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Spatial dynamics of light quasispin vector in an anisotropic medium with twisting

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On the Poincar's sphere the spatial evolution of the light's polarization state has been studied quantitatively in the smoothly inhomogeneous anisotropic media with gyrotropy. The main features of the dependence on the sample?s thickness, the gyration vector and the azimuth of the polarization of the incident linearly polarized wave were revealed. An analytical solution of the system of coupled equations relative to Cartesian components of light's wave electric component obtained by transition to rotating coordinate system has been used for analysis.

Keywords: Poincar' sphere, polarization evolution, medium with gyrotropy, rotating coordinate system.

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1. Introduction

The problem of polarized light propagation in a medium has always been of a high interest in various spheres of physics [1–3]. It is quite clear, because the study of polarization characteristics of light passed through a medium, not only contains information about the medium itself, but is also the basis for a wide spectrum of attachments enabling control of the characteristics of the emission itself [4]. First, these refer to various types of wave plates, which are used for the modulation of the emission polarization state. It should be highlighted that the creation of such control elements is not limited to the optical range only. For example, paper [5] deals with the possibility of creation of achromatic quarter-wave plate in the terahertz range. The working range of 0.2−0.5 THz can be rebuilt within the interval of 0.3−0.7 THz by means of liquid-crystal phase plates, whose anisotropy was regulated by a magnetic field. Paper [6] considered polarizers and phase plates based on polymer dispersed liquid crystals (LC), which were controlled by means of creation of regulated microstructures. Paper [7] considered controllable polarizers of the THz range based on LC in crossed electrical and magnetic static fields. Earlier papers studied adiabatic twisting and achromatic conversion of the light polarization in an heterogeneously anisotropic media [8]. Consideration was performed based on the analogy of equations describing spatial dynamics of light polarization in an heterogeneous anisotropic medium, and the Schrëdinger equation, describing coherent laser excitation of a three-level atom. Broadband polarization converters in media with heterogeneous linear and circular dichroism are also considered.

Apparently, the creation of various kind of controlling elements requires a detailed analysis of the problems of emission propagation in media with different optical characteristics. The coupled-mode approach, considered in [9], allowed to make a deep analogy between the quantum mechanics and optics [10], thus expanding the circle of problems in question, connected, in particular, with the propagation of polarized wave in media with spatially inhomogeneous anisotropy, dissipation and gyrotropy [11,12], and application of the term „quasispin of polarization" allowed not only to considerably simplify solution of a wide range of problems, but also to solve the problems, which are inaccessible for other methods. Such method has been successfully applied also for the study of evolution of the emission polarization in heterogeneously magnetized plasma with dissipation [13], and in [14] — in more common case, heterogeneous, anisotropic, optically active and dichroic plasma.

In [15,16] based on such analogy a series of effects of the polarization optics was qualitatively considered, in particular, the propagation of the polarized light in the presence of external magnetic field (generalized Faraday effect), as well as in case of a gyrotropic medium with modulated anisotropy (optical analog of the magnetic resonance).

The objective of this paper is to theoretically consider the spatial evolution of the light polarization state in the anisotropic medium with twisting of he local optical axis and admixture of gyrotropic molecules. The basis for the analysis refers to a system of coupled-wave equations obtained in the approximation of geometrical optics, and their solution by transition into rotating coordinate system [17,18].

2. Abridged equations system

May a polarized flat monochromatic wave propagate in an optically heterogeneous anisotropic medium with gyrotropy. Not only various mixtures of LC and gyrotropic substances can be considered as such a medium, but also isotropic media in mutually perpendicular heterogeneous electrical and magnetic fields [13–16].

The laboratory system of coordinates is selected so that the axis z matches the wave propagation direction, and the axes x, y are codirectional with main axes of the uniaxial anisotropic medium at the input. As it was shown in [11,12,17], wave equation for the two-dimensional Jones vector

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix},$$

describing the light wave polarization state, can be reduced to two-dimensional form

$$\frac{d^2\mathbf{E}(z)}{dz^2} + \frac{\omega^2}{c^2}\hat{h}(z)\mathbf{E}(z) = 0, \tag{1}$$

where $\hat{h}$ is the tensor describing optical properties of the medium, whose elements are connected with the tensor of permittivity $\tilde{\varepsilon}_{ij}$ of the complex medium by the following ratio:

$$h_{ij}(z) = \tilde{\varepsilon}_{ij} - \frac{\tilde{\varepsilon}_{iz}\tilde{\varepsilon}_{jz}}{\tilde{\varepsilon}_{zz}};\ i, j = x, y. \tag{1a}$$

Note that in case of normal incidence onto the medium the tensor $h_{ij}(z) = \tilde{\varepsilon}_{ij}$.

