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On the possibility of the phonon mechanism of superconductivity in cuprate VTSP

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Some features of the electron-phonon mechanism in cuprate high-temperature superconductors (HTS) are considered to explain their properties. It is shown that the interaction of electrons with weakly fixed oxygen ions in the crystal lattice can lead to a strong electron-phonon interaction, and this explains the high critical temperatures in cuprate HTS. Based on this approach, the presence of a T_c maximum in the homologous series of cuprate HTSPS is explained and a method for searching for new HTSPS with higher critical temperatures is proposed.

Keywords: high-temperature superconductivity, electron–phonon interaction, boundary conditions, Ginzburg–Landau theory, oxygen ions, „jelly“ model.

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1. Introduction

The discovery of high-temperature superconductivity in cuprate compounds by Bednorz and Müller [1] stimulated many theoretical and experimental works dedicated to the study of properties of these superconductors, and has resulted in great progress towards a wide practical use of superconducting devices. Unfortunately, it must be noted that, despite the achieved success, there is still no generally accepted HTS theory, due to which the search for new materials was predominantly empirical. The high-temperature superconductivity mechanism in cuprates remains an exciting research topic. It should be noted that, in our opinion, not enough attention is paid in explaining the HTS properties to the electronic and phononic mechanism developed by Bardeen–Cooper–Schrieffer (BCS) [2]. This can be explained by the following reasons. Firstly, high-temperature superconductivity was discovered in rather an unexpected class of materials — compounds based on transition metal oxides. Secondly, these superconductors manifest several properties which deviate from the BCS theory predictions, which naturally leads to an assumption that some other superconductivity mechanism must act in HTS. For instance, the isotope effect is a proof of the existence of the phononic mechanism of superconductivity. This is explained by the dependence of the critical temperature (T_c) on the Debye temperature (Θ_D), which in its turn depends on superconductor ion mass: $\Theta_D \sim M^{-0.5}$. A dependence of the critical temperature on isotopic mass of the elements, which make up the superconductor crystal lattice, is observed in cuprate HTS. However, it must be noted than the isotope effect in high-temperature superconductors differs greatly from the isotope effect in the conventional low-temperature superconductors (metals, A-15 systems etc.). For example, the index of the isotope effect by oxygen in case of substitution of an O^{16} atom by a heavier O^{18} isotope for the conventional HTS-system $YBa_2Cu_3O_{6+\delta}$ is not 0.5, as predicted by the BCS theory, but only about 0.1 [3]. Thirdly, this is related to the earlier predictions within the framework of the BCS theory that the critical temperature of a superconducting junction cannot exceed 35–40 K [4].

The critical temperature is often estimated within the framework of the phononic mechanism of superconductivity using the formula obtained by MacMillan [4]:

$$T_c = \frac{\Theta_D}{1.45} \exp\left[-\frac{1.04(1+\lambda)}{\lambda - \mu(1+0.62\lambda)}\right], \qquad (1)$$

Here λ is the electron-phonon interaction parameter, μ is the parameter determining the magnitude of the Coulombic interaction between electrons ($\mu \sim 0.1$). The following correlation is true for λ in the phononic mechanism of superconductivity

$$\lambda = 2\int_0^\infty \frac{\alpha^2(\omega)F(\omega)d\omega}{\omega}, \qquad (2)$$

where $F(\omega)$ is the density of phonon state as a function of frequency (ω) and $\alpha^2(\omega)$ is the measure of electron-phonon bond. The product of these functions $\alpha^2(\omega)F(\omega)$ plays an important role in the theory of electron-phonon interaction and it can be considered as an electron-phonon interaction averaged by the Fermi surface. Detailed information about this interaction is given in Eliashberg's paper [5] and monograph [6]. Formula (1) in case of a weak ($\lambda \ll 1$) bond

transforms into a BCS formula for the critical temperature

$$T_c \simeq \Theta_D e^{-1/g}, \quad (3)$$

where $g = \lambda - \mu$. Thus, these assumptions were based on formulas (1), (3) which are true in the case of a relatively weak electron-phonon interaction, which ensures electron attraction and leads to the formation of the so-called Cooper pairs which exist in one and the same quantum-mechanical state in the BCS theory.

