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## Nonlinear effects of spin wave propagation in a bilayer magnon waveguide

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The possibility of frequency-selective propagation of spin waves in a linear and nonlinear mode in a magnon microwave medium consisting of two layers with different values of the magnetization saturation of each layer is demonstrated. It is shown that multimode propagation of spin waves can be carried out inside the two-layer structure in two frequency ranges, with an increase in the power of the input microwave signal leading to a change in the boundaries of both frequency ranges. At the same time, this process is accompanied by a strong nonreciprocity of the spin-wave signal propagation, which manifests itself in a change in the amplitude-frequency characteristics when the direction of the external magnetic field is reversed, with a nonlinear mode change in the frequency bandwidth borders can change with increasing pumping power. The proposed concept of a two-layer spin-wave waveguide can form the basis for the fabrication of nonlinear magnon elements demonstrating interconnection functions with support for multiband modes of operation.

**Keywords:** magnonics, nonlinearity, nonlinear systems, multilayer systems, spin waves.

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### 1. Introduction

For decades, multilayer films based on ferromagnetic materials that support spin-wave signal propagation modes attract a large number of researchers due to the constant development of both technologies for creating magnetic layers on non-magnetic substrates and the development of ideas for using magnetization waves to solve information signal processing problems [1]. Magnetic thin film structures are made in the form of single magnetic films, double magnetic films and multilayer magnetic films, consisting of ferromagnetic (FM), antiferromagnetic (AFM) and non-magnetic (NM) films of various thicknesses and arrangement of layers, among which FM/NM multilayer structures caused a large interest in the last decade [2]. The use of multilayer dielectric films of yttrium iron garnet (YIG) ensures the nonreciprocity effect and at the same time gives a greater advantage over the well-known layered YIG/metal structures due to significantly lower spin-wave losses in a two-layer YIG film consisting of layers with different values of magnetization. In turn, the study of nonlinear processes in width-limited magnon waveguide structures is an interesting task due to the use of microwaveguides as elements of interconnections of functional blocks of magnon networks that perform signal processing functions on the principles of magnon logic [3]. Discussion of nonlinear spin-wave processes in the modes of three-magnon and four-magnon decay in YIG films made a sig-

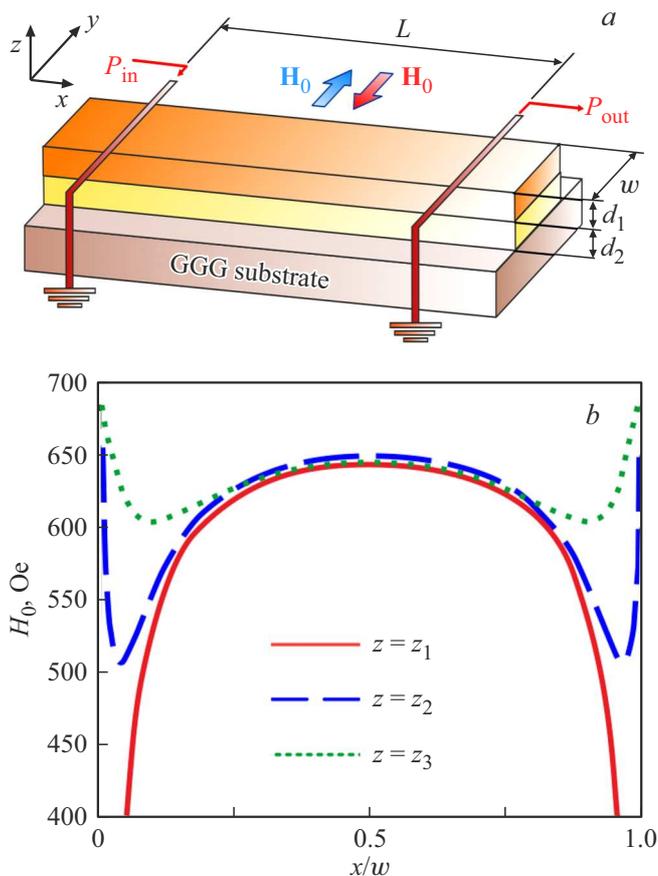
nificant contribution to the theory describing the nonlinear dynamics of dissipative systems [3–5]. Nonlinear modes of signal propagation in thin YIG films were experimentally studied in ferromagnetic waveguides of millimeter width using radiophysical methods [6–8] and the Mandelstam–Brillouin spectroscopy of magnetic materials [9–13]. It was shown that using nonlinear effects in YIG [14], it is possible to create tunable spin-wave devices [15], for example, nonlinear phase shifters, filters [16] and signal switching devices based on magnon crystals [17]. Thus, the use of structured films of ferrite garnets can serve as the basis for next-generation computer technology with a low level of power consumption based on the principles of magnonics [18,3]. It is important to note that for the creation of the Mach–Zehnder type interferometers, the study of SW propagation modes in a finite-width magnon microwaveguide made of a multilayer ferrite film with the ability to control interference modes, including changing the level of the input UHF-signal will be decisive.