In many structures considered the anisotropy heterogeneity at the wavelength is low, and application of the geometrical optics method is quite reasonable. Subject to the above, equation (1) is solved as

$$E_i(z) = F_i(z)e^{i\Phi(z)}. \tag{2}$$

In (2) $\mathbf{F}(z)$ — the complex amplitude, which is changing slowly, and the phase multiplier $\Phi(z)$ in conditions of that problem is

$$\Phi(z) = \frac{\omega}{c}\int n(z)dz, \tag{3}$$

where

$$n(z) = \sqrt{\frac{h_{xx} + h_{yy}}{2}}. \tag{3a}$$

Such substitution in case of propagation in the medium enables to separate relatively slow and fast changes of the wave parameters associated with the medium heterogeneity. Subject to a slow change pace $\mathbf{F}(z)$ (i.e., ignoring low values $\mathbf{F}''(z)$), we obtain the vector equation from (1) and (2)

$$\frac{d\mathbf{J}(z)}{dz} = i\hat{H}(z)\mathbf{J}(z). \tag{4}$$

The following notations are introduced here:

$$\mathbf{J}(z) = \mathbf{F}(z)\sqrt{\phi'(z)}, \tag{4a}$$

$$\hat{H}(z) = \frac{\omega}{c}\left\{\frac{\hat{h}(z)}{2n(z)} - \frac{1}{2}n(z)\hat{I}\right\}. \tag{4b}$$

This is the equation, which describes the spatial evolution of the light polarization in the medium. Formally, it coincides the Schrëdinger equation in quantum mechanics. The matrix $\hat{H}(z)$, being in fact an analog of a Hamiltonian, describes the medium properties (including in the presence of external homogeneous and heterogeneous fields) [19], and own medium polarizations act as two energetic levels. Note that such an analogy could considerably expand the circle of the problems to consider.

3. Permittivity of the medium

Let us consider a composite medium, with the heterogeneity of optical anisotropy and gyrotropy. The permittivity of such medium can be represented [20] as follows:

$$D_i = \tilde{\varepsilon}_{ij}E_j = \varepsilon_{ij}(z)E_j + \gamma_{ijz}(z)\frac{\partial E_j(z)}{\partial z}. \tag{5}$$

Here $\varepsilon_{ij}(z)$ is the permittivity of heterogeneously anisotropic medium, and the second member in (5) considers the gyrotropy. Note that as accepted herein, the medium in the problem in question is homogeneous by coordinates x, y, i.e., $\partial/\partial x = \partial/\partial y = 0$.

Let us make the following additional assumptions on the medium. May the light propagate in the anisotropic medium with twisting of the local optical axis. Represent the tensor of permittivity of such a smoothly heterogeneous anisotropic medium $\varepsilon_{ij}(z)$ [21,22] as

$$\varepsilon_{ij}(z) = \varepsilon_\perp\delta_{ij} + \varepsilon_a m_i(z)m_j(z). \tag{6}$$

Here $\mathbf{m}(z)$ is the unit vector describing the local orientation of the optical axis, δ_{ij} is the Kronecker delta, $\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp$ is the anisotropy. In case of isotropic gyrotropy [20]

$$\frac{\omega}{c}\gamma_{ijz} = ge_{ijz}. \tag{6.1}$$

Here e_{ijk} is the unit antisymmetrical tensor, g is the gyration vector modulus. For the sake of convenience let us transform the permittivity as

$$\tilde{\varepsilon}(z) = \frac{\varepsilon_e + \varepsilon_o}{2}\hat{\sigma}_0 + \frac{\varepsilon_a}{2}\cos 2\psi(z)\hat{\sigma}_1 + \frac{\varepsilon_a}{2}\sin 2\psi(z)\hat{\sigma}_2 - n(z)g\hat{\sigma}_3. \tag{7}$$

In (7) $\hat{\sigma}_i$ — the Pauli matrices [19]:

$$\hat{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\quad \hat{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\hat{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\quad \hat{\sigma}_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}.$$

