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# Magnetovolume effects and thermal expansion in chiral helical ferromagnets $Fe_{1-x}Co_xSi$

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Within the framework of the theory of band magnetism and the Heine model for the volume dependence of the electronic spectrum, an approach is being developed to study magneto-volume effects in chiral helical ferromagnets. Using  $Fe_{1-x}Co_xSi$  as an example, it was found that in the range of long-range order (at temperature  $T < T_c$ ) the magnetovolume effect is determined by the amplitude of helicoidal spin spirals and leads to the experimentally observed negative volume thermal expansion coefficient. In the region of phase transitions of the first order (from Tc to Ts) prolonged in temperature, a new mechanism of the magnetovolume effect is established due to the spatial fluctuations of spin spirals arising due to the difference in the Hubbard potentials of iron and cobalt. It is shown that the considered volume effects lead not only to the experimentally observed negative volume thermal expansion coefficient (VCTE) in the chiral spin short-range order phase, but also to a noticeable increase in the transition temperature to the paramagnetic state ( $T > T_s$ ).

Keywords: helicoidal ferromagnetism, chirality, spin fluctuations, electronic and crystal structure, thermal expansion.

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## 1. Introduction

Chiral helical ferromagnets with structural type B20 are characterized by disturbance of cubic symmetry (distorted structure of NaCl type), which results in disappearance of the inversion center. As a a result of this, the system of strongly correlated electrons includes asymmetric relativistic interaction of Dzyaloshinskii-Moriya (DM), and ferromagnetic helicoid spin spirals are formed [1]. When the sign of mode-mode parameter changes, according to Ginzburg-Landau theory, a magnetic phase transition of the first kind extended by temperature, when chiral spin short-range order occurs as observed in the experiment with fluctuations of spin spirals [2,3]. In particular, such transition in the narrow range of temperatures  $(T_s - T_c) \ll T_c$  occurs in MnSi, where it is accompanied by sharp change of volume and occurrence of lambda — like anomaly of VCTE [4]. A transition extended by temperature related to change of mode-mode parameter sign change in  $Fe_{1-x}Co_xSi$  [5], as a result of fluctuations of intra-atomic potentials of electronelectron repulsion in nodes occupied by iron and cobalt atoms [5], is implemented in a wider range of temperatures  $(T_s - T_c) \sim T_c$ , and in the experiment instead of lambdaanomaly of negative VCTEs, a wide temperature minimum occurs [6].

At the same time, according to [3] in compositions with 0.2 < x < 0.65 at temperatures  $T_c < T < T_s$ , skyrmion microstructures are implemented, related to occurrence of chiral short-range order in the area of temperatures of extended phase transition of the first kind. However, the

developed thermodynamic approach will not explain the considerable magnetic contribution to VCTE observed in the experiment in area  $T_c < T < T_s$ , since it was produced within an assumption on volume continuity. Therefore it is not clear which magnetovolume effects accompany negative magnetic contributions to VCTE, and which impact is provided by volume effects at occurrence of long- and short-range orders in chiral ferromagnets.

In this paper in the model of chiral strongly correlated electronic system with spatial fluctuations of Hubbard potentials in nodes occupied by iron and cobalt, magnetovolume effects and temperature dependences of VCTE are considered in vortex spin structures and microstructures  $Fe_{1-x}Co_xSi$ , occurring at constant pressure.

# 2. Model

Let us consider a strongly correlated electronic system of chiral ferromagnets  $Fe_{1-x}Co_xSi$  with Hamiltonian taking into account the zone motion energy, intra-atomic Coulomb spin and charge correlations, differing in nodes occupied by Fe atoms on Co. At the same time we will keep in mind that use of spin-dependent energy spectrum from first-principles LSDA + U + SO, causes concentration dependences and values of local magnetizations  $Fe_{1-x}Co_xSi$ , not matching with the experimental data [7]. Satisfactory agreement with the experiment is obtained at self-consistent calculation of local magnetization within fluctuation theory of zone magnetism, where the results of the first-principles LDA + U + SO are used only to model electronic structure [5].

An important feature of the considered modification of Hubbard model, together with the difference of potential in intra-atomic electron-electron repulsion in nodes occupied by iron and cobalt, is dependence of the spectrum of strongly correlated *d*-electrons on the volume, which in accordance with formula of V. Heine [8], is described by ratio  $\varepsilon_{\mathbf{k}}(V=\theta^{-1}\varepsilon_{\mathbf{k}}^{(0)})$ , where  $\theta = (V/V_0)^{5/3}$ .

