

## Peculiarities of Polarization Waves Behavior under Excitation of an Extended Resonant Medium by Overlapping Extremely Short Light Pulses

© M.V. Arkhipov<sup>1</sup>, R.M. Arkhipov<sup>1,2</sup>, N.N. Rosanov<sup>1,2</sup>

<sup>1</sup> St. Petersburg State University,  
199034 St. Petersburg, Russia

<sup>2</sup> Ioffe Physico-Technical Institute RAS,  
194021 St. Petersburg, Russia

E-mail: m.arkhipov@spbu.ru, arkhipovrostislav@gmail.com, nnrosanov@mail.ru

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With the coherent interaction of a sequence of extremely short light pulses with a resonant medium (when the duration of the pulses and the delays between them are shorter than the relaxation times of the population difference  $T_1$  and polarization  $T_2$  of the medium), it is possible to create spatial electromagnetically induced gratings (EMIG) of polarization and population difference in the medium, as well as their ultrafast control. The dynamics of such gratings is of interest in connection with the possibility of ultrafast control over the properties of a medium at times of the order of the pulse duration and their influence on the stability of generation of extremely short pulses in a laser with a short linear cavity. In this paper, based on the numerical solution of the system of Maxwell-Bloch equations, an unusual and interesting possibility of the appearance of narrow and alternating sections in the medium is shown and studied in detail, in which harmonic polarization waves of the medium are formed, propagating in opposite directions relative to neighboring sections. This situation occurs when two extremely short attosecond pulses collide in a two-level resonant medium.

**Keywords:** attosecond pulses, electromagnetically induced gratings, polarization waves.

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### Introduction

Obtaining ultrashort electromagnetic pulses of femto- and attosecond duration in recent decades is one of the central topics of modern optics [1–4]. Subfemtosecond pulses are actively used to study and control the dynamics of wave packets in matter [5–8]. These studies are of fundamental interest, as they allow answering questions such as the dynamics of electron motion in atoms, molecules and their ionization on such small time scales of the order of tens to hundreds of attoseconds [9–11]. But the ultrashort femto- and attosecond pulses obtained in practice contain several oscillation cycles [1–4]. The ultimate possibility of shortening the duration of light pulses is the generation of single-cycle pulses containing two half-waves of the field of opposite polarity [12,13]. If one of the half-waves is cut off, then a unipolar half-cycle pulse is obtained, containing one field half-wave and having a non-zero electric area. For such already extremely short pulses a significant difference arises in the nature of the interaction with quantum systems, in contrast to ordinary long multicycle pulses. An important role in the interaction is played by the electric pulse area, if its duration is shorter than the orbital period of an electron in an atom [12–14].

The interest in the extremely short (single-cycle and half-cycle pulses) is due to the fact that they are able to

more quickly control the state of the medium compared to conventional high-cycle pulses, which is important in ultrafast optics. Nevertheless, the practical production of single-cycle and unipolar half-cycle pulses is a difficult experimental problem. Optical one-cycle and half-cycle attosecond pulses are experimentally obtained by Fourier synthesis of broadband pumping [6]. There are a number of theoretical proposals for obtaining half-period optical attosecond pulses by decelerating relativistic electrons in a thin target [15,16]. A cascade scheme for obtaining half-period attosecond pulses with a duration less than 10 as was studied in [17]. Unipolar pulses were obtained experimentally in the THz frequency range [18,19].

Due to the difficulty of experimentally obtaining extremely short (ESP) single-cycle and half-cycle pulses, their coherent propagation in resonant media (when the pulse duration is shorter than the relaxation times of the population difference  $T_1$  and polarization  $T_2$  of the medium) has been poorly studied, to date. The results of numerous theoretical studies show that the dynamics of coherent propagation of low-cycle pulses in resonant media differs significantly from the dynamics of long multi-cycle pulses [20–23]. The coherent interaction of ESP with resonant media leads to the appearance of a number of new and unusual phenomena during the propagation of such pulses in resonant media. In particular, this is the possibility of integrating and

differentiating the temporal form of the pulse field strength, and not their envelope [24]. The possibility of compression of single-cycle bipolar pulses in a two-level medium due to the attraction of their unipolar components propagating coherently in the self-induced transparency mode (SIT) in a resonant medium was shown in [25]. It should also be noted that it is possible to obtain half-cycle attosecond pulses due to the repulsion of these unipolar SIT solitons in a similar problem [26], the phenomenon of self-stopping of a single-cycle pulse in a homogeneous medium [27], the possibility of obtaining single-cycle ESPs in a two-section laser with an ultrashort resonator due to the SIT phenomenon [28] and others.