Moreover, the problem of crystal lattice instability is observed in superconductors with a strong electron-phonon interaction, which is one of the limitations on the maximum possible critical temperature of the superconducting junction. It is seen from formulas (1), (3) that the critical temperature increases with an increase of electron-phonon interaction, that's why a large T_c must be observed in superconductors with a large λ. Thus, the formulas obtained in the limit of a weak electron-phonon interaction are not quite correct for estimating the limit of possible values of T_c. All this stimulates the research dedicated to non-phononic mechanisms of superconductivity [7]. On the other hand, electron-phonon interaction always exists in conductors and, undoubtedly, should be taken into account when analyzing the HTS properties. Moreover, this superconductivity mechanism is developed better than others.

Let us note the most important properties of cuprate HTS. First, these superconductors manifest the usual properties: superconductivity itself, diamagnetism and a second-order phase transitions — the superconducting-normal state. Cuprate superconductors in the normal non-superconducting state usually manifest the hole conduction mechanism. Moreover, the Shapiro steps on I–V dependences of the Josephson junctions on the basis of HTS [8] and flux quantization in these superconductors [9] show that their superconductivity is due to the mating of elementary charge carriers, as in normal superconductors. In cuprate HTS these are holes. The most widespread method for the variation of hole concentration is changing the oxygen content in such compounds. Since cuprate superconductors are complex non-stoichiometric oxides, the oxygen concentration is changed by their heat treatment at different temperatures in an atmosphere with a controlled oxygen pressure and subsequent tempering. It is widely recognized at present that HTS are characterized by a long-range order parameter, caused by condensation of the Cooper pairs, similarly to the one presented in the BCS theory. As distinct from superconducting metals, the Cooper pairs in cuprates consist of two holes [10,11].

Secondly, though there are hundreds of compounds with a high value of T_c, all of them have a layered structure consisting of one or several CuO_2 planes [12,13]. Their ratio of resistivity along the unit cell „c“ axis, perpendicular to the CuO_2 planes, to resistivity along these planes (ρ_c/ρ_{ab}), in the $YBa_2Cu_3O_{7-\delta}$ compound is about 50, while it is about 10^6 in case of the most anisotropic cuprate superconductor $Bi_2Sr_2CaCu_2O_{8+\delta}$. Cuprate superconductors are often classified based on the idea of homologous series of layered copper oxides. These superconductors have blocks of CuO_2 planes, located at a short distance from each other in the unit cell and differ only by their number (n). It is well known that blocks of CuO_2 planes form superconducting layers. While the number of CuO_2 planes in complexes is less than 4 ($n < 4$), an increase of the number of planes leads to an increase of the critical temperature, and the maximum of T_c is achieved for $n = 3$. A decrease of T_c is observed in case of a further increase of n. For instance, the critical temperatures of mercury cuprate compounds $HgBa_2Ca_{n-1}Cu_nO_{2+2n+\delta}$ first increase with an increase of n: $T_c = 97\,\mathrm{K}$ for a single-layer phase ($n = 1$), $T_c = 127\,\mathrm{K}$ for $n = 2$, $T_c = 135\,\mathrm{K}$ for $n = 3$. It should be noted that the latter value of T_c is a record value of the critical temperature when no pressure is applied to the superconductor. With a further increase of the number of CuO_2 planes in the unit cell, the critical temperature decreases: $T_c = 126\,\mathrm{K}$ for $n = 4$, $T_c = 110-114\,\mathrm{K}$ for $n = 5$, $T_c = 96-100\,\mathrm{K}$ for $n = 6$ and $T_c = 88\,\mathrm{K}$ for $n = 7$ [12]. A similar bell-shaped pattern of the $T_c(n)$ dependence is also observed in Bi- and Tl-cuprate superconductors [13,14]. This effect was explained by several phenomenological theories based on an assumption of hole density distribution across CuO_2 planes and based on the consideration of the interaction of these planes with each other and with neighboring non-superconducting planes [15–17]. Thereat, each CuO_2 plane is characterized by its own order parameter and its own critical temperature, which are determined by the hole density in them. In order to explain the bell-shaped pattern of the $T_c(n)$ dependence, the authors of [17] made an additional assumption of charge redistribution among the layers inside a unit cell.