In this paper we study the features occurring in the linear and nonlinear modes of signal propagation in the magnon waveguide formed by a YIG film consisting of layers with different magnetizations. The analytical model is made that describes the spectrum of intrinsic modes of such structure, and mechanisms for controlling the mode spectrum with power change of the spin wave are considered. Based on the experimental study by microwave spectroscopy, the effects of shift of the high-frequency and low-frequency bands of

the spectrum spin waves are found when the power level of the input signal varies, and the polarization of the applied external magnetic field changes. The proposed concept of a two-layer spin-wave waveguide can be the basis of the manufacture of controlled magnon interconnections with support for multiband operation modes.

## 2. Structure under study

Fig. 1, *a* shows the scheme of the two-layer microwaveguide under study. Epitaxially grown single-crystal ferromagnetic two-layer ferrite film YIG [ $\text{Y}_3\text{Fe}_5\text{O}_{12}$ ] with the size  $0.5\text{--}7\text{ mm}^2$ , grown on gallium gadolinium garnet [ $\text{Gd}_3\text{Ga}_5\text{O}_{12}$  (GGG)] substrates, the plane of which coincided with crystallographic plane (111). When creating a film on a GGG substrate, we first grew a layer of pure YIG with a thickness of  $7\text{ }\mu\text{m}$  with a saturation magnetization of  $4\pi M_1 = 1738\text{ G}$  (YIG1), and on it — a YIG layer doped with gallium and lanthanum,  $9\text{ }\mu\text{m}$  thick, with saturation magnetization  $4\pi M_2 = 904\text{ G}$  (YIG2). Next, the waveguide was placed in a uniform external magnetic field  $H_0 = 670\text{ Oe}$ , oriented along the axis  $y$ , and it was possible



**Figure 1.** (a) Scheme of the considered structure. (b) The results of calculating the internal field in the structure under study, in individual layers along the axis  $z$  (red line — upper boundary of the structure, blue dotted line — middle of the structure, green dotted line — lower boundary of the structure).

to change the field direction by 180 degrees. The width of the test sample was  $w = 100\text{ }\mu\text{m}$ . Waveguide length is  $L = 7\text{ mm}$ . The input and output microwave transducers  $30\text{ }\mu\text{m}$  wide were attached to the structure and marked in Fig. 1*b a* as „ $P_{\text{in}}$ “ and „ $P_{\text{out}}$ “ respectively. The geometry of the experiment corresponded to the case of excitation of surface magnetostatic waves (MSWs) in a tangentially magnetized film.