It is known that in the case of coherent interaction of such pulses with a medium, an ultrafast change in the populations of atomic levels is possible due to Rabi oscillations [20]. This leads to the possibility of guidance and ultrafast control of electromagnetically induced gratings (EMIGs) of atomic populations and polarization of the medium using extremely short pulses [29–33]. This situation is possible when pulses do not simultaneously overlap in the medium [29,30,33] or meet inside the medium [31,32]. Previously, the possibility of inducing an EMIG using long multicycle pulses that do not overlap in a medium was studied [34–36]. In the traditional approach, however, EMIGs are created by the interference of two or more long quasi-monochromatic laser radiation beams in a medium [37]. EMIGs created in this way find numerous applications in optics. An overview of the results in this area is given in [36,38] and the cited literature.

In the case of coherent interaction of extremely short pulses with the medium, a rich dynamics of induced gratings of the population difference and polarization difference is possible — the appearance of slow polarization waves, multiplication of the spatial period of the gratings, their erasure, etc. [29,30,38]. The dynamics of polarization and EMIG waves induced by long multicycle laser pulses that do not overlap in a medium was studied in the slow envelope approximation in [39]. However, the scenarios for the formation of the EMIG considered in this work differ from the scenarios analyzed in the studies [29–33,38], in which the possibility of creating and controlling an EMIG using ESPs was studied of used ESPs. The dynamics of polarization waves during the formation of a photon echo in a medium by a sequence of ESPs was theoretically considered in the studies [40–45].

It was noted earlier that when a pair of more unipolar and bipolar extremely short pulses collide in a two-level medium, an unusual situation is possible when sections appear in the medium in which polarization waves (coherence of the medium) are formed with different spatial frequencies or traveling in opposite directions [31,32,38]. In this study, based on the numerical solution of the system of Maxwell-Bloch equations for a two-level medium, an unusual situation is studied in more detail, which consists in the formation of pronounced narrow and successively located sections in the medium, in which polarization waves

arise, running in opposite directions. This situation is most clearly manifested in the coherent interaction of two single-cycle attosecond pulses with a two-level resonant medium, when the pulses simultaneously overlap in the medium. Previously, in the case when pulses do not overlap in the medium, such situations were not found [29,30]. The formation of these polarization gratings is associated with the coherent control of the polarization dynamics of a medium using a sequence of extremely short pulses in a nonlinear resonant medium.

The relevance of the ongoing researches is related to the issue of stability of ESP generation due to SIT with coherent mode locking in a laser with a short linear resonator [28] and the possibility of emitting these gratings, which affects the dynamics of ESP propagation in resonant media.

## Theoretical model and results of numerical simulation

Numerical calculations are based on the well-known system of Maxwell-Bloch equations, which describes the evolution of the off-diagonal element of the density matrix  $\rho_{12}$ , the population difference (inversion)  $n = \rho_{11} - \rho_{22}$ , its polarization  $P$  and electric field strength  $E$  [46]

$$\frac{\partial \rho_{12}(z, t)}{\partial t} = -\frac{\rho_{12}(z, t)}{T_2} + i\omega_0 \rho_{12}(z, t) - \frac{i}{\hbar} d_{12} E(z, t) n(z, t), \quad (1)$$

$$\frac{\partial n(z, t)}{\partial t} = -\frac{n(z, t) - n_0(z)}{T_1} + \frac{4}{\hbar} d_{12} E(z, t) \text{Im} \rho_{12}(z, t), \quad (2)$$

$$P(z, t) = 2N_0 d_{12} \text{Re} \rho_{12}(z, t), \quad (3)$$

$$\frac{\partial^2 E(z, t)}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2 E(z, t)}{\partial t^2} = \frac{4\pi}{c^2} \frac{\partial^2 P(z, t)}{\partial t^2}. \quad (4)$$

In the (1)–(4) system:  $t$  — time,  $z$  — longitudinal coordinate,  $N_0$  — concentration of active centers,  $c$  — speed of light in vacuum,  $\hbar$  — reduced Planck constant,  $\omega_0$  — resonant transition frequency of medium ( $\lambda_0 = 2\pi c/\omega_0$  — wavelength of the resonant transition),  $d_{12}$  — dipole moment of the working transition,  $n_0$  — population difference of two working levels in the absence of electric fields ( $n_0 = 1$  for an absorbing medium). This system of equations does not use the approximation of slowly varying amplitudes and a rotating wave. The applicability of the two-level approximation in such problems, despite the short duration of the pulses, can be substantiated, see remarks in the studies [25,27]. Moreover, the results of theoretical studies show that EMIG guidance in multilevel medium is also possible [30,33,47]. Therefore, we can restrict ourselves to a two-level approximation.