It should be noted that the interest in cuprate HTS did not decrease even after the discovery of superconductivity in sulfur hydride with $T_c = 200\,\mathrm{K}$ and in hydrides of other metals [18,19]. This is related to the fact that all these hydrides achieve their maximum T_c at very high pressures, above 150 GPa, which hinders their application.

2. Ginsburg–Landau theory in HTS

Despite the absence of a generally accepted microscopic theory of HTS, application of the macroscopic Ginsburg–Landau theory (GL) [20] to these superconductors is not contested by anybody. Gorkov's paper [21] gives a strict justification of their rightness within the framework of the electronic-phononic mechanism of device superconductivity, and the GL equations were obtained on the basis of the microscopic BCS theory. The GL theory with account of the HTS layered structure describes the electromagnetic properties of these superconductors well. The system of GL equations for a long and wide superconducting plate without a magnetic field and current reduces to a single equation for the module of the superconducting order parameter (Ψ). Assuming that the „x“ axis is perpendicular

to the surface of the plate with thickness d, and the origin of coordinates is in the plate center, so that $-d/2 \le x \le d/2$, this equation can be written down as

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{1}{\xi(T)^2}(\psi - \psi^3) = 0, \quad (4)$$

where $\xi(T)$ is GL coherence length dependent on temperature (T). This equation uses the normalized order parameter $\psi = \Psi/\Psi_0$, where Ψ_0 is the value of the order parameter in a homogeneous superconductor in a zero magnetic field. This equation was used in paper [22,23] to describe the properties of superconducting CuO_2 layers in HTS, and interaction between them was ignored. Strictly speaking, the GL equations must be modified for cuprate superconductors which are usually considered as a set of quasi-two-dimensional superconducting layers, bound by a weak Josephson interaction. For instance, Lawrence and Doniach developed a model of layered superconductors where the order parameter is determined only in superconducting layers, while the superconductor free energy is equal to the sum of GL free energy for each superconducting layer and interaction energy of the Josephson contacts which provide a bond between these layers [24]. Equation (4) does not take into consideration the Josephson coupling between superconducting layers, since it does not greatly affect the order parameter of the superconductor of which the given contact is made. This is confirmed by the experimental fact that superconductivity in cuprate HTS is maintained in ultrathin films (with the thickness of half the unit cell) and, for instance, T_c of such ultrathin films $Bi_2Sr_2CaCu_2O_8$ is equal to the critical temperature of this superconductor [25]. Therefore, the study of the properties of a thin superconducting layer within the GL theory framework is a topical and important task for understating of the HTS properties. Thus, we can use standard one-dimensional GL equation (4) for a plate made of a hypothetical, homogeneous superconductor with a high T_c. ξ in this case is the GL coherence length in the direction of the „c“ axis. It is well known that $\xi(T = 0)$ is extremely small in superconductors with a high T_c. Different authors have determined it within the range of 2 to 5 Å, i.e., it has an order even less than the interplanar distances in HTS.

One of the HTS peculiarities is the need for the use of new boundary conditions for solving equation (4) in the GL theory:

$$\partial\psi/\partial x = \pm\psi/\Lambda|_s, \quad (5)$$

where Λ is the phenomenological coefficient which is called the extrapolation length in [26]. The need for the use of such boundary conditions has already been stated in the Nobel lecture by Bednorz and Müller [27]. Simonin [28] has demonstrated earlier that this boundary condition is a result of the presence of an additional superficial term for the GL free energy. Extrapolation length for a superconductor with a low T_c is large as compared to the crystal lattice constant, which explains the rightness of the used boundary condition

$$\partial\psi/\partial x = 0|_s. \quad (6)$$

It was found that the new boundary condition causes a decrease of the order parameter and a decrease of the critical temperature $(T_c(d))$ of thin superconducting planes. The plate critical temperature decreases with a decrease of thickness d, and plate superconductivity disappears at temperature T, as soon as $d = d_c(T)$. Equation (4) and boundary condition (5) result in an expression which determines this film thickness

