# Exchange interaction in LaCoO<sub>3</sub> between cobalt ions $Co^{3+}$ in different spin states

### © S.I. Polukeev, V.A. Gavrichkov, S.G. Ovchinnikov

Kirensky Institute of Physics, Federal Research Center KSC SB, Russian Academy of Sciences, Krasnoyarsk, Russia E-mail: psi@iph.krasn.ru

Received July 7, 2022 Revised July 7, 2022 Accepted July 10, 2022

The values and signs of the contributions to the superexchange interaction of  $Co^{3+}$  ions in LaCoO<sub>3</sub> in different spin states, namely, the superexchange of a pair of ions in the intermediate (IS) state and the superexchange of a pair of ions, one of which is in the intermediate state and the other in the high-spin (HS) state, are studied. For this purpose, virtual electron-hole pairs produced in the course of superexchange (the so-called exchange loops) have been studied within the framework of a multiband generalization of the Hubbard model. It is shown that the cobalt ions in the intermediate state are ordered ferromagnetically, while a pair of ions in different HS and IS states makes an AFM contribution to the superexchange.

Keywords: superexchange interaction, magnetic structure, Hubbard model, projection operator method, exchange loop.

DOI: 10.21883/PSS.2022.12.54381.426

## 1. Introduction

Unusual properties of undoped  $LaCoO_3$  with  $Co^{3+}$  ions in the low-spin (LS) state  ${}^{1}A_{1}$  and thermally induced spin states  ${}^{3}T_{1}$  and  ${}^{5}T_{2}$  are of great interest [1–3]. In the experiment, two blurred transitions are observed: from the diamagnetic to the paramagnetic state at  $T_S \approx 120 \,\mathrm{K}$  and the dielectric-metal transition with a further increase in temperature to  $T_{IM} \approx 530 \,\mathrm{K}$  [4–7]. The LS-state existing at low temperatures is characterized by zero spin on the Co<sup>3+</sup> ion and the absence of magnetic moments in the bulk sample. In this state, the LaCoO<sub>3</sub> crystal is a nonmagnetic dielectric [8]. The magnetic contributions detected at temperatures below 35 K are associated with the presence of impurities [6] and crystal lattice [2,9] defects. An increase in temperature to 100 K leads to a transition to a paramagnetic state. There are many works linking this transition with the occupation of medium-spin (IS) [10–15] and high-spin (HS) [16-21] states, but it is still not clear which of them are responsible for paramagnetism. the same time, the energy of IS-states is much higher, and their thermal occupation is unlikely, and HS-states can be populated at finite temperatures or with optical excitation [22]. The study of the dependence of magnetization on temperature leads to the conclusion that there is an antiferromagnetic (AFM) exchange interaction [16]. However, there is experimental evidence of the presence of ferromagnetic order in polycrystalline samples [23], on the surface of single crystals [24] and in thin films LaCoO<sub>3</sub> with a stretchable substrate [25-31].

The magnetic structure of the deformed  $LaCoO_3$  was considered in [32] using density functional theory. It

has been shown that during stretching, competition arises between the ferromagnetic (FM) exchange between the nearest neighbors (the so-called *nn*-bonds) and a stronger AFM interaction between the second neighbors. The study of thin films LaCoO<sub>3</sub> by numerical simulation [33] has shown that the ferromagnetic ordering in such films is due to effective super-exchange interactions between atoms in HS-states, each of which can be considered as a connected pair of two IS-excitons making virtual jumps to neighboring nodes. These fluctuations mediate HS–HS interactions beyond the *nn*-connections.

In the study [8], the interatomic exchange interaction between HS-states in LaCoO<sub>3</sub> was investigated within the framework of a multielectronic approach that allows us to represent the complete exchange interaction as the sum of partial contributions from all the main and excited cation terms [34-36]. This approach is a generalization of the projection operator method for computing the Anderson super-exchange interaction [37]. As a result, an expression was derived for the super-exchange interaction between two Co<sup>3+</sup> ions in excited HS-states, which has the form of the sum of FM and AFM contributions, and the resulting magnitude and sign of the super-exchange depend on the relationship between the intra-atomic Hund interaction  $J_H$ and the effective Hubbard parameter  $U_{eff}$  [8]. The question of the magnitude and sign of the exchange interaction between Co<sup>3+</sup> ions in IS-states has not been considered. In this paper, we generalized the results of [8] to the exchange interaction in LaCoO<sub>3</sub> between cobalt ions Co<sup>3+</sup> in IS-states, as well as to the inhomogeneous case when one of the interacting ions are in the IS- state, and the other ---is in the HS-state.