Therefore, Hamiltonian of the considered system of strongly correlated electrons must be presented as

$$H = H_0 + \delta H_{\text{int}},\tag{1}$$

here  $H_0 = \sum_{\mathbf{k},\sigma} \varepsilon_{\mathbf{k}}(V) a^+_{\mathbf{k},\sigma} a_{\mathbf{k},\sigma}$  — Hamiltonian of zone motion with spectrum  $\varepsilon_{\mathbf{k}}$ , calculated in approximation LDA + U + SO,  $a^+_{\mathbf{k},\sigma}(a_{\mathbf{k},\sigma})$  — operator of electron birth (destruction) with quasipulse  $\mathbf{k}, \sigma$  — spin index.

$$\delta H_{\rm int} = (U_{\rm Fe} - U_{\rm Co}) \langle n \rangle_0 \sum_{\nu} \delta p_{\nu} \delta n_{\nu} / 2$$
$$- \sum_{\nu} (U_{\rm Fe} (1 - p_{\nu}) - U_{\rm Co} p_{\nu}) ((S_{\nu}^{(z)})^2 - (\delta n_{\nu})^2 / 4) \quad (2)$$

— Hamiltonian of intra-atomic correlations in the node, where electronic density fluctuation terms of sum are identified, and difference in parameters of Hubbard interaction is taken into account in nodes occupied by atoms of cobalt or iron ( $U_{\text{Co}}$  and  $U_{\text{Fe}}$  — accordingly),  $\delta p_{\nu} = p_{\nu} - p$ , p concentration of cobalt atoms,  $p_{\nu}$  — projection operator, which may take values 0 on the node occupied by iron, and 1, if the unit is occupied by cobalt ( $p_{\nu}^2 = p_{\nu}$ ),

$$n_{
u} = \sum_{\sigma} n_{
u,\sigma}, \quad n_{
u,\sigma} = a^+_{
u,\sigma} a_{
u,\sigma},$$
 $(z)$   
 $a_{
u}^{(z)} = 2^{-1} \sum_{\sigma} \sigma n_{
u,\sigma}, \quad \delta n_{
u} = n_{
u} - \sum_{\sigma} \langle n_{\sigma} \rangle_0.$ 

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Besides, in the considered chiral ferromagnets with B20 structure Hamiltonian (1) must be supplemented by term of sum of DM–interaction, which in virtue of relativistic minuteness is considered in the mean field approximation

$$H \to H - \sum_{\mathbf{q}} \mathbf{h}_{\mathbf{q}}^{(D)} \mathbf{S}_{-\mathbf{q}}.$$
 (3)

Here  $\mathbf{h}_{\mathbf{q}}^{(D)} = [\mathbf{M}_{\mathbf{q}} \times \mathbf{d}_{-\mathbf{q}}]$  — mean field of Dzyaloshinskii;  $\mathbf{d}_{\mathbf{q}} = i d\mathbf{q}$ , accordingly;  $\mathbf{M}_{\mathbf{q}} (= \langle \mathbf{S}_{\mathbf{q}} \rangle)$  — vector of heterogeneous magnetization in wave vector  $\mathbf{q}$ .

Statistical sum of system with Hamiltonian (3):

$$Z = SpT_{\tau} \left\{ -\int_{0}^{T^{-1}} d\tau H(\tau) \right\},$$
$$H(\tau) = \exp(H_{0}\tau)H\exp(-H_{0}\tau),$$

is to be investigated on the basis of the procedure using Stratonovich-Hubbard transformations [9], which reduce multiple particle interactions (2) to a picture of motion of correlated *d*-electrons in exchange ( $\xi$ ) and charge ( $\eta$ ) fields fluctuating in the space and time. In the considered task "the picture" of fluctuating exchange fields is supplemented by static field of Dzyaloshinskii and concentration fluctuations of exchange fields. At the same time, to determine the statistical sum of the electron system, it is feasible to apply Matsubara technique for complex variables (see [10])

$$Z = \int (d\xi d\eta) (d\Omega)$$
  
 
$$\times \exp\left\{-\sum_{q} |\xi_{q} - \delta_{q,\mathbf{q}} \mathbf{h}_{\mathbf{q}}/c|^{2} - \sum_{q} |\eta_{q}|^{2}\right\} Z(\xi_{q},\eta_{q}), \quad (4)$$

where

$$Z(x,\xi,\rho) = SpT_{\tau} \exp(-T^{-1}H_0(x) - T^{-1}\tilde{\mathscr{H}}_{\text{eff}}),$$
$$(d\xi d\eta) = d\xi_0 d\eta_0 \prod_{q\neq 0, i=1,2} d\xi_q^{(j)} d\eta_q^{(j)}$$