Subject to (2) and low $\gamma_{ijz}(z)F_j'(z)$, after simple transformations the matrix (4b) can be presented in a more useful form:

$$\hat{H}(z) = \frac{\omega}{2cn(z)}\left\{\frac{\varepsilon_{xx}(z) - \varepsilon_{yy}(z)}{2}\hat{\sigma}_1 - \varepsilon_{xy}(z)\hat{\sigma}_2 - n(z)g\hat{\sigma}_3\right\}. \tag{8}$$

Finally, by means of (7) we have

$$\hat{H}(z) = \frac{\omega}{2cn(z)}\left\{\frac{\varepsilon_a}{2}\cos 2\psi\hat{\sigma}_1 + \frac{\varepsilon_a}{2}\sin 2\psi\hat{\sigma}_2 - n(z)g\hat{\sigma}_3\right\}. \tag{8a}$$

Unfortunately, even a simplified system of equations (4) with (8a) not always has a complete analytical solution.

4. Solution of the abridged equations system

As it was demonstrated in [17,23], in case of the twist orientation of the optical axis, the system of equation (4) can be solved analytically, if we refer to the rotating coordinates system. Subject to the fact that the presence of constant gyrotropy does not change the symmetry of the medium, for the analytical solution of the system (4) we proceed with the rotating coordinates system [17] by means of substitution

$$\mathbf{J}(z) = \hat{R}^{-1}\mathbf{A}(z), \quad \hat{H}(z) = \hat{R}^{-1}\hat{H}_0\hat{R}, \tag{9}$$

where

$$\mathbf{A} = \begin{pmatrix} A_\xi \\ A_\eta \end{pmatrix}$$

— the Jones vector in a rotating coordinates system, $\hat{R}(\psi)$ — rotation matrix of the coordinate axes:

$$\hat{R}(\psi) = \begin{pmatrix} \cos\psi(z) & \sin\psi(z) \\ -\sin\psi(z) & \cos\psi(z) \end{pmatrix},$$

and $\hat{H}_0$ — „Hamiltonian" of the heterogeneously anisotropic medium in the local coordinates system. After transformation we obtain a system of coupled equations relative to $\mathbf{A}(z)$ in the rotating coordinates system:

$$\frac{d\mathbf{A}(z)}{dz} - i\omega_a\hat{\sigma}_1\mathbf{A}(z) = i(\psi' - \omega_g)\hat{\sigma}_3\mathbf{A}(z). \tag{10}$$

Here $\omega_a = \pi\varepsilon_a/2\lambda n_0$, $\omega_g = \pi g/\lambda$. In case of the linear angle change law $\psi(z) = \alpha z$, this is a system of equations with constant coefficients, whose solution is easy to find. Having solved this system of equations with limit conditions

$$E_x(0) = \frac{1}{\sqrt{\phi'(0)}}J_x(0) = A_0, \ E_y(0) = \frac{1}{\sqrt{\phi'(0)}}J_y(0) = B_0$$

and having executed reverse transition into the laboratory system of coordinates, the final result is:

$$E_i(z) = \frac{i}{2\Omega}\left[A_i(z)e^{-i\Omega z} + B_i(z)e^{i\Omega z}\right]e^{i\frac{2\pi}{\lambda}n_0 z}, \tag{11}$$

$$A_x(z) = \left(B_0 - i\frac{\alpha - \omega_g}{\omega_a + \Omega}A_0\right) \times \left[(\alpha - \omega_g)\cos\psi(z) + i(\omega_a + \Omega)\sin\psi(z)\right],$$

$$B_x(z) = \left[-i(\omega_a + \Omega)A_0 - (\alpha - \omega_g)B_0\right] \times \left[\cos\psi(z) - i\frac{\alpha - \omega_g}{\omega_a + \Omega}\sin\psi(z)\right],$$

