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Estimation of possibilities of calculations based on simplified models of the leak location in gas pipelines

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For gas transportation through pipes in normal and emergency modes, a comparison of calculations based on mathematical models of various degrees of generality is presented. A mathematical model of a non-isothermal steady flow of a mixture of gases and its simplified versions are studied. For simplified options, simple analytical dependencies were obtained for calculating flow characteristics and calculating the location of an emergency gas leak. Examples of calculations of pressure distributions, temperature and leakage coordinates in gas pipelines of medium pressures according to the general and simplified models are provided. The examples cover the parameter change area of practical interest. The conditions for the admissibility of using simplified models for calculating the coordinates of a leak of medium intensity and different locations are determined.

Keywords: gas pipelines, adequacy of the model, simplifications, calculation of the place of emergency leakage, compressibility factor.

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Introduction

Numerous publications have been devoted to calculations of the leak location in gas pipelines and wells based on a mathematical model of gas flow, starting with the fundamental work of Vasiliev, Bondarev, Voevodin and Kanibolotsky [1], up to the present, for example, papers [2–5]. Success in solving this problem is impossible without creating an adequate mathematical model of gas flow under the conditions under study. Despite the availability of well-known software systems (OLGA, PIPESIM, a number of domestic software systems, for example, SINF), it is premature to consider as completed the creation of the adequate mathematical model and the program for calculating gas transportation in the general case. The reason for this is as follows. The adequacy of the mathematical model is ensured by a reasonable choice in the problem under study of such hard-to-determine variables as the hydraulic resistance coefficient λ and the total heat transfer coefficient β. The universal method for calculating λ and β under actual conditions is the method of identifying these parameters from experimental data [1]. The above software systems do not include the method of identifying the λ and β parameters. For the adequacy of the mathematical model of gas transportation, the choice of the equation of the gas mixture state in the studied range of pressure p and temperature T changes is also of great importance. Many universal equations of state are known, for example, the American Gas Association equation recommended in [6]. However, the cumbersomeness of this universal dependence of the compressibility factor $Z(p, T)$ calls into question the expediency of its use in problems where the range of p and T changes is obviously limited.

For main and offshore gas pipelines our book [7] provides a solution to the problem of identifying the parameters λ and β, based on the iterative Bellman quasilinearization method [8]. Besides, in book [7] the effect of errors in setting these parameters on the flow characteristics was studied. In paper [9] a method for plotting the dependence $Z(p, T)$ from experimental data is proposed, and its efficiency is demonstrated in the region of superhigh pressures. Paper [9] also presents a solution of the problem of calculating the coordinates of a stationary gas leak by iteration using quasilinearization for superhigh pressures (under the assumption of a stable flow pattern both before the leak and some time after its occurrence). The solution of the similar problem for medium pressures is given in [5]. In this paper we consider the admissibility of using relatively simple models for calculating the coordinate of the stationary leak in gas pipelines, the pressure in which does not exceed 100 atm. As it is well known [1], at such pressures the Berthelot equation has proven itself well for calculating the compressibility coefficient.

1. Mathematical model of non-isothermal stable flow of mixture of gases through a pipeline of constant circular cross-section [1,7]

$$\rho u S = Q^0, \tag{1}$$

$$\frac{d}{dz}\left(p + \rho u^2\right) = -\frac{\lambda \rho u |u|}{4R} + \rho g \cos\alpha(z), \tag{2}$$

$$\rho u c_p \frac{dT}{dz} = \rho u T \left(\frac{\partial(1/\rho)}{\partial T}\right)_p \frac{dp}{dz} + \frac{\lambda \rho u^2 |u|}{4R} - 2\beta \frac{T - T^*}{R}, \tag{3}$$

$$pV = R_g T Z(p, T),$$

$$Z(p, T) = 1 + 0.07 \frac{p}{p_c} \frac{T_c}{T} \left(1 - 6 \frac{T_c^2}{T^2}\right), \tag{4}$$

$$V = 1/\rho.$$

In the system of equations (1)−(4) V is specific volume of gas; R_g, p_c, T_c are gas constant, critical pressure and temperature of the mixture of gases of a given composition; z, L are coordinate along the pipeline axis and its length; ρ, p, T, u are density, pressure, temperature and gas velocity, respectively; R, S are internal radius and cross-sectional area of the gas pipeline; g is free fall acceleration; $\alpha(z)$ is angle between gravity direction and gas pipeline axis in z-th section; λ is hydraulic resistance coefficient; β is total heat transfer coefficient; c_p is specific heat of gas mixture at constant pressure;