(index *j* numbers real and imaginary parts of stochastic  $\xi$ and  $\eta$ -fields), *T* — temperature  $T_{\tau}$  — operator of alignment by "imaginary" Matsubara time  $\tau$ 

$$\tilde{\mathscr{H}}_{\text{eff}} = 2\sum_{q} \mathbf{S}_{q} \boldsymbol{\xi}_{-q} + i 2^{-1} \sum_{q} n_{q} \eta_{q}^{(j)},$$

 $\mathbf{S}_q$  — Fourier transform of spin density operator on the node, recorded in representation of interaction,  $q = (\mathbf{q}, \omega_{2m})$  — four-vector, including  $\omega_{2m}$  (Matsubara Bose frequency, *m* — integer number),

$$\xi_q = (\xi_q)^* = c \left( \xi_q e_q + (2U)^{-1} (U_{\text{Co}} - U_{\text{Fe}}) \sum_{q'} \delta p_{q+q'} \xi_{q'} \right).$$

Calculation (4) is reduced to procedure of quantumstatistic averaging of spin and charge density operators with summation of the row by degrees of exchange and charge fields. Besides, taking into account abnormally large periods of spin superstructures  $Fe_{1-x}Co_xSi$  and abnormally strong dependence of Stoner factors on quasipulses and frequency, one may limit oneself to long-wave approximation.

Methods to assess functional intervals for statistical sum (4) may be based on using saddle point approximation. In the considered model

$$\xi_0^{(\gamma)} \equiv \operatorname{Re} \xi_0^{(\gamma)}(\operatorname{Im} \xi_0^{(\gamma)} = 0), \ \operatorname{Re} \xi_q^{(\gamma)} \text{ and } \operatorname{Im} \xi_q^{(\gamma)} \text{ with } \mathbf{q} \neq 0,$$
$$|\xi_q^{(\gamma)}| \text{ with } q = (\mathbf{q}, \omega_{2n}) \text{ at } \omega_{2n} \neq 0, \eta_0$$
(5)

and are related to magnetization by ratios

$$M_{\mathbf{q}}^{(\gamma)} = (c/U)\xi_{\mathbf{q}}^{(\gamma)} - h_{\mathbf{q}}^{(\gamma)}$$

and

$$|\xi_q^{(\gamma)}|^2 = 2^{-1} \Big( \langle T_\tau | S_q^{(\gamma)} |^2 \rangle + 1 \Big), \ \gamma = (x, y, z).$$

### 3. Free energy at constant pressure

Going to thermodynamic potentials, let us consider the known ratio between statistical sum and free energy  $(F = T \ln Z - \mu N)$ , we will consider the terms of saddlepoint for functional integrals (4) by variables (5). At the same time the free energy of the system of correlated electrons with the volume-dependent energy spectrum  $\varepsilon_k(V)$ will be supplemented by the term of sum related to elastic deformation  $P = -K\Delta V$  (K — isothermal compressibility).

As a result, for the free energy at constant pressure we get the equation

$$F = F_{mag} + F_{el} + F_{fl} - K\Delta V^2 / 2,$$
 (6)

where single-electron contribution becomes renormalized as a result of electron terms splitting by fluctuating exchange fields

$$F_{el}/U = \sum_{lpha(=\pm 1)} \int heta g^{(0)}(arepsilon + lpha heta \langle m 
angle_T) 
onumber \ imes \ln(1 + \exp T^{-1}(\mu - arepsilon)) darepsilon,$$
 (6a)

magnetic contribution includes terms of sum in mode-tomode interaction

$$F_{mag}/U = \sum_{\mathbf{q}} \left( 1 - U\chi^{\perp}(V/V_0) \right) |\mathbf{M}_{\mathbf{q}}|^2 + \sum_{\mathbf{q}} X(\mathbf{q}, 0) |\mathbf{M}_{\mathbf{q}}|^2$$
$$- U^{-1} \sum_{\mathbf{q}} \mathbf{h}_{\mathbf{q}} \mathbf{M}_{\mathbf{q}} + \kappa \sum_{\mathbf{q}_1 \neq \mathbf{q}_2, \mathbf{q} \neq \mathbf{q}_4} \left( \mathbf{M}_{\mathbf{q}_1} \mathbf{M}_{\mathbf{q}_2} \right) \left( \mathbf{M}_{\mathbf{q}_3} \mathbf{M}_{\mathbf{q}_4} \right)$$
$$\times \delta_{\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3 + \mathbf{q}_4; \mathbf{0}} (V/V_0) + F_{mfl}, \tag{6b}$$