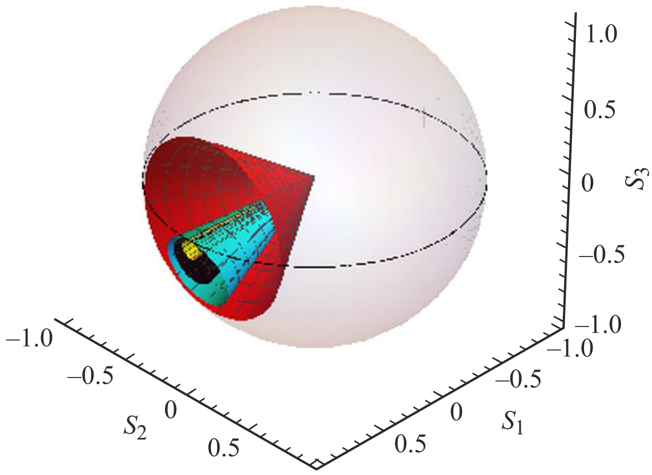

Figure 1. Polarization run-outs in gyrotropic medium with homogeneous anisotropy. The values of parameters are the following: $\varepsilon_e = 2.43$; $\varepsilon_0 = 2.42$; $\lambda = 0.5\,\mu$m; layer thickness is $200\,\mu$m; $g = 0$ ($\beta = 0.1$, red cone), $0.2 \cdot 10^{-3}$ ($\beta = 0$, yellow cone), $0.5 \cdot 10^{-3}$ ($\beta = 0$, black cone), $0.8 \cdot 10^{-3}$ ($\beta = 0$, blue cone).

$$A_y(z) = \left(B_0 - i\frac{\alpha - \omega_g}{\omega_a + \Omega}A_0\right) \times \left[(\alpha - \omega_g)\sin\psi(z) - i(\omega_a + \Omega)\cos\psi(z)\right],$$

$$B_y(z) = \left[-i(\omega_a + \Omega)A_0 - (\alpha - \omega_g)B_0\right] \times \left[\sin\psi(z) + i\frac{\alpha - \omega_g}{\omega_a + \Omega}\cos\psi(z)\right].$$

$$\Omega^2 = \omega_a^2 + (\alpha - \omega_g)^2, \ A_0 = E_x(z = 0), \ B_0 = E_y(z = 0).$$

The values A_0 and B_0 can be complex (for the possibility of analysis of elliptically polarized input wave). It is easy to verify that equations (11) in the limit α, $g \to 0$ are transformed to well known ones [21,22]. Obtained analytical expressions (11) allow to fully quantitatively explain the behavior of the polarized light in twist-structures with the admixture of gyrotropic molecules. Such an approach can especially be fruitful also in the media with the time-dependent anisotropy for the purpose of creation of polarization devices with time control [24]. From this point of view paper [25] is also interesting, where, based on the analog of the phenomenon of the linear anisotropy compensation in the presence of harmonically modulated gyrotropy, the existence of similar effect is predicted in a homogeneous magnetic field with time modulation. Since the linear anisotropy compensation phenomenon in presence of spatial harmonically modulated gyrotropy is studied in details in paper [26] in the approximation of a rotating wave, it should be expected that the universality of the obtained solution will allow to study similar problems.

