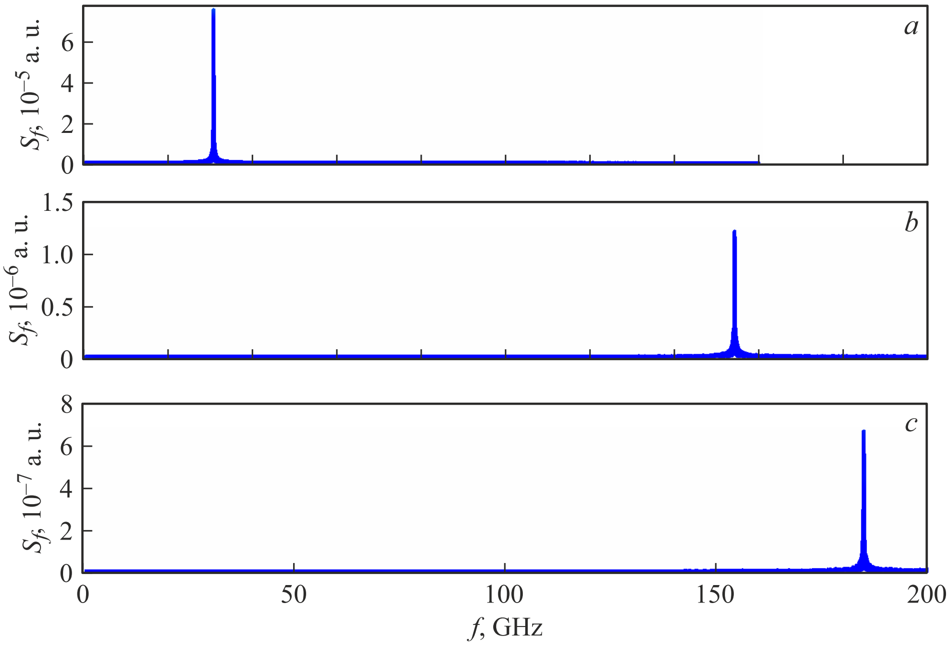

Figure 3. Results of the 3D PIC simulation. The spectrum of the gyrotron output radiation at modes $TE_{-3,2}$ (*a*), $TE_{-15,7}$ (*b*) and $TE_{-18,8}$ (*c*).

a power may be provided in the specified frequency range only by free-electron lasers that are rather massive and high-cost facilities.

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Conflict of interests

The authors declare that they have no conflict of interests.

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