**Figure 1.** Scheme of neutral, hole and electron terms of  $Co^{3+}$  ions. Two exchange loops  $J_{2T_2, 4T_1}^{FM}(a)$  and  $J_{2T_2, 2T_1}^{AFM}(b)$ , giving the main contribution to the super-exchange interaction for IS-states in LaCoO<sub>3</sub>.

# 2. Exchange interaction in LaCoO<sub>3</sub> between cobalt ions Co<sup>3+</sup> in intermediate IS-state

The calculation of super-exchange interaction is based on a multi-band generalization of the Hubbard model. With the help of a unitary transformation, a second-order perturbation theory is constructed, where interatomic jumps of electrons of an interband nature are distinguished, giving an expression of the super-exchange Hamiltonian [34-37]. This is done using the projection operator method developed in [37] for the Hubbard single-band model and generalized to the case of an arbitrary spectrum of virtual excitations [34-36]. In the standard Hubbard model with one electron per atom, the super-exchange interaction is formed as a result of the birth and subsequent destruction of virtual electron-hole pairs. At the initial moment, two ions are in the configuration  $d^1$  (let us call them electroneutral). As a result of the birth of an electron-hole pair, virtual hole terms  $d^0$  and electronic terms  $d^2$  appear. In the multi-band case, the principle is the same, but the neutral ion  $d^n$  with the number of electrons  $n_0 = 6$  for  $Co^{3+}$ , as well as the hole c  $n_h = n_0 - 1$  and the electron  $d^{n+1}$  with  $n_e = n_0 + 1$ can be in various multiplet states. The representation of the X-Hubbard operators makes it possible to analyze partial contributions to the exchange from various neutral ion terms, both basic and excited. The resulting Hamiltonian of the super-exchange interaction

$$\hat{H}_{S} = -\sum_{i \neq j} J_{ij}^{tot} \left( \hat{S}_{in_{0}} \hat{S}_{jn_{0}} - \frac{1}{4} \hat{n}_{in_{0}}^{(h)} \hat{n}_{jn_{0}}^{(e)} \right), \tag{1}$$

where  $J_{ij}^{tot}$  — full node pair exchange,  $\hat{S}_{in_0}$  — spin operator on *i*-m node,  $\hat{n}_{in_0}^{(h,e)}$  — quasiparticle number operators for hole (*h*) and electron (*e*) terms, is the sum of contributions from all ground and excited states, each of which can be graphically represented as a virtual electronhole pair or a so-called exchange loop [38]. The sign of each contribution is determined by the ratio of the spins  $S(d^{n+1})$  and  $S(d^{n-1})$  of the electron and hole terms if  $S(d^{n-1}) = S(d^{n+1})$ , then the interaction is antiferromagnetic. When  $S(d^{n-1}) = S(d^{n+1}) \pm 1$ , then a ferromagnetic exchange is formed. The main contribution to the super-exchange IS of states in LaCoO<sub>3</sub> is given by two exchange loops of the opposite sign, shown in Fig. 1.

According to Fig. 1, *a* the ferromagnetic contribution to the super-exchange is given by the exchange loop  $J_{2T_2,4T_1}^{FM}$ . The value of this contribution is equal to

$$J_{2T_{2,}4T_{1}}^{FM} = \frac{1}{(2S_{h}+1)} \frac{1}{(2S_{n_{0}}+1)} \frac{2t^{2}}{(^{2}T2, \,^{4}T_{1})}, \qquad (2)$$

where  $S_h, S_{n0}$  — the spins of the neutral and hole terms, t — the hopping integral, and  $\Delta = E(d^5) + E(d^7)$  $-2E(d^6)$  — the energy denominator, similar to the parameter U in the Hubbard model. The energies of the hole, electroneutral and electron terms participating in this exchange loop are equal

$$E(d^5) = -20Dq - 4J_H + 10U, (3)$$

$$E(d^6) = -14Dq - 7J_H + 15U, (4)$$

$$E(d^7) = -8Dq - 11J_H + 21U, (5)$$

where  $J_H$  and Dq are the parameters of the Hund interaction and the crystal field, respectively. Hence  $\Delta = U - J_H$ . Thus, the contribution from the ferromagnetic loop to the super exchange is equal to

$$J_{2T_{2},4T_{1}}^{FM} = \frac{1}{2} \frac{1}{3} \frac{2t^{2}}{U - J_{H}}.$$
 (6)

