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The magnetic field force calculation in magnetic systems with stationary current

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Three different approaches to calculating the forces acting on ferromagnets in a stationary magnetic field are considered. The results of calculating of the magnetic field forces by these methods on a test model are compared with the measurement data obtained using an experimental unit.

Keywords: numerical simulation, magnetostatics, calculation of magnetic force, Maxwell tensor.

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The calculation of magnetic field forces is required in the design and fabrication of large electromagnets (specifically, those used in particle detectors). The mass of such magnets is as high as several tens (or even hundreds) of tons. Magnetic forces in them reach comparable values, thus inducing significant additional stress and deformation of structural elements of magnets. Knowing the magnitudes and directions of magnetic forces, one may design unstressed magnet structures within specified tolerances.

The force acting on ferromagnets in a magnetic field may be calculated in various ways. Two calculation methods were compared in [1]. The first of them is related to the virtual work principle. Within this principle, the force is regarded as a derivative of the total magnetic field energy under an infinitesimal displacement of an object to which this force is applied. The second method of calculation of the force acting on a ferromagnetic object (Maxwell stress tensor method) relies on integration of the Maxwell tensor over an arbitrary surface that encloses this object and does not transect ferromagnets. The efficiency and accuracy of methods in numerical modeling of a magnetic system for calculation of both local and global magnetic field forces was compared.

The feasibility of application of the Maxwell tensor in calculation of the magnetic field force in the hybrid finite element-boundary integral formulation was demonstrated in [2]. The methods of force calculation via integration over a surface enclosing a volume to which this force is applied were discussed in [3]. The key theoretical formulae and their practical application were examined. Magnetic force calculations with the Maxwell tensor method in the finite element implementation as applied to magnetostatics were considered in [4]. The details of application of this method to different types of magnets were covered in [5]. The calculation error of the considered methods is the key issue even in simple examples with a nonlinear dependence of permeability on the field in iron.

The calculation of forces with the use of the Maxwell tensor is performed in the following way. Magnetic forces acting on volume V may be reduced to an equivalent system of surface tensions that is represented by the Maxwell tension tensor [2–6]. Components of this tensor for surface S, which encloses volume V and does not transect ferromagnets, may be written as

$$\mathbf{T}=\begin{pmatrix} \mu_0(H_x^2-\mathbf{H}\cdot\mathbf{H}/2) & \mu_0 H_x H_y & \mu_0 H_x H_z \\ \mu_0 H_x H_y & \mu_0(H_y^2-\mathbf{H}\cdot\mathbf{H}/2) & \mu_0 H_y H_z \\ \mu_0 H_x H_z & \mu_0 H_y H_z & \mu_0(H_z^2-\mathbf{H}\cdot\mathbf{H}/2) \end{pmatrix}, \quad (1)$$

where $\mathbf{H}$ is the magnetic field vector and μ_0 is the permeability of vacuum.

The components of resulting force vector $\mathbf{F}$ are then expressed in terms of components of tension tensor $\mathbf{T}$ in the following way:

$$\mathbf{F} = \oint_S \mathbf{T}\cdot\mathbf{n}dS, \quad (2)$$

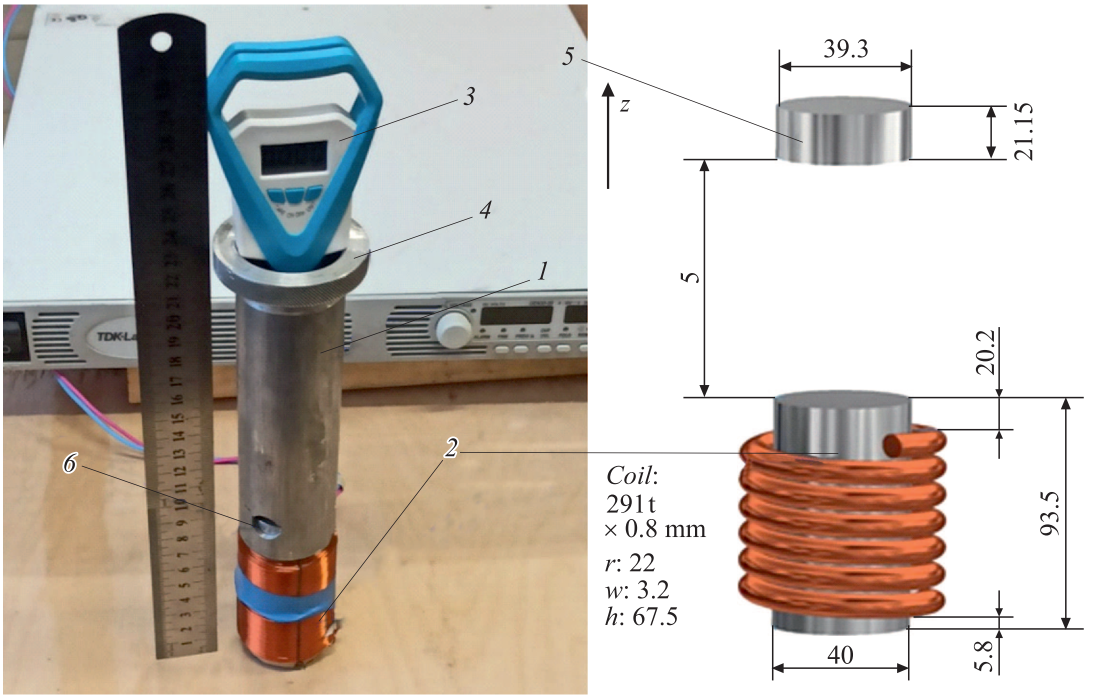
where $\mathbf{n}$ is an outward normal to surface S.