and magnetic fluctuation contribution is described by the equation

$$F_{fl} = \sum_{\mathbf{q}} \int_{0}^{\infty} \operatorname{cth}(\omega/2T) \operatorname{Im} \ln \left( D^{-1} + 2\kappa |M_{\mathbf{q},\gamma}|^{2} + X(\mathbf{q},\omega) \right) d\omega.$$

Besides,  $g^{(0)}(\varepsilon)$  — calculated in the method LDA + U + SO — density of electron *d*-states,  $D(V) = (1 - U\theta\chi^{(\perp)} + 3^{-1}\theta\kappa\langle m\rangle_T^2)^{-1}$  — factor of exchange amplification of magnetic susceptibility, in the mean field approximation matching Stoner factor,  $\kappa = U\langle m\rangle_T^{-2}(\chi^{(\perp)} - \chi^{(\parallel)})$  — mode-mode coupling coefficient, which in the mean field approximation is reduced to coefficient at the fourth degree of order parameter in Ginzburg-Landau expansion, dependence of which on magnetization is determined by magnetovolume effect

$$\chi^{(\perp)}(V) = \lim_{q \to 0} \sum \left( f\left(\varepsilon_{k,\alpha}(V) - \mu\right) - f\left(\varepsilon_{k+q;-\alpha}(V) - \mu\right) / \left(\varepsilon_{k\alpha}(V) - \varepsilon_{k+q-\alpha}(V)\right) \right),$$

$$\chi^{(\parallel)}(V) = \lim_{q \to 0} \sum_{\alpha} \left( f\left(\varepsilon_{k,\alpha}(V) - \mu\right) - f\left(\varepsilon_{k+q;\alpha}(V) - \mu\right) / \left(\varepsilon_{k\alpha}(V) - \varepsilon_{k+q-\alpha}(V)\right) \right)$$

— transverse and longitudinal susceptibilities as functions of Matsubara frequency,  $\varepsilon_{k\alpha}(V) = \varepsilon_k - \alpha \Theta U \langle m \rangle_T$ ,  $\alpha = \pm 1, \mu$  — chemical potential, amplitude of thermal fluctuations

$$\langle m^2 \rangle_T = (3T^2/4U^2)\Theta C \left\{ (D^{-1} + 2\kappa(V)M_S^2)^2 + \Theta^2 A^2/2 \right\}_{(7)}^{-1},$$

A and C — values of coefficients at the second degree of wave vector and first degree of frequency in Lindhard function expansion (see, for example, [9]) accordingly.

## 4. State equations

Possible spin configurations and related volume effects may be produced by minimizing (9) simultaneously by local magnetization and volume. At the same time we get that magnetic state equation

$$M_{\mathbf{q}_{0}}^{(\gamma)} \left( D^{-1}(V) + \kappa(V/V_{0}) \left( 1 + x(1-x)(2U)^{-1} \right) \times \left( U_{\mathrm{Co}} - U_{\mathrm{Fe}} \right) \sum_{\mathbf{q}=\pm\mathbf{q}_{0}} |M_{\mathbf{q}}^{(\gamma)}|^{2} + X(\mathbf{q}_{0},0) \right) = 2h_{\mathbf{q}_{0},\gamma}/U$$
(8)

is to be supplemented by ratio for equilibrium volume  $(V = V_0 + \Delta V)$ , related to magnetovolume effect

$$\Delta V(T)/V_0 = K^{-1} U^{-1} \Big( \mathbf{M}_{\mathbf{q}_0}^2 + (2U)^{-2} (U_{\rm Co} - U_{\rm Fe})^2 \\ \times \sum_{q'} \langle \delta p_{q_0 + q'}^2 \rangle |M_{q'}|^2 + \langle m^2 \rangle_T \Big).$$
(9)