## 3. Methods of theoretical and numerical study

To make static profiles of the distribution of the internal magnetic field in the structure under study, the micromagnetic modeling was carried out in the program MuMax3 [19], based on the numerical solution of the Landau–Lifshitz–Gilbert equation by the Dorman–Prince method, which describes the precession of the magnetic moment  $M$  in the effective magnetic field  $H_{\text{eff}} = H_0 + H_{\text{demag}} + H_{\text{ex}} + H_a$ , where  $H_0$  is external magnetic field,  $H_{\text{demag}}$  is demagnetization field,  $H_{\text{ex}}$  is exchange field,  $H_a$  is anisotropy field. At the same time the anisotropy field was assumed equal to  $H_a = 0$ , since the equilibrium magnetization vector is directed along the symmetry axes of YIG. According to papers [20,21] the doped YIG with saturation magnetization  $4\pi M = 904\text{ G}$  can have perpendicular magnetic anisotropy (PMA). During numerical simulation, we did not consider the effect of the PMA of each of the YIG layers, because in this study only the case of tangential magnetization of the film is considered, and the constructed model indicates the occurrence of a nonreciprocal nature for propagating spin waves. To reduce the signal reflections from the calculated area boundaries in the numerical simulation the regions with geometrically increasing decay coefficient  $\alpha = 10^{-5}$  at the beginning and at the end of exit sections of a waveguide structure were introduced.

Fig. 1, *b* shows the results of calculation of the internal magnetic field inside the structure under study obtained using the finite difference method in the MuMax3 program. The calculation region in the direction of the axis  $z$  was divided into 16 layers, while it turned out to be possible to show how the distribution of the internal magnetic field changes in the center of the YIG1 first layer ( $z = z_1$ ), in the center of the YIG1 second layer ( $z = z_3$ ) and on the boundary between the layers ( $z = z_2$ ). It can be seen that the internal magnetic fields are strongly heterogeneous both in the direction of the axis  $y$  and in the direction of the axis  $z$ , which in turn strongly affects the propagation spectra of spin waves. In this case, the heterogeneous propagation of the internal magnetic field leads to a more pronounced nonreciprocal behavior of the spin-wave signal.

The propagation of spin waves in plane-layered structures with different values of the magnetization of the layers and at different orientations of the external magnetic field was theoretically and experimentally studied in papers [22–24].

In magnetic structures of a limited width the propagation of spin waves was discussed, for example, in [23,24]. In this paper, we consider the propagation of dipole-dipole spin waves in two transversely limited magnetic waveguides with different saturation magnetizations  $M_1$  and  $M_2$ , thicknesses  $d_1$  and  $d_2$ , of the same width  $w$ , separated by a non-magnetic layer with a thickness  $t$ . The magnetic properties of media in waveguides are described by magnetic permeability tensors of the form

$$\bar{\mu}_i = \begin{pmatrix} \mu_i & j\mu_i^a & 0 \\ -j\mu_i^a & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where  $i = 1, 2$  is layer number,  $\mu(\omega) := 1 + \frac{\omega h \cdot \omega m}{\omega h^2 - \omega^2}$ ,  $\mu a(\omega) := 1 + \frac{\omega m \cdot \omega}{\omega h^2 - \omega^2}$ .

Outside the magnetic layers, the magnetostatic potential satisfies the Laplace equation:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = 0. \quad (1)$$

Representing the potentials as plane waves inside and outside the waveguides as  $\psi(x, y, z) = e^{-j(k_x x + k_y y + k_z z)}$  and  $\psi_i(x, y, z) = e^{-j(k_x x + k_y y + k_z z)}$ , respectively, from (1) we obtained relations for wave numbers in all spatial regions  $k_x^2 + k_y^2 + k_z^2 = 0$ ,  $\mu_i k_x^2 + \mu_i (k_y^i)^2 + k_z^2 = 0$ . Boundary conditions on the side surfaces of the waveguides (where  $z = -w/2$  and  $z = w/2$ ) were taken into account by setting boundary conditions of the „magnetic wall“ type [4], at that the wave numbers  $kz$  are determined from the relation:  $k_z = \frac{n\pi}{w}$ ,  $n = 1, 2, 3, \dots$ , where  $n$  is index of the transverse (width) mode of the spin wave [10]. The solution to the problem of wave propagation in a two-layer magnon microwaveguide is sought in the form of propagating waves with the heterogeneous distribution of the magnetostatic potential  $\Psi(y)$  along the axis  $y$ :  $\psi(x, y, z) = \Psi(y) \sin(k_z z) e^{-jk_x x}$ . In waveguides the potential distribution is sought in the form:  $\Psi_i(y) = A_i e^{-jk_y^i y} + B_i e^{jk_y^i y}$ . Equating the potentials and y-components of the magnetic induction vector at the interfaces between magnetic and non-magnetic media, we obtain a system of equations for unknown coefficients. The dispersion equation obtained in this way turns out to be rather cumbersome and is not given further in the text of the work.