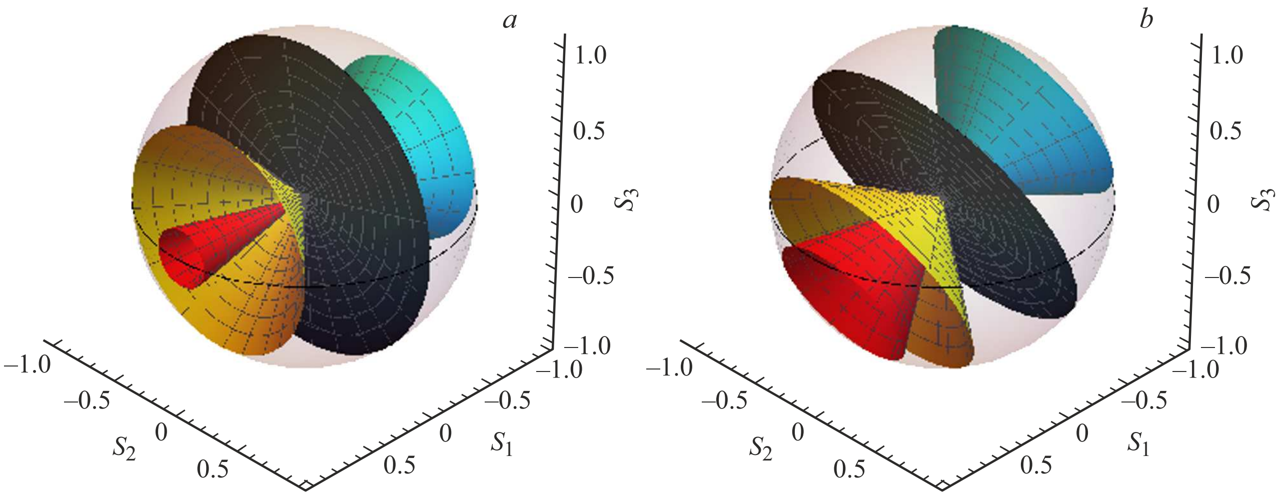

Figure 2. Polarization run-outs in gyrotropic medium with homogeneous anisotropy. The values of parameters are the following: $g = 0$ (*a*), 0.0015 (*b*); $\varepsilon_e = 2.43$; $\varepsilon_0 = 2.42$; $\lambda = 0.5\,\mu$m; the layer thickness is $200\,\mu$m; $\beta = 0.03\pi$ (red cone), 0.15π (yellow cone), 0.25π (black cone), 0.6π (blue cone).

5. Discussion

Let us perform the analysis of passage of a flat linearly polarized wave through the layer of an uniaxial anisotropic medium with twisting of the optical axis and admixture of optically active molecules. For the sake of certainty let us assume the linearly polarized wave incidence with the azimuthal angle β. Then, limit conditions will be

$$A_0(z = 0) = \cos\beta, \; B_0(z = 0) = \sin\beta. \tag{12}$$

According to (11), in particular case, when $\alpha = \omega_g$, we have

$$A_x(z) = 2i\omega_a B_0 \sin\psi(z), \;\; B_x(z) = -2i\omega_a A_0 \cos\psi(z),$$

$$A_y(z) = 2iB_0\omega_a \cos\psi(z), \;\; B_y(z) = -2i\omega_a A_0 \sin\psi(z).$$

If the rising wave is linearly polarized along the axis x of the laboratory system of coordinates (i.e. $\beta = 0$), then $A_0 = E_x(z = 0) = 1$, $B_0 = E_y(z = 0) = 0$. Therefore, for the final solution we have

$$\left.\begin{aligned} E_x(z) &= A_0 \cos\psi(z) \exp\left\{ i\,\frac{2\pi}{\lambda}\, n_0 \left(1 + \frac{\varepsilon_a}{4n_0^2}\right) z \right\} \\ E_y(z) &= A_0 \sin\psi(z) \exp\left\{ i\,\frac{2\pi}{\lambda}\, n_0 \left(1 + \frac{\varepsilon_a}{4n_0^2}\right) z \right\} \end{aligned}\right\}$$

or

$$\frac{E_y(z)}{E_x(z)} = \mathrm{tg}\,\psi(z).$$

It means that in such conditions the light wave polarization adiabatically follows the optical axis rotation. With that gyration vector value it is easy to find also the relevant thickness of the sample. The condition $\alpha = \omega_g$ results in $L = 2\lambda/g$.

For the sake of clarity of further results, we perform the analysis on the Poincar' sphere. Therefore, we will determine [19] also the Stokes parameters (or the quasispin vector) by using the ratios

$$\mathbf{S}(z) = \mathbf{E}^{+}(z)\hat{\sigma}\mathbf{E}(z). \tag{13}$$

Note, that due to (12) the quasispin vector is normalized to one.

5.1. Gyrotropic medium with homogeneous anisotropy

Let us assume that there is no twisting in the medium, i.e., it is homogeneously anisotropic. We select the coordinate axes S_1, S_2, S_3 so that the equatorial plane match the linear polarization. Figs. 1 and 2 represent the behavior of the polarization state in case of propagation in such medium. As we can see in Fig. 1, the quasispin vector in this case is rotating over the cone surface (quasispin vector precession) as follows:

- in the absence of gyration the quasispin vector rotates on the cone surface symmetrically arranged relative to the equatorial plane (red cone);
- in case when gyration is other than zero, the axis rotates relative to that plane to an angle, which is pro rata of the gyration [27];
- when gyrotropy is included, the cone apex angle rises as far as the gyration constant is increased;
- with the constant value of gyration, the change of the input polarization azimuth affects only the apex angle, such behavior is given in Fig. 2 — as we can see, the presence of gyrotropy changes only the cone axes angle relative to the equatorial plane.