In the general case, the variables λ, β, c_p, and T^* can be functions of z coordinate, in particular, they can depend on the pressure and temperature of the gas, on the design parameters of the gas pipeline, and on external conditions. The adequacy of the stationary mathematical model (1)−(4), as noted, depends on the choice of the variables λ, β, c_p and on the choice of the type of dependence $Z(p, T)$ of the compressibility coefficient in the investigated range of pressure, temperature and flow changes. The system of equations (1)−(4) is supplemented by the boundary condition

$$z = 0: \quad p = P(0), \quad T = T(0), \tag{5}$$

where $P(0)$, $T(0)$ are dimensional pressure and temperature at the gas pipeline inlet.

The solution of the system of equations (1)−(4) under the boundary condition (5) exists and is unique in a wide range of Q^0, $P(0)$, $T(0)$, R and L. The numerical solution can be obtained with high accuracy, for example, by the Runge−Kutta method. The solution of the problem of identifying the parameters λ and β from experimental data on the pressure and flow rate at the outlet of the gas pipeline section, within which these variables can be considered constant, is given in [7].

2. Calculation of value $c_p(p, T)$

The specific heat c_p of a gas mixture at constant pressure can be calculated using the well-known formula [10]:

$$c_p(p, T) = c_p^0(T) - T \int_{p^0}^{p} \left(\partial^2 V/\partial T^2\right)_p dp. \tag{6}$$

Here $c_p^0(T)$ is temperature dependence of the specific heat capacity of the gas mixture at a constant pressure p^0, at which the mixture behaves like an ideal gas. In the temperature range from 2 to 40°C, the $c_p^0(T)$ dependence can be considered linear. The second derivative of V with respect to T and the integral in equation (6) for the Berthelot equation (4) are found analytically, resulting in the following expression for the dependence of the isobaric specific heat c_p on pressure p and temperature T:

$$c_p(p, T) = k_1 + k_2 \frac{T}{T_c} + k_3 \frac{T_c^3}{T^3} \left(\frac{p}{p_c} - \frac{p^0}{p_c}\right). \tag{7}$$

The values of the dimensional constants k_1, k_2, k_3, depending on the composition of the gas mixture, are given below (10). In the general case, the system of equations (1)−(4) must be solved taking into account the dependence $c_p(p, T)$ (7). When solving problems in which the range of pressure and temperature changes is not very large, one can restrict oneself to the mean integral variable $\langle c_p \rangle$ defined by the equality:

$$\langle c_p \rangle = \frac{1}{(P(0) - P_L)(T(0) - T_L)} \int_{T_L}^{T(0)} \int_{P_L}^{P(0)} c_p(p, T) dp dT. \tag{8}$$

In terms of dimensionless quantities $T_0 = T(0)/T_c$, $T_1 = T_L/T_c$, $p_0 = P(0)/p_c$, $p_L = P_L/p_c$, $p_c^0 = p^0/p_c$ the mean value of $\langle c_p \rangle$ calculated by formula (8) is equal to

$$\langle c_p \rangle = k_1 + k_2 \left(\frac{T_0 + T_1}{2}\right) + k_3 \frac{(T_0 + T_1)/2}{T_0^2 T_1^2} \left(\frac{p_0 + p_L}{2} - p_c^0\right). \tag{9}$$