On medium, from left to right, at the initial moment of time, a single-cycle pulse was launched from vacuum in the form

$$E(0, t) = E_{01} e^{-\frac{(t-\tau_1)^2}{\tau^2}} \sin[\omega_0(t - \tau_1)]. \quad (5)$$

Parameters used in the numerical calculation

Resonance transition wavelength	$\lambda_0 = 700 \text{ nm}$
Dipole transition moment	$d_{12} = 20 \text{ Debye}$
Inversion relaxation time	$T_1 = 1 \text{ ns}$
Polarization relaxation time	$T_2 = 1 \text{ ps}$
Atom concentration	$N_0 = 5 \cdot 10^{14} \text{ cm}^{-3}$
Filed amplitude 1	$E_{01} = 9.55 \cdot 10^4 \text{ ESU}$
Pulse duration 1,2	$\tau = 388.88 \text{ as}$
Filed amplitude 2	$E_{02} = 2E_{01}$
Delay parameter	$\tau_1 = \tau_2 = 2.5\tau$

And from right to left, an impulse began to propagate with an amplitude slightly different from its value for the first impulse

$$E(L, t) = E_{02}e^{-\frac{(t-\tau_2)}{\tau^2}} \sin[\omega_0(t - \tau_2)]. \quad (6)$$

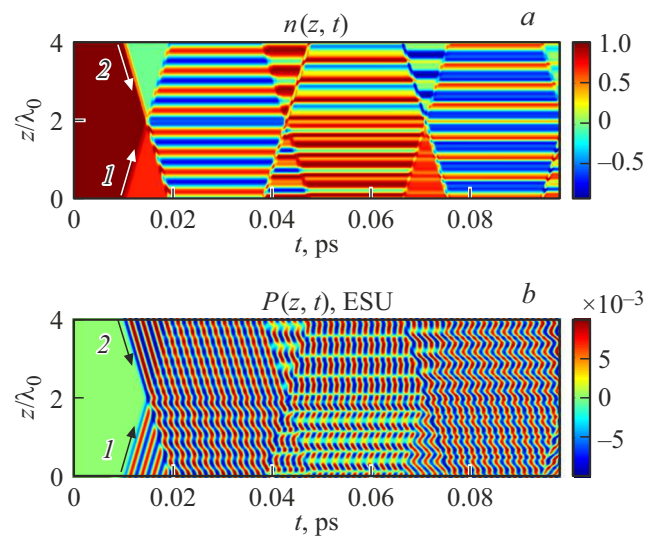
Here  $\tau_{1,2}$  — delays.

The numerical solution of the system of equations (1)–(4) was carried out with the parameters when the effect under study was manifested most clearly (table). The spatial region of integration had a length of about twelve wavelengths of the resonant transition of the medium,  $L = 12\lambda_0$ . And the resonant medium itself was located along the  $z$  axis in the center of the region between the points  $z_1 = 4\lambda_0$  and  $z_2 = 8\lambda_0$ . To create a sequence of exciting attosecond pulses in numerical calculations, zero boundary conditions were set for the field values at the ends of the integration region. The calculation parameters are given in the table.

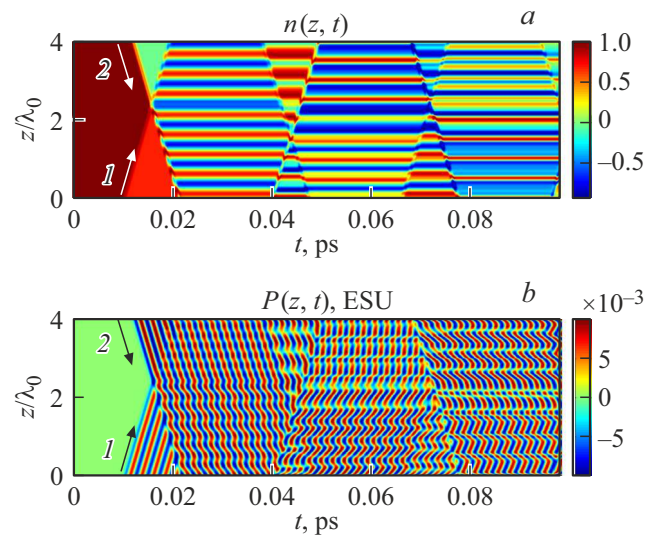
The field amplitude 1 was chosen in such a way that the first pulse acted approximately as a  $\pi/4$  pulse, and the second as a  $\pi/2$  pulse, i.e. left behind the medium in a state with zero inversion  $n = 0$ . The results of the numerical calculation for the given parameters are shown in Fig. 1. Fig. 1, *a* illustrates the dynamics of the population difference  $n(z, t)$ , Fig. 1, *b* illustrates the dynamics of the  $P(z, t)$  polarization. The arrows and numbers indicate the direction of propagation of impulses 1 and 2.