$$d_c(T) = 2\xi(T)\tan^{-1}[\xi(T)/\Lambda]. \quad (7)$$

Obviously, this formula implicitly determines the $T_c(d)$ dependence. Thus, $T_c(d)$ is below the critical temperature of a bulk superconductor. The new boundary condition was applied to thin films made of metal superconductors, where it successfully explained the critical temperature decrease observed in thin Nb, Pb and Bi films. Moreover, these boundary conditions were also applied to complexes of CuO_2 planes, which are the main structural elements accountable for superconductivity in cuprate compounds with a high T_c [22,23]. Thereat, the order parameter was considerably suppressed by non-superconducting layers which separate the CuO_2 planes. As with superconducting metal films, this boundary impact must reduce the critical temperature of these superconductors. The authors of [22,23] made a conclusion that the number of CuO_2 planes, located at a short distance from each other in a unit cell, must be increased in order to increase T_c of cuprate superconductors. This tendency is explained by the fact that the influence of non-superconducting interlayers decreases with an increase of the thickness of superconducting layers, and the order parameter increases. This approach was used to demonstrate that T_c in cuprate HTS must tend to 155 K upon an increase of the number of CuO_2 $n > 3$, and it must tend to 190 K in mercury-based cuprate superconductors when pressure T_c is applied.

Unfortunately, this contradicts the experimental situation when the maximum T_c is attained for $n = 3$ and then there is a decrease of the critical temperature with an increase of the number of CuO_2 planes in the superconducting complexes. Evidently, the explanation of this contradiction requires a detailed consideration of the processes in cuprate superconductors, first of all, the processes on boundaries which separate the superconducting and non-superconducting layers (SN). One of the possible reasons to explain this decrease is the fact that an increase of the number of CuO_2 planes leads to a decrease of the density of charge carriers (holes) in them; the main source of charge carriers is non-superconducting interlayers between superconducting complexes. However, there are many methods to solve this problem. That's why the absence of progress in the increase of T_c of the HTS with $n > 3$ indicates the need for finding out another reason of the decrease of the critical temperature with an increase of

superconducting layer thickness, in other words, with an increase of the number of CuO_2 planes in superconducting layers. It can be assumed that such a decrease indicates the presence of processes, which stimulate superconductivity, on the SN boundaries. This is evidently very important for understanding of the high-temperature superconductivity mechanism.

3. Role of boundary phenomena in cuprate HTS

Cuprate superconductors have several peculiarities of particular importance from the viewpoint of the phononic mechanism of superconductivity. As an example, Fig. 1, *a, b* shows the unit cells of $YBa_2Cu_3O_{6+\delta}$ and $HgBa_2Ca2Cu_3O_9$. The crystal structure of these superconductors contains many oxygen atoms whose atomic mass is significantly less than the atomic mass of ions of the conventional metal superconductors: Nb, Pb, Sn etc. There are particularly many oxygen ions near the superconducting layers. Similar planes also exist in Tl- and Bi-cuprate superconductors. These planes differ in different cuprate superconductors only by the type of ions located between oxygen ions. The large number of oxygen ions in the crystal structure of cuprate HTS is important for the phononic mechanism of attraction of charge carriers: electrons or holes in them. Such a charge transmits a certain pulse **p** to an immobile ion when flying near it. Thus, the ion gets kinetic energy $p^2/2M$ from the flying charge as a result of collision. Thereat, ions are exposed to elastic forces from the neighboring ions which prevent the motion of ions. In other words, the kinetic energy turns into potential energy in compliance with the Hooke law, and ion shift (y) from the equilibrium position is determined by correlation

$$\frac{p^2}{2M} = \frac{ky^2}{2}, \tag{8}$$

where k is the elastic constant. As can be seen from this correlation, the shift y is inversely proportional to $M^{-0.5}$, i.e., this shift is particularly large for oxygen ions in cuprate HTS. The larger the ion shift, the larger the originating excess positive charge in case of a moving electron or negative charge in case of a moving hole and, consequently, the greater the bond energy of two electrons or holes.