In this paper all calculations were carried out for the gas mixture of 12 components with a predominance of methane; the composition and parameters of the mixture are given in book [7]. For this mixture of gases, the critical parameters and variables k_1, k_2, k_3 are:

$$k_1 = 1803.7\,\text{J/(kg}\cdot\text{K)}, \quad k_2 = (1.5 T_c)\,\text{J/(kg}\cdot\text{K)},$$

$$k_3 = 2.52 R_g\,\text{J/(kg}\cdot\text{K)},$$

$$T_c = 193.698\,\text{K}, \quad p_c = 4.5978\,\text{MPa},$$

$$R_g = 493.501\,\text{J/(kg}\cdot\text{K)}. \tag{10}$$

Expression (9) for the variable $\langle c_p \rangle$ includes the values of temperature and pressure at the outlet. These variables either are measured experimentally or can be calculated using the system of equations (1)−(4) with boundary condition (5). In the second case, the variable $\langle c_p \rangle$ is calculated iteratively. In a zero approximation a reasonable value $\langle c_p \rangle^{(0)}$ is given, from the solution of the system of equations (1)−(4) the variables $P_L^{(0)}$ and $T_1^{(0)}$ in the zero approximation are determined, and by formula (9) the value of $\langle c_p \rangle^{(1)}$ is calculated in the first iteration and etc. In the examples below three iterations were sufficient to calculate the variable $\langle c_p \rangle$ with an accuracy of 10^{-4} J/(kg · K).

Further, in all calculations, the average value $\langle c_p \rangle$, denoted as c_p, was used.

3. Simplified models

The solution of the problem of calculating the coordinates of the gas leak according to model (1)–(4) is given in [5]. The analysis of these calculations showed that there is a range of parameters Q^0, $P(0)$, $T(0)$, R, L, β, λ, T^* change, in which, with an acceptable accuracy, it is possible to calculate the pressure and temperature distributions in the gas flow using simplified models. It is essential that the simplified models lead to simple formulas for calculating the coordinates of the stationary gas leak.

About possible simplifications . For many problems the pipeline route can be considered horizontal. The inlet pressure, gas flow rate and inner radius of the gas pipeline in actual problems are chosen in such a way that the flow velocity at any point of the gas pipeline does not exceed the critical value at which the pipeline begins to vibrate. This leads to the fact that in actual problems of gas transportation, the inertial forces in the flow are by several orders of magnitude smaller than the pressure forces, and the term ρu^2 in the left side of the equation of motion can be neglected compared to pressure. Thus, under the conditions

$$\forall z \in [0, L]$$

1) horizontal route,

$$2)\ p \gg \rho u^2 \rightarrow \frac{d}{dz}(p + \rho u^2) = \frac{dp}{dz} \tag{11}$$

the equation of motion and the thermal equation of model (1)–(4) are simplified and written as follows:

$$\frac{dp}{dz} = -\frac{\lambda \rho u|u|}{4R}, \tag{12}$$

$$\frac{dT}{dz} = \frac{1}{c_p}\left(T\left(\frac{\partial V}{\partial T}\right)_p - V\right)\frac{dp}{dz} - \frac{2\beta}{R\rho u c_p}(T - T^*). \tag{13}$$

The multiplier in front of $\frac{dp}{dz}$ in equation (13) is known [10] Joule–Thomson coefficient $\mathscr{D}_*$, this allows us to represent thermal equation (13) in the form

$$\frac{dT}{dz} = \mathscr{D}_* \frac{dp}{dz} - \frac{2\beta}{R\rho u c_p}(T - T^*). \tag{14}$$

It follows from equation (14) that at $T > T^*$ at any point of the gas pipeline the temperature drop is due to two reasons: gas cooling due to pressure drop (the so-called „throttle effect"), and gas cooling as the result of heat exchange with the environment.