Similarly for the antiferromagnetic exchange loop  $J_{2T_2,2T_1}^{ATM}$ , shown in Fig. 2, *b*, the energy of the terms forming it and the energy denominator are equal

$$E(d^5) = -20Dq - 4J_H + 10U, (7)$$

Physics of the Solid State, 2022, Vol. 64, No. 12



**Figure 2.** Exchange loops that make up the main contribution to the exchange between  $\text{Co}^{3+}$  ions in HS- and IS-states: loop  $J_{2T_2, 4T_2}^{AFM}$ , which gives a contribution to the exchange in the case when the HS-state is *j*-th ion (*a*), and two loops  $J_{4T_1, 4T_1}^{FM}$ ,  $J_{4T_1, 2T_1}^{AFM}$ , giving the main contribution to the exchange when the HS-state is *i*-th ion (*b*).

$$E(d^{6}) = -14Dq - 7J_{H} + 15U, \qquad (8)$$

$$E(d^7) = -8Dq - 9J_H + 21U, (9)$$

$$\Delta = U + J_H,\tag{10}$$

where does the value of the AFM contribution to the super exchange come from

$$J_{2_{T_2},2_{T_1}}^{AFM} = -\frac{1}{2} \frac{1}{3} \frac{2t^2}{U + J_H}.$$
 (11)

Summing up the expressions obtained, we obtain the total contribution from the IS-states to the complete superexchange interaction of  $Co^{3+}$  ions, which is ferromagnetic:

$$J_{tot}^{FM} = \frac{t^2}{3} \left\{ \frac{2J_H}{(U^2 - J^2)} \right\} = \frac{2}{3} \frac{4}{100} \frac{1}{16 - 1}$$
$$= 0.002 \,\text{eV} = 2 \,\text{meV}. \tag{12}$$

The following parameter values characteristic of transition metal oxides were used for numerical evaluation:  $J_H = 1 \text{ eV}$ , U = 4 eV, t = 0.2 eV. It is important that the total ferromagnetic contribution is obtained for any parameter values, since  $U > J_H$ .

# Exchange interaction in LaCoO<sub>3</sub> between cobalt ions Co<sup>3+</sup> in intermediate ISand high spin HS-states

A pair of Co<sup>3+</sup> ions, one of which is in the high-spin HS-, and the other — in the intermediate IS-states, are connected by a super-exchange consisting of three exchange loops: two antiferromagnetic  $J_{^{2}T_{2},^{4}T_{1}}^{AFM}$ ,  $J_{^{4}T_{1},^{2}T_{1}}^{AFM}$  and one ferromagnetic  $J_{^{4}T_{1},^{4}T_{1}}^{FM}$  (Fig. 2). The situation in this case depends on which of the nodes

The situation in this case depends on which of the nodes goes into a state with an additional electron, and which — into a state with an additional hole. There is one exchange

loop  $J_{2T_2, 4T_2}^{AFM}$ , corresponding to the birth of a virtual electron on a high-spin ion and a hole on an ion with an intermediate spin (see Fig. 2, *a*). To calculate the exchange value, it is necessary to take into account the energies of the HS- and IS-states:

$$E(d^5) = -20Dq - 4J_H + 10U, (13)$$

$$E_{IS}(d^6) = -14Dq - 7J_H + 15U, (14)$$

$$E_{HS}(d^6) = -4Dq - 10J_H + 15U, \tag{15}$$

$$E(d^7) = 2Dq - 11J_H + 21U, (16)$$

$$\Delta = U + J_H,\tag{17}$$

$$J_{2T_2,4T_2}^{AFM} = -\frac{1}{2} \frac{1}{5} \frac{2t^2}{U+2J_h}.$$
 (18)