Some peculiarities of these processes in cuprate superconductors can be analyzed using permittivity, which is a function of frequency within the framework of the plasma „jelly" model [6,29]. Two interacting charge systems are considered within the framework of this model: electronic and ionic plasma formed by positive ions with an unshielded Coulombic interaction. Ions are considered as a liquid, that is, the substance crystal structure is not taken into account. The natural vibration modes of such a system are plasma vibrations. A description of an electron-ion system using this approach was formerly widely used in the theory of superconductivity. Within the framework of this model, electron attraction originates due to a negative permittivity in frequency region of $0 < \omega < \omega_D$, ω_D is the Debye frequency.

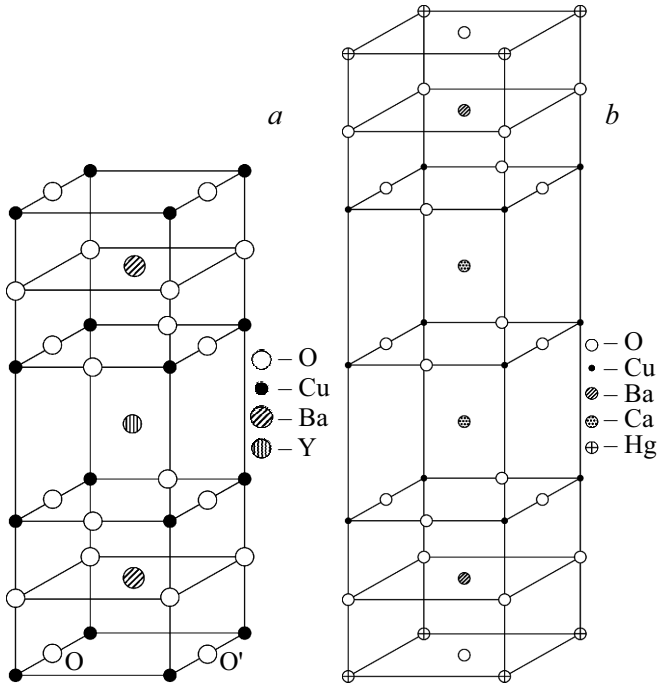

Figure 1. *a* — unit cell of $YBa_2Cu_3O_{6+\delta}$. Both positions O and O′ for $\delta = 1$ are occupied by oxygen ions. Either O or O′ is occupied for $\delta = 0.5$. *b* — unit cell of $HgBa_2Ca_2Cu_3O_9$.

Cuprate HTS usually have the hole conduction, that's why two interacting systems of charges must be considered within the framework of the „jelly" model: holes and ions, first of all, these are negative oxygen ions. For instance, the $YBa_2Cu_3O_{6+\delta}$ superconductor contains both positive ions Y^{3+}, Ba^{2+}, Cu^{2+} (the most stable among the copper ions) and negative ions — O^{2-}. It should be noted that with $\delta = 0.5$ a superconductor must not contain holes in view of the simple correlation for a unit cell: $1 * 3 + 2 * 2 + 3 * 2 - 6.5 * 2 = 0$.

Superconductivity is usually observed at $\delta > 0.5$, i.e., in case of an excess of negative oxygen ions, which causes the hole conduction in the given material. Thus, if $YBa_2Cu_3O_{6+\delta}$, non-superconducting interlayer between superconducting complexes indeed determine the charge carrier density in the given superconductor. Thereat, plasma vibrations of positively charged metal ions are not taken into consideration for a qualitative understanding of the occurring processes. There are particularly many oxygen ions near the superconducting planes of CuO_2. The neighboring unit cells in superconductors $YBa_2Cu_3O_{6+\delta}$ may contain a different number of oxygen ions. Either 1 or 2 ions are possible in case of $0.5 < \delta < 1$ in oxygen chains. They are marked with letters O and O' in Fig. 1, *a*. Oxygen ions in the plane where these chains are located can form ordered structures, the form of which depends on δ, while