The right side of equation of motion (12), taking into account equations (1) and (4), can be expressed in terms of T, p and $Z(p, T)$:

$$\frac{dp}{dz} = -\left(\frac{\lambda {Q^0}^2 R_g}{4RS^2}\right)\frac{TZ(p, t)}{p}. \tag{15}$$

The simplified system of equations (1), (15), (14) and (4), which follows from general system (1)–(4) under conditions (11), is much simpler than system (1)–(4), however, it also does not admit simple analytical solutions. The situation is greatly simplified if we assume that the compressibility factor $Z(p, T)$ can be considered as a constant value:

$$Z(p, T) = \text{const} = Z^*. \tag{16}$$

Direct simplification (16), of course, remains undone for non-ideal gases. The admissibility of its use in actual problems can only be justified by the coincidence with the required accuracy of calculations of the pressure and temperature distributions, as well as the coordinates of the gas leak according to the general and simplified models. Under condition (16) the Joule–Thomson coefficient $\mathscr{D}_*$ becomes zero, which allows us to split the closed system of equations (14), (15). Thermal equation (14), when condition (16) is satisfied, is simplified

$$\frac{dT}{dz} = -\frac{2\beta}{R\rho u c_p}(T - T^*) \tag{17}$$

and is easily integrated. The integral of equation (17) is well-known [11] V.G. Shukhov's formula. When condition (16) is satisfied, the equation of motion is also integrated analytically.

Let us write the general and simplified models in dimensionless form. Let us introduce dimensionless variables by the formulas:

$$\tilde{z} = \frac{z}{L},\ \tilde{T} = \frac{T}{T_c},\ \tilde{p} = \frac{p}{p_c},\ \rho_c = \frac{p_c}{R_g T_c Z_c},$$

$$Z_c = Z(p_c, T_c) = 0.65,\ \tilde{\rho} = \frac{\rho}{\rho_c},\ \tilde{u} = \frac{u\rho_c S}{Q^0},$$

$$T_s = \frac{T^*}{T_c},\ p_0 = \frac{P(0)}{p_c},\ T_0 = \frac{T(0)}{T_c},\ p_L = \frac{P_L}{p_c}. \tag{18}$$

The wave indication in dimensionless variables is omitted wherever this does not lead to ambiguity.

For a horizontal route the general system of equations (1)–(4) and boundary condition (5) in dimensionless form have the form

Model I

$$\begin{cases} \rho u = 1, \\ \dfrac{d}{dz}\left(p + m_1 \dfrac{TZ}{p}\right) = -m_2 \dfrac{TZ}{p}, \\ \dfrac{dT}{dz} = m_3 \dfrac{T}{p}\left(Z + T\dfrac{\partial Z}{\partial T}\right)\dfrac{dp}{dz} + m_2 m_3\left(\dfrac{TZ}{p}\right)^2 - m_4(T - T_s), \\ p = \rho TZ/Z_c, \quad Z = 1 + 0.07\dfrac{p}{T} - 0.42\dfrac{p}{T^3}. \end{cases} \tag{I}$$

$$z = 0:\ p = p_0,\ T = T_0. \tag{19}$$

When conditions (11), (16) are met, model (I) is transformed into a simplified model.

Model II

$$\begin{cases} \rho u = 1, \\ \dfrac{dp}{dz} = -m_2 \dfrac{TZ^*}{p}, \\ \dfrac{dT}{dz} = -m_4(T - T_s), \\ \rho = \dfrac{pZ_c}{TZ(p, T)}. \end{cases} \qquad \text{(II)}$$

In model II it is more accurate to calculate the dimensionless density not at constant compressibility factor, but at the compressibility factor determined by Berthelot equation (4), in which the pressure and temperature values are found by integrating the simplified system of equations. The calculation of the effective constant compressibility factor Z^* is given below (23).

The dimensionless complexes included in models I and II are equal to

$$m_1 = \left(\frac{Q^0}{S}\right)^2 \frac{R_g T_c}{p_c^2}, \quad m_2 = m_1 \frac{\lambda L}{4R}, \quad m_3 = \frac{R_g}{c_p},$$

$$m_4 = \frac{2L\beta S}{Rc_p Q^0}, \quad S = \pi R^2. \qquad (20)$$

Calculation of pressure and temperature distributions by model II

The temperature distribution is determined by integrating the thermal equation of model II under boundary condition (19):

$$T(z) = T_s + (T_0 - T_s)\exp(-m_4 z). \qquad (21)$$

The pressure distribution is found as a result of integrating the equation of motion of model II with the found dependence $T(z)$ (21) under boundary condition (19):

$$p(z) = \left[p_0^2 - 2m_2 Z^* T_s z + 2\frac{m_2}{m_4} Z^* \big(T_0 - T_s\big) \times \big(\exp(-m_4 z) - 1\big)\right]^{1/2}. \qquad (22)$$