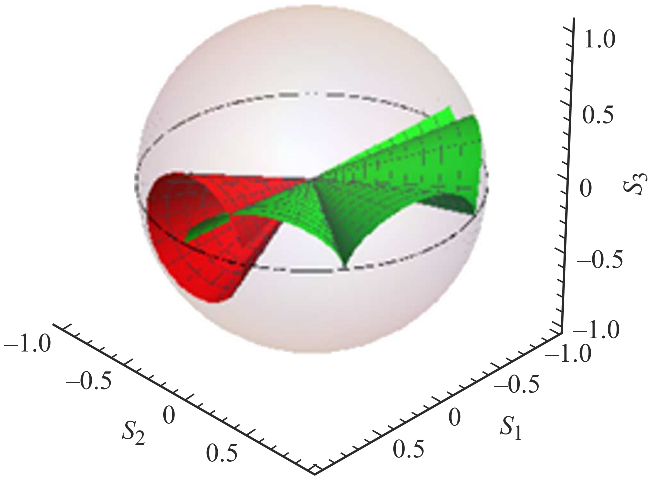

Figure 3. Evolution of the light polarization state in anisotropic medium with optical axis twisting. The values of parameters are the following: $\varepsilon_e = 2.43$, $\varepsilon_0 = 2.42$, $q = 0$, $\lambda = 0.5\,\mu$m, the layer thickness is $500\,\mu$m. $\psi(L) = 0$ (red image), $\pi/2$ (green image).

5.2. Heterogeneously anisotropic medium

Consideration of the optical axis twisting changes the situation drastically. Figure 3 demonstrates the optical axis twisting effect for the quasispin behavior. As we can see in that figure, in this case the precession of the polarization quasispin changes in nutation.

Such analysis allows to perform the quantitative study of the adiabatic tracking phenomenon. As known, the adiabatic tracking phenomenon is observed in an anisotropic media with optical axis twisting. That is to say, if an incident wave at the medium input is linearly polarized along one of the normal axes, then the polarization vector of the light wave tracks the rotation of main axes, provided that the twisting coefficient is low [21,22]. Such a property is widely used for the creation of controllable light valves (in particular, on LC [22]). However, such a qualitative description does not fully represent the real situation of the light polarization state evolution in media with optical axis twisting. As an example, Fig. 4 represents the behavior of polarized light in such medium at three different thicknesses. As we can see in the figure, the light polarization state evolution in the layer is heavily dependent also on the sample thickness. The following relationship is observed. With the increase of the layer thickness the heterogeneous anisotropic medium with twisting of the optical axis is broken down into elementary semiwave plates, whose middle optical axes are turned relative to each other (Fig. 4). Another relationship is observed as well. With the increase of the sample thickness the amplitude of spatial oscillations is decreased (ellipticity $e \sim S_3 \to 0$) and the quasispin vector is virtually rotating in the equatorial plane.

It means that the input linearly polarized wave adiabatically follows the optical axis rotation, meanwhile being a linearly polarized wave.

5.3. Gyrotropic medium with heterogeneous anisotropy

There is a feature in case of the simultaneous presence of isotropic gyrotropy and the heterogeneity of anisotropy. Figs. 5 and 6 demonstrated that feature for two different sample thicknesses. As we can see in these figures, the increase of the gyration value decreases the value of ellipticity, the same as in case of a pure heterogeneously anisotropic medium (Fig. 3), and in case of complying with the condition $\alpha = \omega_g$ the quasispin vector rotates in the

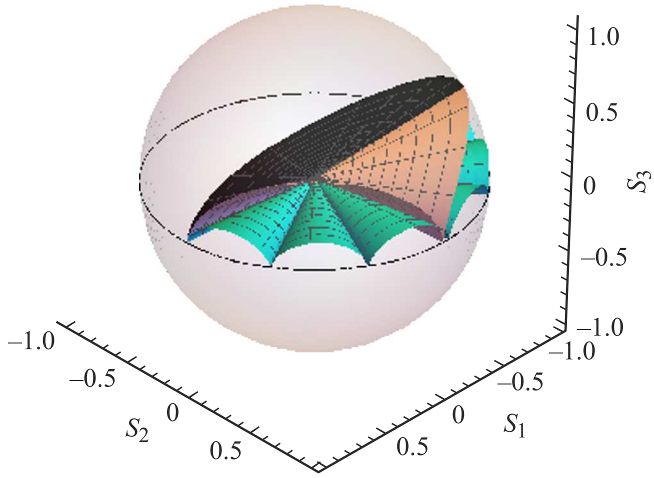

Figure 4. Evolution of the light polarization state in anisotropic medium with optical axis twisting. The values of parameters are the following: $\varepsilon_e = 2.43$, $\varepsilon_0 = 2.42$, $q = 0$, $\lambda = 0.5\,\mu$m; thickness of layer is 150 (black image), 300 (orange image), $900\,\mu$m (green image).