Solutions of equations (8), (9) take into account interdependences of volume and magnetization. In the area of temperature values  $T < T_c$  and volumes  $V < V_c$ , for which the condition  $D^{-1} < -5d\Theta/(4AU^2)$  is met, left and right chiral spin spirals are possible

$$M_{\nu}^{(\pm)} = \pm M_S \exp(\mathbf{q}_0 \nu), \qquad (10a)$$

besides

$$\Delta V(T)/V_0 = K^{-1} U^{-1} (M_S^2(T, V) + \langle m^2 \rangle_T), \qquad (10b)$$

$$M_{S}(T,V)^{2} = (2\kappa(T,V))^{-1} ((D^{-1}(T,V) + X_{q})^{2} - (d|\mathbf{q}_{0}|/U)^{2}).$$
(11)

At  $0 > D^{-1} > -5d\Theta/(4AU^2)$ , in case of negative values of mode-mode parameter  $R_C \sim \chi (U + x(1-x) \times (2)^{-1}(U_{\rm Co} - U_{\rm Fe}))$  the spin spiral fluctuations occur, the



**Figure 1.** Densities of electron states of  $Fe_{1-x}Co_xSi$  alloys, calculated in method LDA + U + SO. Chemical potential position matches the start of energy count. Density of *sp*-electron states is tripled. Parameters of Hubbard interaction were calculated in approximation of virtual crystal:  $U = (1 - x)U_{Fe} + xU_{Co}$ ,  $U_{Co} = 2.4 \text{ eV}$ ,  $U_{Fe} = 1.2 \text{ eV}$ , x — cobalt concentration.

initial phases of which are determined by Berry phase differences:

$$M_{\nu}^{(\pm)} = \pm M_S \exp(\mathbf{q}_0 \nu + \varphi), \qquad (12a)$$

$$\Delta V/V_0 \approx (3KU/5)^{-1} \left( \langle m \rangle_T^2 - U^{-1} (U_{\rm Co} - U_{\rm Fe}) x (1-x) M_S^2 \right).$$
(12b)

At the same time the wave vector of spin spirals and fluctuations of spirals is determined by a single equation

$$|\mathbf{q}_0(V)| = d/(U^2\theta g_0(\theta \varepsilon_F)),$$

and is weakly dependent on the volume and temperature.

## 5. Results of numerical analysis

2

For numerical analysis of magnetic phase transitions based on the conditions of free energy minimization we shall use the results of first-principles LDA + U + SO calculations DOS  $Fe_{1-x}Co_xSi$ . In Fig. 1 these results are given for x = 0.3, 0.4, 0.5 and 0.6, calculated at atmospheric pressure. Dependences of chemical potential of compositions with various *x* on temperature and volume are determined from the conditions of electric neutrality (which for *d*-electrons meets the saddle-point condition by charge variable  $\eta_0$ ) with account of calculations of DOS *s*-, *p*- and *d*-electrons

$$N = \int d\varepsilon g^{(s,p)}(\varepsilon) f_F(\varepsilon - \mu) + \theta \sum_{\alpha} \int d\varepsilon g^{(0)}(\varepsilon + \alpha \theta \langle m \rangle_T) f_F(\varepsilon - \mu).$$
(13)

Temperature variation of magnetization at phase transitions extended by temperature in alloys  $Fe_{1-x}Co_xSi$ with 0.2 < x < 0.6 together with the results of magnetic contribution calculations to VCTE is given on inserts to Fig. 2, 3 and 4. Besides, it is shown that magnetovolume effects and thermal expansion result in increase of  $T_s$ and temperature interval of short-range order with nonzero local magnetization. Influence of volume effects here manifests itself through dependence on the volume of exchange interaction factor D(T, V), which near the point of disappearance of local magnetization  $(T_s)$  must be abnormally strong, whereas the influence of volume at  $T_c$ and  $R_c$  may be neglected.

Magnetovolume effects arising in the field of both shortand long-range orders result in magnetic contribution to VCTE of alloys  $Fe_{1-x}Co_xSi$ , and determine temperature variation of nonframework VCTE on the background of smooth (approximately to law  $T^3$ ) temperature variation of framework component. Magnet-electron contribution to VCTE:  $\beta = \partial \omega / \partial T = \beta_{el} + \beta_{mag}$ . Besides, magnetic contribution is determined by local magnetization  $M_s$  and mean square magnetic torque

$$\langle M^2 \rangle = \langle m^2 \rangle_T + \frac{U_{\rm Co} - U_{\rm Fe}}{U} x (1-x) M_S^2.$$

In the area of long-range order  $(T < T_c)$ 

$$\beta_m = 10(3K)^{-1}U(\partial M_S^2/\partial T)(2\langle m \rangle_T^2 + M_S^2), \qquad (14a)$$



**Figure 2.** Temperature dependence of VCTE in alloy  $Fe_{0.7}Co_{0.3}Si: I - nonframework contribution to VCTE produced in [11] after processing of experimental data, <math>2 - calculation in this paper$ . On the insert: temperature dependence of magnetization.