The birth of an electron on the IS-ion and holes on the HS-ion are answered by two exchange loops of the opposite sign  $J_{4T_{1},2T_{1}}^{AFM}$  and  $J_{4T_{1},4T_{1}}^{FM}$  (Fig. 2, *b*). A similar calculation gives the following values for these loops:

$$J_{{}^{4}T_{1},{}^{2}T_{1}}^{4FM} = -\frac{1}{4} \frac{1}{3} \frac{2t^{2}}{U+2J_{h}},$$
(19)

$$J_{{}^{FM}_{T_1,{}^{4}T_1}}^{FM} = \frac{1}{4} \frac{1}{3} \frac{2t^2}{U}.$$
 (20)

The total value of the exchange interaction is equal to the sum of the contributions from the three exchange loops and has an antiferromagnetic sign

$$J_{tot}^{AFM} = J_{2T_2, 4T_2}^{AFM} + J_{4T_1, 2T_1}^{AFM} + J_{4T_1, 4T_1}^{FM}$$
  
\$\sim -3 \cdot 10^{-4} eV = -0.3 meV. (21)

## 4. Conclusion

Thus, cobalt ions  $Co^{3+}$  in the same IS-states are bound by the FM super-exchange  $J_{tot}^{FM} = 2 \text{ MeV}$ , and ions in different IS- and HS-states — AFM interaction  $J_{tot}^{AFM} = 0.5 \text{ MeV}$ , however, the latter is inferior to the first in magnitude, and the ratio between them does not depend on the magnitude of the crystal field 10Dq. Taking into account the results of the work [8] on the AFM interaction between Co<sup>2+</sup> ions in the same HS-states, we identify the observed FM interaction in some cobaltites as signs of the presence of cobalt ions Co<sup>3+</sup> in the intermediate spin state of IS.

#### Funding

The work was carried out with the financial support of the Russian Foundation for Basic Research (project No. 20-42-240016).

#### Conflict of interest

The authors declare that they have no conflict of interest.

## References

- [1] J.B. Goodenough. Prog. Solid State Chem. 5, 145 (1971).
- [2] M.A. Senaris-Rodriguez, J.B. Goodenough. J. Solid State Chem. 116, 2, 224 (1995).
- [3] N.B. Ivanova, S.G. Ovchinnikov, M.M. Korshunov, I.M. Eremin, N.V. Kazak. UFN 179, 8, 837 (2009). (in Russian).
- [4] S. Yamaguchi, Y. Okimoto, H. Taniguchi, Y. Tokura. Phys. Rev. B 53, 2926R (1996).
- [5] S. Tajima, A. Masaki, S. Uchida, T. Matsuura, K. Fueki, S. Sugai. J. Phys. C 20, 3469 (1987).
- [6] S. Yamaguchi, Y. Okimoto, Y. Tokura. Phys. Rev. B 55, 14, 8666R (1997).
- [7] A. Ishikawa, J. Nohara, S. Sugai. Phys. Rev. Lett. 93, 136401 (2004).
- [8] Yu.S. Orlov, S.V. Nikolaev, V.A. Gavrichkov, S.G. Ovchinnikov. Comput. Mater. Sci. 204, 111134 (2022).
- [9] S. Zhou, L. Shi, J. Zhao, L. He, H. Yang, S. Zhang. Phys. Rev. B 76, 17, 172407 (2007).
- [10] K. Asai, A. Yoneda, O. Yokokura, J. Tranquada, G. Shirane, K. Kohn. J. Phys. Soc. Jpn. 67, 1, 290 (1998).
- [11] T. Saitoh, T. Mizokawa, A. Fujimori, M. Abbate, Y. Takeda, M. Takano. Phys. Rev. B 55, 4257 (1997).
- [12] R.F. Klie, J.C. Zheng, Y. Zhu, M. Varela, J. Wu, C. Leighton. Phys. Rev. Lett. 99, 047203 (2007).
- [13] C. Zobel, M. Kriener, D. Bruns, J. Baier, M. Grüninger, T. Lorenz, P. Reutler, A. Revcolevschi. Phys. Rev. B 66, 020402 (2002).
- [14] C. Zobel, M. Kriener, D. Bruns, J. Baier, M. Grüninger, T. Lorenz, P. Reutler, A. Revcolevschi. Phys. Rev. B 71, 019902 (2005).
- [15] P.G. Radaelli, S.-W. Cheong. Phys. Rev. B 66, 094408 (2002).
- [16] M.J.R. Hoch, S. Nellutla, J. van Tol, E.S. Choi, J. Lu, H. Zheng, J.F. Mitchell. Phys. Rev. B 79, 21, 214421 (2009).
- [17] S. Noguchi, S. Kawamata, K. Okuda, H. Nojiri, M. Motokawa. Phys. Rev. B 66, 9, 094404 (2002).
- [18] M.W. Haverkort, Z. Hu, J.C. Cezar, T. Burnus, H. Hartmann, M. Reuther, C. Zobel, T. Lorenz, A. Tanaka, N.B. Brookes, H.H. Hsieh, H.-J. Lin, C.T. Chen, L.H. Tjeng. Phys. Rev. Lett. 97, 176405 (2006).