The force value is theoretically independent of the choice of surface S, but this choice in actual practice is affected significantly by the possible presence of subdomains of the calculation domain where an error of magnetic field calculation, which is noticeable in integration, is present (see [7,8]).

The calculation of the magnetic force acting on a ferromagnetic object in terms of displacements may be performed in accordance with the formula

$$F_r = -\lim_{t\to 0}\frac{W_r - W}{t}, \quad (3)$$

where $\mathbf{r}$ is the displacement direction, W is the total energy in the initial position, and W_r is the total energy in the case



Experimental setup for calculation of the magnetic field force. *1* — Stand, *2* — coil with a cylindrical core, *3* — spring balance, *4* — upper part of the stand with adjustable height, *5* — upper cylinder (attached to the spring balance and positioned inside the stand), and *6* — lateral opening.

of object displacement in direction **r** over distance t. The magnetic field energy is then calculated as

$$W = 1/2 \int_V \mathbf{H} \cdot \mathbf{B}\, dV, \tag{4}$$

where V is the entire calculation domain. Note that the considered method provides an opportunity to calculate only one component of the force vector per a single displacement.

The third method of force calculation is volume integration of the specific force. The following formula may be used in this case:

$$\mathbf{F} = \mu_0 \int_\Omega \mathbf{M} \cdot \nabla(\mathbf{H} + \mathbf{M}) d\Omega, \tag{5}$$

where **M** is the magnetization, **H** is the magnetic field intensity, and Ω is a region enclosing the body for which the force is calculated. Since magnetization **M** is a discontinuous function at the interface of two media, the calculation of integral (5) evolves into calculation of a sum of integral (5) over volume $\Omega\backslash\Gamma$ (Γ is the surface of **M** discontinuity) and an additional integral over surface Γ, the form of which depends on the specifics of representation of magnetization in the used model of the magnetic field. For example, in the case when magnetization is expressed in terms of relative permeability $\hat{\mu}$ as $\mathbf{M} = (\hat{\mu} - 1)\mathbf{H}$, this integral takes the form

$$\frac{\mu_0}{2} \int_\Gamma (\hat{\mu}_2 - \hat{\mu}_1)(\hat{\mu}_2 + \hat{\mu}_1 - 2)(\mathbf{H} \times \mathbf{n})^2 \mathbf{n} d\Gamma, \tag{6}$$

where **n** is a normal to surface Γ and $\hat{\mu}_1$ and $\hat{\mu}_2$ are relative permeabilities at the negative and positive sides of surface Γ (in accordance with the direction of normal **n**). Note that this force calculation technique is technically more complex for finite-element modeling, since it requires calculating the magnetization gradient in integral (5).

The experimental setup assembled for measuring the attractive force produced by a magnetic field is shown in the figure. This device has two key physical parts joined together by a special stand in the form of a hollow nonmagnetic metal tube. The lower part is a coil with a cylindrical Armco steel core. The upper part features a spring balance, which is mounted on the stand, and a second metallic cylinder that is attached to the balance and positioned inside the stand. The upper cylinder is also made from Armco steel. A 5 mm gap between the cylinders was set with the use of thin plates through a lateral opening in the stand.

The magnetic force acting on the upper cylinder was measured for different values of current in the coil.