**Figure 3.** Temperature dependence of VCTE in alloy Fe<sub>0.5</sub>Co<sub>0.5</sub>Si: I — nonframework contribution to VCTE produced in [11] after processing of experimental data, 2 — calculation in this paper. On the insert: n — temperature dependence of magnetization.



**Figure 4.** Results of modeling temperature dependence of VCTE in alloys  $Fe_{1-x}Co_xSi \ x = 0.4$  (axis on the left), 0.6 (axis on the right). On the insert: temperature dependence of magnetization.

and in the interval of temperatures of phase transition is determined by the equation

$$\beta_m = 10(3K)^{-1}Ux(1-x)$$
$$\times \left( (\partial M_S^2/\partial T) \langle M^2 \rangle + AR_C^{-1}(\partial M_S^2/\partial T) \right), \quad (14b)$$

electron contribution to VCTE, related to Fermi excitation

$$egin{aligned} eta_{el} &= \ - \, rac{5}{3K} \, T^{-2} \sum_{lpha = \pm 1} \int g_0(arepsilon) (arepsilon - \mu - lpha Um)^2 \ & imes f'(arepsilon - \mu - lpha Um) darepsilon pprox rac{5}{3K} \, T g_0(\mu), \end{aligned}$$

turns out to be negligibly low. Results of calculations of temperature dependence of VCTE compared to experimental data are presented in Fig. 2, 3 and 4. It also includes calculated values of temperatures  $T_c$  and  $T_s$ . The conducted numeric analysis shows that the negative contribution to VCTE observed in the experiment is increased by the module up to temperature  $T_c$  (see (14a)). Then as a result of sharp (but continuous!) change of temperature dependence of local magnetization (inserts to Fig. 2, 3, 4) and sign of mode-mode parameter, the mechanism of VCTE temperature dependence is implemented related to fluctuation long-range order (14b) (Fig. 2, 3, 4). As a result of disappearance of local magnetization in point  $T_s$ , in paramagnetic zone VCTE sign turns out to be positive.

## 6. Conclusion

Therefore, change of the parameter sign of mode-mode interaction and fluctuation of Hubbard potentials in chiral ferromagnetic quasibinary alloys with B20 structure result in phase transitions extended by temperature, when selfconsistent change of local magnetization and volume takes place (magnetovolume effect). Besides, if in the area of long-range order the magnetovolume effect is determined by amplitude of helicoid spin spirals, in the area of short-range order this effect is related to amplitude of spin spiral fluctuations (which are implemented in space areas of spin correlation radius  $R_c$ ). Value of magnetovolume effect is one of factors determining upper border ( $T_s$ ) of temperature interval of chiral short-range order.

The reason for negative VCTEs are magnetovolume effects. Therefore, negative VCTEs occur not only in the field of helicoid ferromagnetic alignment, but in the conditions of unstable ferromagnetism (negative mode-mode parameter), when vortex spin microstructures occur. Same as in invar ferromagnets (such as nickel-iron alloys [12]), volume instability of microstructures arises here, and magnetovolume effects turn out to be provided by fluctuations of Hubbard potentials of the intra-atomic electron-electron repulsion. At the same time the impact of volume effects at magnetic characteristics, in contrast to invar alloys, in topologically protected microstructures is weak, except for temperatures close to  $T_s$ , when contribution to free energy related to DM-interaction and local magnetization disappear.

Study of volume effects and spin skyrmions with extended magnetic phase transitions in alloys based on monogermanides of transition metals (for example, FeGe [13]) is of special interest, which, as it is known, may possess significantly higher (compared to solid solutions of iron, cobalt and manganese mono-silicides) values of Curie–Neel temperatures. However, for such systems the study of long-range order nature requires separate consideration, since the need occurs to study the picture of spin spirals twisting [14], being the cause for higher values  $T_c$  in helicoid ferromagnets.

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#### **Conflict of interest**

The authors declare that they have no conflict of interest.

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