the position of other ions in the crystal structure does not change [30], which must lead to a greater lattice stability under vibrations of oxygen ions. It is to be recalled that lattice stability in relation to a transition to another non-superconducting phase under a strong electron-phonon interaction is an important limitation in the electronic-phononic mechanism of superconductivity [6]. On the other hand, this must lead to greater mobility of oxygen ions as compared to metal ions in crystal structures of simple superconductors. Moreover, as demonstrated in [31], the vibration amplitude of oxygen ions in chains is several times greater than the vibration amplitudes of other ions. These properties of oxygen ions in cuprate HTS are an additional argument in favor of applicability of the „jelly" model for the analysis of HTS processes. Thus, such heterogeneity of the crystal structure must evidently lead to a decrease of the value of the elastic constant k in such chains, which, according to correlation (8), must lead to an increase of the energy of attraction between moving holes. We hereinafter assume in the present paper that hole plasma in cuprate superconductors is located in CuO_2 planes, while ion plasma is formed by oxygen ions of a particularly high density on the boundaries of these planes. The plasma model is widely used to analyze the properties of superconducting metals. As demonstrated in this model, permittivity can take on negative values in the frequency range of $0 < \omega < \omega_i$ due to the screening effect. Hereω_i is the plasma frequency of ions, which determines the average ion vibration frequency; the latter is an equivalent of the Debye frequency

$$\omega_i = \left(\frac{4\pi n_i e_i^2}{M}\right)^{0.5}, \qquad (9)$$

where n_i and e_i are ion concentration and charge. Negative permittivity causes the mutual attraction of an electron pair in superconducting metals or a pair of holes in the case of cuprate HTS. As demonstrated in monograph [6], the electron interaction energy in the „jelly" model is determined by the following correlation:

$$U_0(\gamma) = \frac{\beta}{2(1-\gamma^{-2})} \cdot \ln\left|1 + \frac{1}{\beta}(1-\gamma^{-2})\right|, \qquad (10)$$

where $\beta = \frac{e^2 m}{(2\pi^2 n_c)^{1/3}\pi\hbar^2}$, e — electron charge, $\hbar$ — Planck constant, m — electron (hole) mass, n_c — electron (hole) concentration and $\gamma = \omega/\omega_i$.

Fig. 2 shows this interaction energy calculated using formula (10) for different values of parameter β. Evidently, the plasma frequency increases with a decrease of ion mass, i.e., ω_i of oxygen plasma is considerably higher than the plasma frequency of ions of simple metal superconductors such as Nb, Pb and Sn. A large plasma frequency according to formulas (1), (3) must lead to higher T_c. It follows from these formulas that the critical temperature of superconductors also increases with an increase of the electron-phonon interaction, which is determined by parameter λ. An

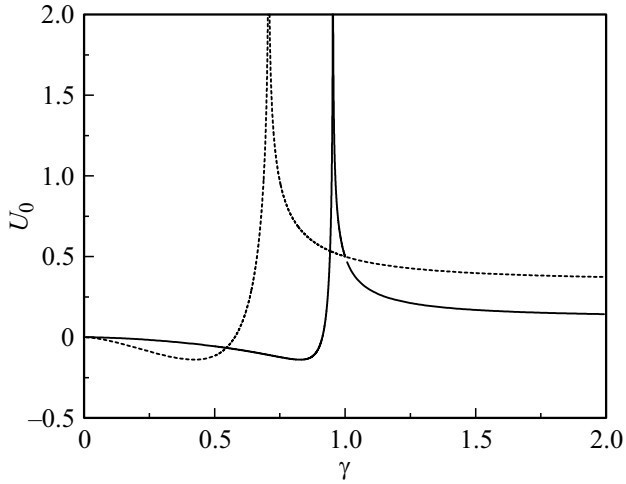

Figure 2. Interaction energy for the „jelly" model, determined by correlation (10) for two parameters β: solid line $\beta = 0.1$, dashed line $\beta = 1$.