Nonlinearity in the proposed model was considered on the assumption of the saturation magnetization value decreasing with the increasing of the magnetization precession amplitude

$$M \approx M_0 \left[ 1 - \frac{m_x^2 + m_z^2}{2M_0^2} \right] = M_0(1 - \varphi^2/2).$$

In this case, the power of spin waves can be estimated asymptotically as [23,24]  $P_{in} \approx |\varphi|^2 M_0^2 v_g \omega_{eff} t_1$ , where

$v_g$  is the group velocity of the SMSW in the magnetic band of effective width  $\omega_{eff}$ ;  $\varphi = \varphi_0$  is initial amplitude of the SMSW. Taking into account the resistance of the lowest transverse mode of the SMSW [4,25], excited by the microstrip transmission line, it is possible to estimate the value of  $P_{in}$  from the experimental values  $P_0$ .

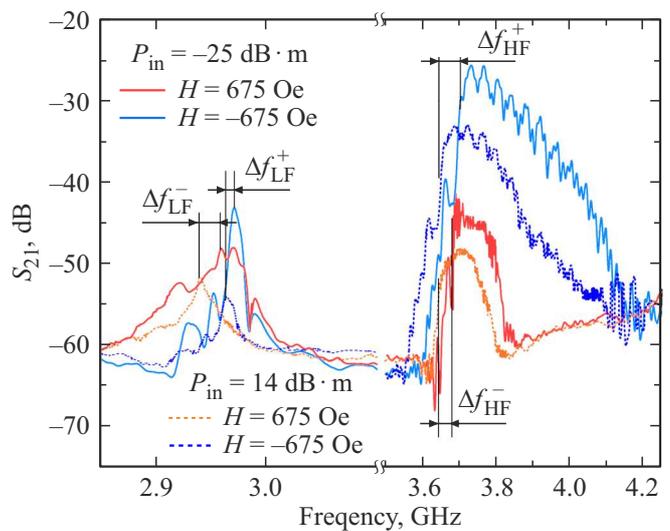
Let us estimate the effective saturation magnetization  $\delta M_s$  decreasing with magnetization amplitude  $h_0$  increasing [26,27]:

$$\delta M_s = M_0 - M_s \approx M_0 \gamma^2 h_0^2 / (2\alpha^2 \omega^2),$$

where  $\alpha = \frac{1}{\omega} \left| \frac{\delta \omega}{\delta H_i} \right| \frac{\Delta H}{2} = 1.15 \cdot 10^{-5}$  the Hilbert damping parameter [3],  $h(x, z, t) = h_0 \sin(2\pi f t) [h_x(x, z) \mathbf{x}_0 + h_z(x, z) \mathbf{z}_0]$  — magnetic field excited by UHF current with frequency  $f$ .

## 4. Experimental study

To study the effect of changing the UHF signal power on the properties of spin waves in a two-layer microwaveguide, a microstrip antenna  $30 \mu\text{m}$  wide was placed in the input section of the structure under study, to which a UHF signal was applied. The structure was tangentially magnetized using an external magnetic field created by a GMW 3472-70 electromagnet, the field was directed along the axis  $y$  to excite the surface magnetostatic wave (SMSW). The signal was received during the microwave experiment by an output antenna. Fig. 2, it shows the frequency dependence of the complex gain modulus  $S_{21}$  measured with the E8362C vector network analyzer. The shown amplitude-frequency characteristic (AFC) shows the presence of two transmission zones for two-layer magnon waveguide in low-frequency (LF) (2.92–3.01 GHz) and high-frequency (HF) (3.61–4.0 GHz) bands. It can be seen that when the