The value of the effective compressibility factor Z^* can be determined in different ways. Calculations showed that it is expedient to determine Z^* by the value of the pressure at the outlet of the gas pipeline, measured experimentally or calculated using the general model of processes. In this case Z^* is defined by the equality

$$Z^* = \frac{p_0^2 - p_L^2}{2m_2\Big(T_s - \big(T_0 - T_s\big)\big(\exp(-m_4) - 1\big)/m_4\Big)}. \qquad (23)$$

Calculation of coordinates of gas leak according to model II

In this paper, stable gas flow modes are considered, and it is assumed that some time after the start of the stationary gas leak δQ of low or medium intensity, a new stable flow mode is established in the gas pipeline, in which the place of gas leakage can be considered as a specific point in flow distribution.

Let us denote by z_a the dimensional coordinate of the cross-section, in which the gas leak occurs, by l — the dimensionless coordinate of the gas leak ($l = z_a/L$). We will divide the pipeline into two sections:

first section: $z \in [0, l]$, *second section*: $z \in [l, 1]$.

The gas flow rate in the first section is equal to the flow rate Q^0 at the gas pipeline inlet. The gas flow rate in the second section in the new stable state is equal to the outlet flow rate: $Q_L = Q^0 - \delta Q$.

The distributions of pressure and temperature in the first section are determined by dependences (21), (22). In particular, the pressure p_a and the temperature T_a at the boundary of the first section (at $z = l$) are:

$$p_a^2 = p_0^2 - 2m_2 Z^* T_s l + 2\frac{m_2}{m_4} Z^* \big(T_0 - T_s\big)\big(\exp(-m_4 l) - 1\big), \qquad (24)$$

$$T_a = T_s + (T_0 - T_s)\exp(-m_4 l). \qquad (25)$$

For the second section in the new stable state with a known flow rate Q_L at the outlet in simplified model (II) the values of the dimensionless complexes $m_2 \to \hat{m}_2$, $m_4 \to \hat{m}_4$ and the boundary condition change:

$$z = l: \quad p = p_a, \ T = T_a, \qquad (26)$$

$$\hat{m}_2 = \left(\frac{Q_L}{S}\right)^2 \frac{R_g T_c}{p_c^2} \frac{\lambda L}{4R}, \quad \hat{m}_4 = \frac{2L\beta S}{Rc_p Q_L}. \qquad (27)$$

As a result of integrating the thermal equation of model II under boundary condition (26), we find the temperature distribution in the second section:

$$z \in [l, 1]: \quad T(z) = T_s + (T_0 - T_s)\exp\big(-l(m_4 - \hat{m}_4)\big) \times \exp(-\hat{m}_4 z). \qquad (28)$$

As the result of integrating the equation of motion of model II at $T(z)$ (28) under boundary condition (26), we find the pressure distribution in the second section:

$$z \in [l, 1]: \ p(z) = \left[p_a^2 - 2\hat{m}_2 Z^* T_s (z - l) + 2\frac{\hat{m}_2}{\hat{m}_4} Z^* (T_0 - T_s) \times \exp\big(-l(m_4 - \hat{m}_4)\big)\big(\exp(-\hat{m}_4 z) - \exp(-\hat{m}_4 l)\big)\right]^{1/2}. \qquad (29)$$

In equality (29), the value p_a^2 is defined by equality (24). Let the value of the dimensional pressure P_L at the output be experimentally determined.