- [19] A. Podlesnyak, S. Streule, J. Mesot, M. Medarde, E. Pomjakushina, K. Conder, A. Tanaka, M.W. Haverkort, D.I. Khomskii. Phys. Rev. Lett. 97, 247208 (2006).
- [20] A. Podlesnyak, K. Conder, E. Pomjakushina, A. Mirmelstein, P. Allenspach, D. Khomskii. J. Magn. Magn. Mater. 310, 2, 1552 (2007).
- [21] M. Rotter, Z.-S. Wang, A.T. Boothroyd, D. Prabhakaran, A. Tanaka, M. Doerr. Sci. Rep. 4, 7003 (2014).
- [22] T. Ishikawa, K. Yamada, H. Itoh, S. Iwai, T. Arima, S. Yamada, T. Sasaki. EPJ Web Conf. 41, 03013 (2013).
- [23] J. Androulakis, N. Katsarakis, J. Giapintzakis. Phys. Rev. B 64, 17, 174401 (2001).
- [24] J.-Q. Yan, J.-S. Zhou, J.B. Goodenough. Phys. Rev. B 70, 1, 014402 (2004).
- [25] A. Harada, T. Taniyama, Y. Takeuchi, T. Sato, T. Kyomen, M. Itoh. Phys. Rev. B 75, 18, 184426 (2007).
- [26] D. Fuchs, C. Pinta, T. Schwarz, P. Schweiss, P. Nagel, S. Schuppler, R. Schneider, M. Merz, G. Roth, H.V. Lohneysen. Phys. Rev. B 75, 14, 144402 (2007).
- [27] A. Herklotz, A.D. Rata, L. Schultz, K. Dorr. Phys. Rev. B 79, 9, 092409 (2009).
- [28] V.V. Mehta, M. Liberati, F.J. Wong, R.V. Chopdekar, E. Arenholz, Y. Suzuki. J. Appl. Phys. **105**, *7*, 07E503 (2009).
- [29] N. Biskup, J. Salafranca, V. Mehta, M.P. Oxley, Y. Suzuki, S.J. Pennycook, S.T. Pantelides, M. Varela. Phys. Rev. Lett. 112, 8, 087202 (2014).
- [30] H. Liu, L. Shi, Y. Guo, S. Zhou, J. Zhao, C. Wang, L. He, Y. Li. J. Alloys Compd. 594, 158 (2014).
- [31] Q. Feng, D. Meng, H. Zhou, G. Liang, Z. Cui, H. Huang, J. Wang, J. Guo, C. Ma, X. Zhai, Q. Lu, Y. Lu. Phys. Rev. Mater. 3, 7, 074406 (2019).
- [32] H. Seo, A. Posadas, A.A. Demkov. Phys. Rev. B 86, 014430 (2012).
- [33] A. Sotnikov, K.-H. Ahn, J. Kunes. Sci. Post. Phys. 8, 082, 1 (2020).
- [34] V.A. Gavrichkov, S.I. Polukeev, S.G. Ovchinnikov. Phys. Rev. B 95, 14, 144424 (2017).
- [35] V.A. Gavrichkov, S.I. Polukeev, S.G. Ovchinnikov. ZHETF (in Russian) 154, 4, 835 (2018).
- [36] R.V. Mikhaylovskiy, T.J. Huisman, V.A. Gavrichkov, S.I. Polukeev, S.G. Ovchinnikov, D. Afanasiev, R.V. Pisarev, T. Rasing, A.V. Kimel. Phys. Rev. Lett. **125**, *15*, 157201 (2020).
- [37] K.A. Chao, J. Spalek, A.M. Oles. J. Phys. C 10, 10, L271 (1977).
- [38] V.A. Gavrichkov, S.I. Polukeev, S.G. Ovchinnikov. Phys. Rev. B 101, 10, 094409 (2020).