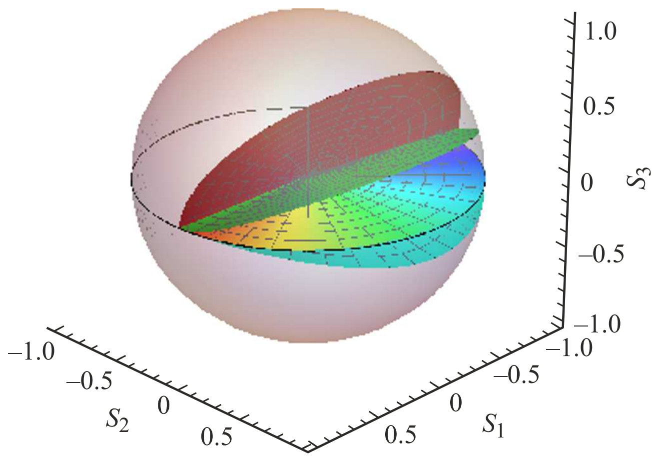

Figure 5. The light polarization state evolution in heterogeneous anisotropic medium with admixture of gyrotropic molecules. The values of parameters are the following: $\varepsilon_e = 2.43$, $\varepsilon_0 = 2.42$, $\lambda = 0.5\,\mu$m; the layer thickness is $150\,\mu$m; $g = 0$ (red image), $1.0 \cdot 10^{-3}$ (green image), $1.65 \cdot 10^{-3}$ (rainbow image), $2.0 \cdot 10^{-3}$ (blue image).

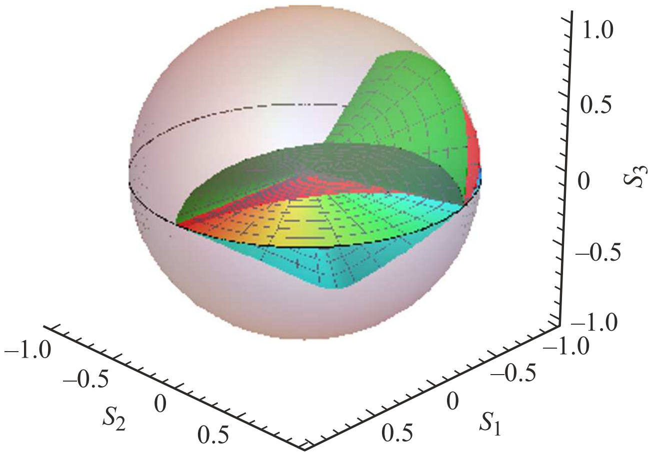

Figure 6. The light polarization state evolution in heterogeneous anisotropic medium with admixture of gyrotropic molecules. The values of parameters are the following: $\varepsilon_e = 2.43$, $\varepsilon_0 = 2.42$, $\lambda = 0.5\,\mu$m; the layer thickness is $250\,\mu$m; $g = 0$ (green image), $0.5 \cdot 10^{-3}$ (red image), $g = 1.0 \cdot 10^{-3}$ (rainbow image), $1.5 \cdot 10^{-3}$ (blue image).

equatorial plane. Such behavior, according to Figs. 5, 6, is observed regardless of the thickness.

Therefore, as a summary of the analysis, the following can be concluded.

• In combined media with heterogeneous anisotropy (rotation of the optical axis according to the linear law) the transition into rotating coordinates system allows to solve the problem analytically. This conclusion correlates also with paper [23], where a similar problem is considered, but with dichroic molecules. It should be expected that transition into the rotating coordinates system can be useful in the study of other (not only optical) problems.

• The presence in the mixture of the component with constant gyrotropy does not change qualitatively the nature of spatial dynamics of the light polarization (Fig. 4). The gyration value decreases ellipticity, and when $\alpha = \omega_g$, the light is virtually not affected by the medium anisotropy.

We emphasize that the clarity of consideration of the known problems allows to conclude that within the framework of such approach we could study the specifics of polarization transformation of the light in other optically heterogeneous linear and non-linear media as well.

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