Transcendental equation for determining the dimensionless coordinate l of stationary gas leak from the measured variables Q_L and P_L follows from equation (29) at

$z = 1$, $p_L = p(1) = P_L/p_c$. This equation has the following form:

$$p_0^2 - 2m_2 T_s Z^* l + 2\frac{m_2}{m_4} Z^*(T_0 - T_s)\left(\exp(-m_4 l) - 1\right)$$

$$- 2\hat{m}_2 Z^* T_s(1-l) + 2\frac{\hat{m}_2}{\hat{m}_4} Z^*(T_0 - T_s)\exp\left(-l(m_4 - \hat{m}_4)\right)$$

$$\times \left(\exp(-\hat{m}_4) - \exp(-\hat{m}_4 l)\right) - p_L^2 = 0. \qquad (30)$$

The solution of transcendental equation (30) is not difficult. The dimensional coordinate z_a of the stationary gas leak is determined by the found variable l: $z_a = l \cdot L$.

Simplified model III for thermal insulated gas pipeline

For a thermal insulated gas pipeline, the total heat transfer coefficient β is equal to zero. The dimensionless complexes m_4 (20) and $\hat{m}_4$ (27) become zero. In thermal equation (14) at $\beta = 0$, the only mechanism of gas cooling in the flow is the temperature drop due to the pressure drop. If compressibility factor (16) is constant, the Joule–Thomson coefficient becomes zero, which leads to the constant temperature in model III. Under boundary condition (19) the following equality follows for the gas temperature: $T = \text{const} = T_0$.

Pressure distribution (22) at $m_4 = 0$ taking into account the passage to the limit

$$\lim_{m_4 \to 0} \frac{\exp(-m_4 z) - 1}{m_4} = -z$$

is written in the form

$$p(z) = \left[p_0^2 - 2m_2 Z^* T_0 z\right]^{1/2}. \qquad (31)$$

Equality (23) for $m_4 = 0$ leads to the following expression for the effective compressibility factor Z_0^*:

$$Z_0^* = \frac{p_0^2 - p_L^2}{2m_2 T_0}. \qquad (32)$$

In the presence of the stationary gas leak, the pressure distribution in the first section is given by formula (31), in particular, for $z = l$, the pressure p_a in model III is equal to

$$p_a^2 = p_0^2 - 2m_2 Z_0^* T_0 l. \qquad (33)$$

In the second section, after the establishment of a new stable mode in model III at $m_4 \to 0$, $\hat{m}_4 \to 0$, from formula (29) by passing to the limit we obtain the following expression for the pressure distribution:

$$p(z) = \left[p_a^2 - 2\hat{m}_2 Z_0^* T_0 (z - l)\right]^{1/2}, \qquad (34)$$

in which the variable p_a is defined by equality (33).

As in the derivation of equation (30), we set $z = 1$: $p_L = p(1) = P_L/p_c$ and obtain the following simple *analytical formula for calculating the dimensionless coordinate l of the gas leak in models III:*

$$l = \frac{p_0^2 - p_L^2 - 2\hat{m}_2 Z_0^* T_0}{2Z_0^* T_0 (m_2 - \hat{m}_2)}. \qquad (35)$$

Formula (35), as follows from its derivation, can be used to calculate the location of the stationary gas leak after reaching a new stable flow mode at $\beta \to 0$ and under conditions (11), (16).

4. Examples of test calculations

Let us give examples of calculations using general model I and simplified models II and III for four options of the problem parameters. **For all options** we take the following parameters as unchanged:

$$R = 0.5\,\text{m}, \quad T(0) = 308.15\,\text{K},$$

$$T^* = 278.15\,\text{K}, \quad L = 5 \cdot 10^4\,\text{m},$$

$$T_c = 193.698\,\text{K}, \; p_c = 4.598\,\text{MPa}, \; R_g = 493.501\,\text{J/(kg} \cdot \text{K)}. \qquad (36)$$

Option 1.

$$P(0) = 60\,\text{atm}, \quad Q^0 = 250\,\text{kg/s}, \quad \delta Q = 25\,\text{kg/s},$$

$$\beta = 5\,\text{W/(m}^2 \cdot \text{K)}, \quad c_p = \langle c_p \rangle = 2649.24\,\text{J/(kg} \cdot \text{K)},$$

$$\lambda = 0.0087. \qquad (36.1)$$

Option 2.