The TELMA software suite, which is being developed at the Novosibirsk State Technical University under the supervision of one of the authors of the present study, was used for numerical modeling. This suite provides an opportunity to use a hybrid finite-boundary element

Calculation results and measurement data (the air gap is 5 mm)

I, A	F_d, kg	F_e, N	F_c, N		
			Maxwell tensor	Energy derivative	Integration
6	0.15	1.470	1.542	1.542	1.529
7	0.21	2.058	2.099	2.099	2.083
8	0.28	2.744	2.742	2.742	2.721
10	0.45	4.410	4.284	4.288	4.254
12	0.63	6.174	6.170	6.178	6.128

formulation in solving magnetostastics problems [9–11]. In addition, the approach utilizing total and partial scalar potentials [12,13] is applied. This makes it possible to calculate the magnetic field of a coil with current separate from ferromagnets [14].

Note that the upper cylinder was shifted only along axis z in implementing the above-described method for force calculation via the energy derivative, since only one force component was calculated.

The problem of force calculation via the Maxwell tensor comes down in this case to integrating over a surface enclosing the upper cylinder. A force of the same magnitude may be obtained by integrating over a surface enclosing the lower cylinder with the coil. In the software implementation, the integral was calculated with the use of Gaussian quadratures over the surface of an enclosing cylinder on which nested meshes were plotted to achieve the needed accuracy of integration.

The values of force acting on the upper cylinder at different currents in the coil are listed in the table. The spring balance readings in their original form (in kilograms) are presented in column F_d. The experimental data converted to newtons (column F_e) can be compared to the results of computer calculations F_c performed using the three methods considered above. The strength of current in the coil was restricted to 6–12 A in the experiment, since the used spring balance had a limited accuracy (a scale value of 0.01 kg).

It follows from the table that the difference between calculated and measured values does not exceed 5% within the entire range of currents for all three calculation methods. The indicated discrepancy is generally comparable to the experimental error. Thus, all the considered techniques yield fairly accurate results. The results of integration deviate slightly from those provided by the other methods, which is attributable both to the error of additional smoothing of the field for calculations (5) and to the error of calculation of tangential components of the field intensity on the ferromagnet surface in (6). The considered methods also differ in terms of computational cost; the most computationally intensive one is the calculation of the total field energy. Therefore, the procedure of force calculation via the Maxwell tensor is the most efficient in the case of geometrically complex electromagnets.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] Z. Ren, IEEE Trans. Magn., **30** (5), 3471 (1994). DOI: 10.1109/20.312686
[2] Z. Ren, A. Razek, IEEE Trans. Magn., **26** (5), 2774 (1990). DOI: 10.1109/20.104869
[3] C.J. Carpenter, Proc. of the IEE — Part C: Monographs, **107** (11), 19 (1960). DOI: 10.1049/pi-c.1960.0005
[4] W. Boughanmi, F. Henrotte, A. Benabou, Y. Le Menach, IEEE Trans. Magn., **52** (3), 7402604 (2015). DOI: 10.1109/TMAG.2015.2492018
[5] A. Bermúdez, A.L. Rodrıguez, I. Villar, IEEE Trans. Magn., **53** (4), 7200409 (2016). DOI: 10.1109/TMAG.2016.2633316
[6] I.E. Tamm, *Osnovy teorii elektrichestva* (Fizmatlit, M., 2003) (in Russian).
[7] F. Freschi, M. Repetto, IEEE Trans. Magn., **49** (5), 1717 (2013). DOI: 10.1109/TMAG.2013.2240668
[8] M. Li, D. Lowther, IEEE Trans. Magn., **45** (3), 1344 (2009). DOI: 10.1109/TMAG.2009.2012621
[9] M.E. Royak, I.M. Stupakov, N.S. Kondratyeva, in *13th Int. Scientific-Technical Conf. on actual problems of electronics instrument engineering (APEIE)* (IEEE, 2016), vol. 2, p. 330. DOI: 10.1109/APEIE.2016.7806481
[10] R. Beatson, L. Greengard, in *Wavelets, multilevel methods and elliptic PDEs* (Clarendon Press, 1997), vol. 1, p. 1–37.
[11] I.M. Stupakov, M.E. Royak, P.A. Bubley, in *XIV Int. Scientific-Technical Conf. on actual problems of electronics instrument engineering (APEIE)* (IEEE, 2018), p. 307. DOI: 10.1109/APEIE.2018.8545530
[12] J. Simkin, C.W. Trowbridge, Int. J. Numer. Meth. Eng., **14** (3), 423 (1979). DOI: 10.1002/NME. 1620140308
[13] J. Simkin, C.W. Trowbridge, IEE Proc. B (Electric Power Applications), **127** (6), 368 (1980). DOI: 10.1049/ip-b.1980.0052
[14] I.M. Stupakov, M.E. Royak, N.S. Kondratyeva, in *13th Int. Scientific-Technical Conf. on actual problems of electronics instrument engineering* (APEIE) (IEEE, 2016), vol. 2, p. 347. DOI: 10.1109/APEIE.2016.7806485