**Figure 2.** Frequency dependence of the modulus of the complex transfer coefficient  $S_{21}$  in the structure under study, obtained with the help of experimental study.

direction of the external magnetic field changes, one can see how the bandwidth changes in both LF and HF regions. In turn, one can observe a shift in the AFC depending on the applied power of the signal. The nonlinear shift of frequency in the HF region ( $\Delta f_{\text{HF}}$ ) changes with the field polarization change; in the LF region ( $\Delta f_{\text{LF}}$ ) this shift also changes, but in the LF region the shift is higher than that in the HF region. This effect manifests itself, among other things, due to the constant location of the microantenna on the surface of one of the layers when the field polarization changes. Thus, in addition to the fact that the spin wave amplitude decreases in the case of negative direction of the external magnetic field because the microstrip transducer was located on one side of the sample, the nonlinear effects are most pronounced in the system in the layer where the power of the spin wave is greater. And by changing the field polarization, the nonlinear shift in the LF region becomes larger than in the HF region ( $\Delta f_{\text{LF}}^- > \Delta f_{\text{HF}}^-$  and  $\Delta f_{\text{LF}}^+ < \Delta f_{\text{HF}}^+$ ).

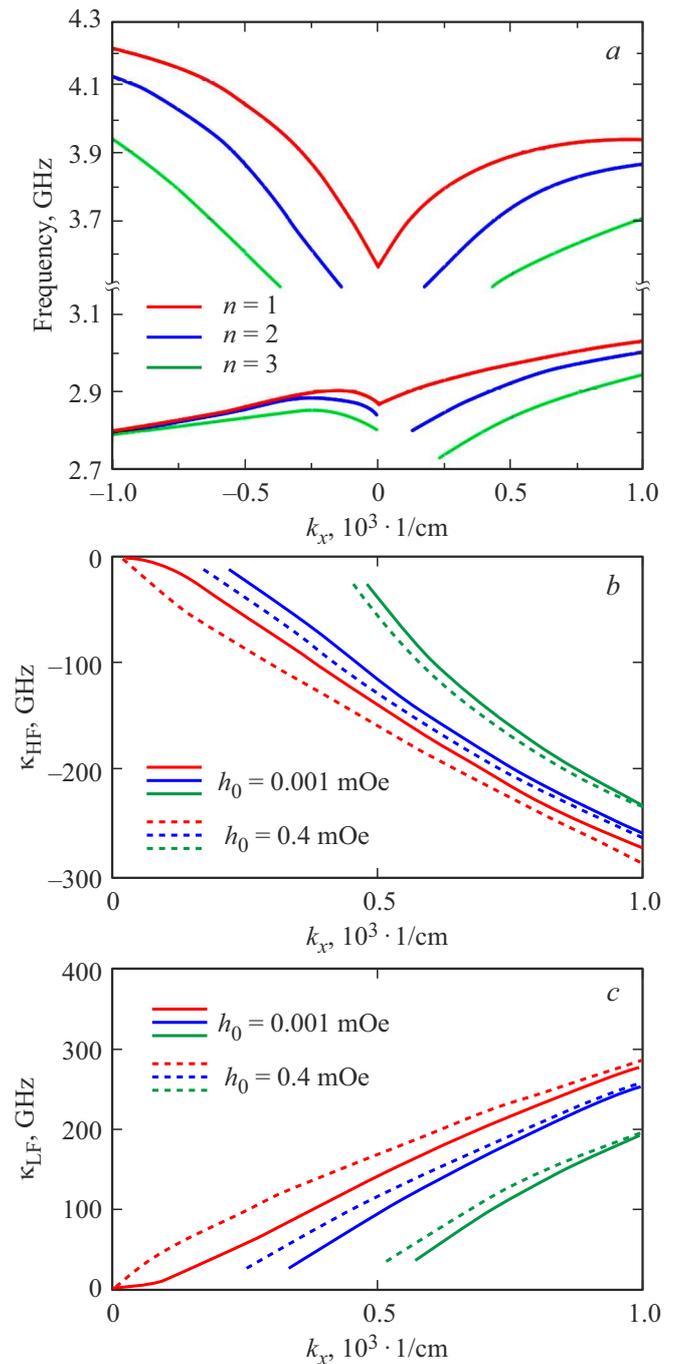
## 5. Results of numerical modelling

Fig. 3, *a* shows the dispersion characteristics of the first three SW modes ( $n = 1, 2, 3$ ) propagating in the two-layer magnon waveguide under study, which were obtained by solving the dispersion equations described above. It can be seen that with a change in the field polarization, the dispersion characteristics change strongly, especially in the low-frequency range, which shows the strongly nonreciprocal behavior of the SW in the two-layer structure.

To estimate the nonreciprocity phenomenon, we use the nonreciprocity coefficient as  $\kappa_{\text{HF}} = f_+ - f_-$ , where  $f_+$  is the SW propagation frequency in the positive direction of the axis  $y$ , and  $f_-$  is the SW propagation frequency in the negative direction of the axis  $x$  with the same wavenumber  $k$ . The nonreciprocity coefficient for the lower branch of the dispersion characteristic  $\kappa_{\text{LF}} = f_+ - f_-$  is also determined. Both coefficients are shown in Fig. 3, *b*, *c* at  $h_0 = 0.001$  mOe and  $h_0 = 0.4$  mOe, which corresponds to the power  $P_{\text{in}} = -25$  dBm and  $P_{\text{in}} = 14$  dBm, respectively. Thus, with the wave number increasing the nonreciprocal coefficient decreases for the upper branch of the dispersion characteristic and increases for the lower branch, while power increasing affects the nonreciprocity coefficient in such a way that in the LF region this leads to  $\kappa_{\text{LF}}$  decreasing, and in HF-region, alternatively to  $\kappa_{\text{HF}}$  increasing.

## 6. Conclusion

A study was made of the modes of the spin-wave signal propagating in a two-layer magnon waveguide. The magnetostatic approach and numerical simulation of the eigenproblem demonstrated the phenomenon of nonreciprocity in the frequencies of spin waves. The distribution profiles of the internal field value and the nonreciprocity coefficient for a two-layer structure were calculated. The transformation



**Figure 3.** (a) Dispersion characteristics of the first three SW modes (red line —  $n = 1$ , blue line —  $n = 2$ , green line —  $n = 3$ ) obtained using the analytical model; (b) nonreciprocity coefficient  $\kappa_{\text{HF}}$  for the HF region at different values of  $h_0$ ; (c) nonreciprocity coefficient  $\kappa_{\text{LF}}$  for the HF region at different values of  $h_0$ .

of dispersion curves propagating in two opposite directions is revealed. On the other hand, it is shown that two-layer structures support two frequency bands for the spin waves propagation. With the help of the made model and the solution of nonlinear dispersion equations the nonlinear dynamics of waves propagating in the structure under study

was shown. The data obtained are in good agreement with the results of the experimental study, which, in turn, confirms the possibility of spin wave propagation in the low-frequency and high-frequency regions, as well as frequency shifts in these regions with the signal power change. These results open up new ways to fabricate nonreciprocal magnon devices that also use the nonlinear properties of spin waves.

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### Conflict of interest

The authors declare that they have no conflict of interest.