$$P(0) = 90\,\text{atm}, \quad Q^0 = 250\,\text{kg/s}, \quad \delta Q = 25\,\text{kg/s},$$

$$\beta = 5\,\text{W/(m}^2 \cdot \text{K)}, \quad c_p = \langle c_p \rangle = 2881.11\,\text{J/(kg} \cdot \text{K)},$$

$$\lambda = 0.0087. \qquad (36.2)$$

Option 3.

$$P(0) = 90\,\text{atm}, \quad Q^0 = 400\,\text{kg/s}, \quad \delta Q = 40\,\text{kg/s},$$

$$\beta = 5\,\text{W/(m}^2 \cdot \text{K)}, \quad c_p = \langle c_p \rangle = 2853.69\,\text{J/(kg} \cdot \text{K)},$$

$$\lambda = 0.0085. \qquad (36.3)$$

Sets (36)–(36.3) contain rounded values of the variables; their exact values were used in the calculations. The average variable $\langle c_p \rangle$ of the specific isobaric heat capacity for all options was calculated using general model I in accordance with iterative algorithm given above (9). The value of the hydraulic resistance coefficient λ in real problems

Table 1. Option 1 (60 atm, 250 kg/s)

z, km	10	20	30	40	50
P_{II}	59.017	58.036	57.053	56.065	55.069
δP	0.0115	0.0159	0.0147	0.0090	0
T_{II}	28.665	23.669	19.727	16.617	14.165
δT	0.309	0.565	0.777	0.954	1.104

should, as noted, be determined from the solution of the inverse problem of identifying model parameters. In the above test options, the hydraulic resistance coefficient λ was calculated using the Haaland formula, which is included, for example, in the software system OLGA, the equivalent uniform-grain roughness coefficient was assumed to be 10^{-5} m, the dynamic viscosity coefficient, which is included in the expression for the Reynolds number, was calculated by the Dean–Steele formula [7] for above gas mixture (10). The results of calculations for models I and II for options (36.1)–(36.3) are presented in Tables 1–3.

The following designations are accepted: P, P_{II} — pressure [atm] calculated by model I and by simplified model II, respectively, $\delta P = (P_{\text{II}} - P)$ [atm] — error of pressure calculation by model II; T, T_{II} — temperature [°C] calculated from model I and simplified model II, respectively, $\delta T = (T_{\text{II}} - T)$ [°C] — error of temperature calculation by model II; z [km] — coordinate along the pipeline axis.

For each option of the test problem, for the given flow rate and pressure at the outlet of the gas pipeline, the coordinate z_k [m] of the gas leak was calculated (k — number of the parameter option, $k = 1, 2, 3$).

In view of the lack of experimental data for determining the outlet pressure, for a given set of parameters, with a given gas leak δQ and a given location z_a [m], the outlet pressure at gas pipeline outlet was calculated, which was then taken as experimentally measured. Further, in accordance with simplified model II, from the solution of transcendental equation (30), the dimensionless value l_k for the k-th option was determined, and the dimensional coordinate z_k [m] of the gas leak was found from it, then the error in determining the leak location was calculated using simplified model II, characterized by the value δz_k [m], equal to: $\delta z_k = (z_k - z_a)$ [m], k =1,2,3 (Table 4).

Example of calculation by model III

Table 2. Option 2 (90 atm, 250 kg/s)

z, km	10	20	30	40	50
P_{II}	89.381	88.769	88.162	87.557	86.954
δP	0.0116	0.0163	0.0153	0.0095	0
T_{II}	29.122	24.395	20.595	17.539	15.082
δT	0.181	0.331	0.454	0.556	0.641

Table 3. Option 3 (90 atm, 400 kg/s)

z, km	10	20	30	40	50
P_{II}	88.434	86.859	85.272	83.670	82.049
δP	0.0196	0.0282	0.0269	0.0171	0
T_{II}	31.143	27.782	24.853	22.301	20.076
δT	0.476	0.905	1.293	1.646	1.968

Table 4. Error of calculation of coordinate of gas leak

z_a, m	10^4	$2 \cdot 10^4$	$3 \cdot 10^4$	$4 \cdot 10^4$	$4.5 \cdot 10^4$
δz_1, m	−81.3	58.8	105.9	76.6	45.1
δz_2, m	−89.2	119.2	189.4	142.5	81.8
δz_3, m	−103.9	57.0	117.1	92.4	53.6