analysis of hole interaction using formulas (2), (10) shows that such interaction in cuprate HTS can be stronger than in the conventional metal superconductors. As shown in Fig. 2, the minimum of the $U_0(\gamma)$ dependence shifts to the lower-frequency region with an increase of parameter β. Taking into account the dependence of this parameter on charge carrier concentration, it follows from formula (10) that the minimum shifts to the lower-frequency region with a decrease of concentration (n_c), while its amplitude does not change. In this respect, it should be noted that HTS are characterized by a low concentration of charge carriers as compared to superconducting metals. For instance, it can vary in $YBa_2Cu_3O_{6+\delta}$ and $HgBa_2Ca_{n-1}Cu_nO_{2+2n+\delta}$ with a change in parameter δ, i.e., with a change of oxygen concentration, and in $La_{2-x}Sr_xCuO_4$ with a change in strontium concentration. Thereat, the crystal structure of these superconductors and, consequently, their phonon spectrum do not change greatly. Evidently, the measure of electron to phonon bond ($\alpha^2(\omega)$) must similarly change with a change of the type of electron-phonon dependence $U_0(\gamma)$; the bond must be more efficient in the low-frequency region. This, as follows from formula (2), must lead to an increase of parameter λ and, consequently, to a higher critical temperature in accordance with formulas (1), (3). Moreover, one of the reasons for strengthening of hole attraction is the fact that a low hole concentration leads to a large distance between them (as compared to the distance between conduction electrons in metal superconductors). This, in its turn, must result in a decrease of the Coulomb repulsion between holes, which determines the parameter μ. In accordance with formulas (1), (3), a decrease of μ must also lead to the rise of the critical temperature.

Qualitative reasoning about the structure of charge density in cuprate HTS are based on the previous band calculations [32]. As shown in Fig. 1, *b*, the central CuO_2 layers in cuprate HTS at $n \geq 3$ border on planes formed by calcium

cations, as distinct from the endmost CuO_2 planes which border (from the outside) on oxygen-enriched planes. These cation planes try to localize the motion of holes in the corresponding CuO_2 planes. Evidently, electron-phonon interaction in the central planes is less efficient than in the two endmost ones, which border on the planes enriched with oxygen ions. Thus, the averaged electronic-phononic interaction decreases with an increase of the quantity of the central CuO_2 planes, i.e., with an increase of n at $n > 3$; this must result in a decrease of T_c in homologous series of cuprate HTS.

The given approach can explain the small value of the isotope effect index by oxygen for $YBa_2Cu_3O_{6+\delta}$, which is only about 0.1 [3]. The Debye frequency decreases with an increase of the mass of oxygen ions in accordance with formula (9), and at the same time the minimum of the $U_0(\gamma)$ dependence must also shift to the lower-frequency region ω. The latter, in view of correlation (2), must lead to an increase of electron-phonon interaction and, consequently, to an increase of T_c. Thus, influence of a Debye frequency decrease which, according to correlations (1), (3), leads to a decrease of the critical temperature, is compensated by a shift of the minimum $U_0(\gamma)$ to the lower-frequency region and, consequently, to a decrease of the isotope effect index by oxygen.

4. Conclusion

It has previously been assumed (based on the Ginzburg–Landau theory) that higher critical temperatures must be in cuprate superconductors with a large number of CuO_2 planes, which form superconducting layers [22,23], but the role of oxygen-enriched planes decreases with an increase of the number of CuO_2 planes. It is shown in the present paper that such planes stimulate superconductivity in cuprate superconductors within the framework of the electronic-phononic mechanism of superconductivity. Thus, electron-phonon interaction in HTS can be more efficient than in the conventional low-temperature superconductors. A combination of high deformation potentials and high vibration frequencies of oxygen ions are observed in cuprate HTS. The analysis performed in the present paper, which is based on use of the GL theory for describing the properties of individual superconducting layers, shows that this peculiarity of the crystal structure can explain the high critical temperatures of cuprate HTS. As distinct from the boundary CuO_2 planes in Tl, Hg and in Bi-cuprate superconductors, the internal planes at $n \geq 3$ border on the planes formed by Ca cations where oxygen ions are absent. Evidently, the electronic-phononic mechanism of superconductivity in such planes is less efficient than this mechanism in the boundary ones. The number of internal CuO_2 planes increases with an increase of the number of CuO_2 planes in Tl, Hg and in Bi-cuprate superconductors at $n \geq 3$. Such an increase must lead to a decrease of the role of boundary planes and to a decrease of integral electronic-phononic interaction, which explains the decrease of T_c with an increase of the number of the CuO_2 planes at $n > 3$ in homologous series of cuprate HTS. Thus, it can be assumed than an increase of T_c in such superconductors requires not only an increase of the number of CuO_2 planes ($n > 3$) in superconducting layer, but also an addition of planes with a large content of oxygen ions to these layers.

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Conflict of interest

The author declares that he has no conflict of interest.

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