### References

- [1] S.A. Nikitov, A.R. Safin, D.V. Kalyabin, A.V. Sadovnikov, E.N. Beginin, M.V. Logunov, M.A. Morozova, S.A. Odintsov, S.A. Osokin, A.Yu. Sharaevskaya, Yu.P. Sharaevskiy, A.I. Kirilyuk. *UFN*, **190**, 1009 (2020) (in Russian).
- [2] I.V. Vetrova, M. Zelent, J. Šolýs, V.A. Gubanov, A.V. Sadovnikov, T. Šcepka, J. Dérer, R. Stoklas, V. Cambel, M. Mruzckiewicz. *Appl. Phys.Lett.* **118**, 212409 (2021)
- [3] A.G. Gurevich, G.A. Melkov. *Magnetization Oscillations and Waves*. CRC Press, London (1996).
- [4] D.D. Stancil, A. Prabhakar. *Spin Waves: Theory and Applications*. Springer, (2009).
- [5] N.N. Rozanov. Dissipativnye opticheskie solitony. *UFN* **170**, 4, 462 (2000) (in Russian).
- [6] P.E. Zilberman, S.A. Nikitov, A.G. Timiryazev. *Pisma v ZhTF*, **42**, 3, 82 (1985) (in Russian).
- [7] A.D. Boardman, S.A. Nikitov, N. Waby. *Phys. Rev. B* **48**, 13602 (1993).
- [8] M. Chen, M. Tsankov, J. Nash, C. Patton. *Phys. Rev. Lett.* **70**, 1707 (1993).
- [9] R.W. Damon, J.R. Eshbach. *J. Phys. Chem. Solids*. **19**, 308 (1961).
- [10] T.W. O’Keeffe, R.W. Patterson. *J. Appl. Phys.* **49**, 4886 (1978).
- [11] S.N. Bajpai. *J. Appl. Phys.* **58**, 910 (1985).
- [12] M.A. Morozova, S.V. Grishin, A.V. Sadovnikov, D.V. Romanenko, Yu.P. Sharaevskii, S.A. Nikitov. *Appl. Phys. Lett.* **107**, 242402 (2015).
- [13] A.V. Chumak, P. Kabos, M. Wu, C. Abert, C. Adelman, A. Adeyeye, J. Akerman, F.G. Aliev, A. Anane, A. Awad. Roadmap on Spin-Wave Computing, *IEEE Transactions on Magnetics* (2002). DOI:10.1109/TMAG.2022.3149664.
- [14] H. Suhl. *J. Phys. Chem. Solids* **1**, 209 (1957).
- [15] V.E. Demidov, M. Evelt, V. Bessonov, S.O. Demokritov, J.L. Prieto, M. Munoz, J. Ben Youssef, V.V. Naletov, G. de Loubens, O. Klein, M. Collet, P. Bortolotti, V. Cros, A. Anane. *Sci. Rep.* **6**, 32781 (2016).
- [16] U.-H. Hansen, V.E. Demidov, S.O. Demokritov. *Appl.Phys. Lett.* **94**, 252502 (2009).
- [17] A.V. Sadovnikov, E.N. Beginin, M.A. Morozova, Yu.P. Sharaevskii, S.V. Grishin, S.E. Sheshukova, S.A. Nikitov. *Appl. Phys. Lett.* **109**, 042407 (2016).
- [18] S.O. Demokritov. In: *Topology in Magnetism*. Springer Series in Solid-State Sciences / Eds J. Zang, V. Cros, A.Hoffmann. Springer, Cham (2018). V. 192.
- [19] A. Vansteenkiste, J. Leliaert, M. Dvornik, M. Helsen, F. Garcia-Sanchez, B.V. Waeyenberge. *AIP Advances* **4**, 10, 107133 (2014).
- [20] N. Vukadinovic, J. Ben Youssef, V. Castel, M. Labruno. *Phys. Rev. B* **79**, 184405 (2009).
- [21] C. Yang, H. Kuihua T. Zhaocai, L. Jinxiao, Ji Tongtong, Z. Xian. *Alloys Compd.* **860**, 158235 (2021).
- [22] J.P. Parekh, L.-P. Peng, H.S. Tuan. *J. Appl. Phys.* **85**, 4862 (1999).
- [23] T. Wolfram. *J. Appl. Phys.* **41**, 11, 4748 (1970).
- [24] M.S. Sodha, N.C. Srivastava. *Microwave Propagation in Ferrimagnetics*. Springer US (1981).
- [25] A.K. Zvezdin, A.F. Popkov. *ZhETF*, **84**, 2, 606 (1988) (in Russian).
- [26] B.A. Kalinikos, N.G. Kovshikov, A.N. Slavin. *ZhETF* **94**, 2, 159 (1983) (in Russian).
- [27] P.R. Emtage. *J. Appl. Phys.* **49**, 4475 (1978).

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