Single-cycle attosecond pulses 1 and 2 in the form (5) and (6) enter the medium and induce polarization traveling waves in it (vertical oblique lines in Fig. 1, *b* in the time interval 0.01–0.02 ps). Then, having collided in the center of the medium at the point  $z_0 = 2\lambda_0$ , the pulses begin to interact with these polarization waves running towards them. An EMIG of the population difference arises in the medium (Fig. 1, *a*). But as a result of coherent control of the polarization (coherence) oscillations of the medium, the induced early harmonic polarization waves change their shape. This is most clearly seen in the area to the left of the dot  $z_0$ . Clearly visible zigzag breaks appear in it (Fig. 1, *b*) in the time interval 0.02–0.04 ps. These are the alternating sections of interest to us, in which polarization waves arise, running in opposite directions.

After the pulses reach the boundary of the integration region, they return to the medium and again control the



**Figure 1.** (a) Dynamics of the population difference  $n(z, t)$ , (b) dynamics of the polarization  $P(z, t)$  under the action of single-cycle attosecond pulses 1 and 2 in the form (5) and (6) colliding in the center of the medium at the point  $z/\lambda_0 = 2$ . The calculation parameters are given in the table.



**Figure 2.** (a) Dynamics of the population difference  $n(z, t)$ , (b) dynamics of the polarization  $P(z, t)$  under the action of single-cycle attosecond pulses 1 and 2 in the form (5) and (6) colliding in the center of the medium at the point  $z_0 = 2.5\lambda_0$ . Parameter  $\tau_2 = 7.5\tau$ . Other calculation parameters are given in the table.

polarization and inversion gratings induced at previous times. As a result of such coherent control, characteristic small zones again arise in the medium with polarization waves that run in opposite directions. They again appear in the form of zigzag structures already in the entire medium, starting from the time 0.07 ps in Fig. 1, *b* and further. When unipolar pulses collide, the effect is weakly pronounced.

The behavior of these structures is affected by the amplitude of the incident pulses and the delay between them  $\tau_{1,2}$ , which indicates the coherent control of these gratings. Example of the dynamics of polarization and inversion gratings, when  $\tau_2 = 7.5\tau$ , is shown in Fig. 2. Other parameters are the same as in Fig. 1. Change in delay  $\tau_2$  leads to a shift in the pulse overlap region. With these parameters the pulses occur  $z_0 = 2.5\lambda_0$ . It can be seen that with a change in the data delay, these polarization structures clearly manifest themselves in all regions of the medium over the entire time interval after 0.02 ps (Fig. 2, *b*).

## Conclusion

Based on the numerical solution of the system of the Maxwell-Bloch equation, an unusual situation is shown and studied in this study, which consists in the appearance of alternating small sections in the medium, in which the waves of polarization (coherence) of the medium propagating in opposite directions are formed. This phenomenon occurs during the coherent propagation of a pair of one-cycle attosecond pulses in a two-level resonant medium, when the pulses overlap in the medium.

These gratings of polarization (coherence) of the medium arise due to the coherent control of oscillations of the polarization of the medium by exciting pulses. The predicted effect is of interest in the physics of coherent interaction of extremely short pulses with resonant media and in ultrafast optics, since it demonstrates the possibility of ultrafast control and switching of the state of the medium at ultrasmall times of the order of the pulse duration. This, in particular, makes it possible to control the radiation of the medium by changing the direction of motion of the polarization wave, making it counter to the pump wave.

Also, the radiation dynamics of the polarization gratings and the population difference must be taken into account in the problems of obtaining ESPs in lasers with a linear resonator due to the SIT phenomenon, since EMIGs affect the stability of the generation mode [28].

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## Conflict of interest

The authors declare that they have no conflict of interest.

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