Table 5. Option 4 (90 atm, 400 kg/s, $\beta = 0$)

z, km	10	20	30	40	50
P	88.400	86.766	85.103	83.407	81.677
δP	0.0007	0.0010	0.0010	0.0007	0
δT	0.51	1.04	1.57	2.12	2.69

Table 6. Error of calculation of gas leak place by model III

z_a, m	10^4	$2 \cdot 10^4$	$3 \cdot 10^4$	$4 \cdot 10^4$	$4.5 \cdot 10^4$
δz_4, m	2.04	1.73	1.33	0.76	0.40

Consider option 3, setting $\beta = 0$ in it. As before, in Table 5 we denote by P, P_{III} the pressure [atm] calculated by model I and by simplified model III, respectively, $\delta P = (P_{\text{III}} - P)$ [atm] is error of pressure calculation by model III; T is temperature [°C] calculated by model I at $\beta = 0$ for option of parameters 3 in simplified model III the temperature is constant and equals to: $T = \text{const} = T_0$. Let us denote $\delta T = (T_0 - T)$ [°C] is the error of temperature calculation by model III; z [km] is coordinate along the pipeline axis.

Option 4.

$$P(0) = 90\,\text{atm}, \quad Q^0 = 400\,\text{kg/s}, \quad \delta Q = 40\,\text{kg/s},$$

$$\beta = 0, \quad c_p = \langle c_p \rangle = 2821.45\,\text{J/(kg} \cdot \text{K)}, \quad \lambda = 0.0085.$$

Table 6 shows the error values for calculating the coordinates of the gas leak according to formula (35) of simplified model III, $z_4 = l_4 \cdot L$ [m], $\delta z_4 = (z_4 - z_a)$ [m].

Conclusions The performed calculations, some of which are presented above, showed that there is a range of problem parameters change, in which the coordinate of the stationary gas leak can be calculated with an acceptable accuracy from the transcendental equation obtained, which follows from the simplified flow model. The data in Tables 3 and 4 indicate that an increasing of the inaccuracy of temperature calculation (option of parameters 3) practically does not affect the error of the location calculation of the gas leak. This conclusion is also confirmed by calculations for other options of parameters; it also follows from the data of calculating the place of the stationary gas leak (Table 6) in the simplified isothermal model, for which the temperature differs from the actual temperature calculated according to the general model by more than 2°C (Table 5).

Conclusion

The aim of the paper was to compare the accuracy of calculation of the location of stationary gas leak in the studied area based on the different degree of generality of the mathematical models of stable gas transportation.

Analytical dependences on pressure and temperature of the isobaric specific heat and its average integral value in the studied area are obtained for the gas mixture meeting the equation of state with the Berthelot compressibility coefficient. These dependences, along with the algorithm for identifying hard-to-determine model parameters, make it possible to ensure the adequacy of the gas transportation model in the stable state under actual conditions. The paper uses the assumption that the processes are stationary both before the formation of the stationary gas leak and some time after it. This assumption narrows the area of practical application of the obtained results. For a simplified stationary model, which assumes the small inertia forces and the predominant effect of heat exchange with the environment on the gas temperature, a transcendental equation is obtained for calculating the coordinates of the stationary gas leak of low and medium intensity according to experimental data of the pressure and flow rate at the outlet of the studied section of the gas pipeline. The results of a comparison of the flow characteristics calculated by the general and simplified stationary models for different options of parameters of practical interest are presented. The conclusion is substantiated that the inaccuracy of calculating gas temperature according to the simplified model has an insignificant effect on the error in calculating the coordinates of the stationary gas leak using the simplified model.

Simple analytical formulas obtained in the framework of the simplified models can serve as a good initial approximation when calculating the coordinate of the stationary gas leak of low and medium intensity in the iterative algorithm given earlier for solving the problem of identifying the stationary gas leak based on the general stationary model.

Conflict of interest

The authors declare that they have no conflict of interest.

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