Interaction of an electromagnetic H-wave with a semiconductor nanolayer

© I.A. Kuznetsova, O.V. Savenko

Demidov State University,
150003 Yaroslavl, Russia

E-mail: kuz@uniyar.ac.ru

Received March 2, 2022
Revised March 25, 2022
Accepted March 25, 2022

A theoretical model of the electromagnetic H-wave interaction with a semiconductor nanolayer, the thickness of which can be comparable to or less than charge carrier de Broglie wavelength is constructed. We assume the frequency range of the electromagnetic wave to be much less than the plasma frequency. Analytical expressions are derived for optical coefficients as the functions of the dimensionless thickness, electromagnetic wave frequency and incidence angle, chemical potential, and surface roughness parameters. The results derived for the limiting cases of degenerate and nondegenerate electron gas are analyzed.

Keywords: nanolayer, Liouville equation, de Broglie wavelength, Soffer model, optical coefficients.

DOI: 10.21883/SC.2022.08.54116.33

1. Introduction

Studying the optical characteristics of the semiconductor layers is of considerable interest for researchers due to rapid development of the nanotechnologies in recent decades. The developments are taken to increase the efficiency and the energy efficiency of multi-layer solar elements [1–3]. It includes active development of the technologies of manufacturing and growing lamellar nanostructures, which can create layers of the thickness of several atomic layers. In this regard, there is an increasing number of the studies dedicated to theoretical and experimental research of the resonance transmission of the charge carriers through the multi-layer quantum-sized structures [4–8]. The gas of the free charge carriers in layers of the thickness of about an atomic one can be regarded as a quasi-two-dimensional gas contained in a special well with endlessly high walls. With small thicknesses, the surface irregularity at the atomic level substantially affects the transport of the charge carriers in the nanolayer. Thus, there is interest in the studies of transfer phenomena in the nanolayers taking into account quantization of the energy spectrum of the charge carriers and the surface scattering.

The first known scientific studies, which examine the effects of dimensional quantization in the semiconductor and half-metallic films, were published in the middle of the 20-th century [9–12]. They substantiate in detail the causes and conditions of occurrence of the quantum dimensional effect. There are studies, whose authors used the various methods taking into account the roughness of the surface for solving the quantum problem on the static conductivity of the metallic film: the method of Green's functions [13–15] and direct calculation of the transfer probability of the charge carrier as a result of scattering processes [16–18].The studies [19,20] have solved the above-mentioned problem using the boundary conditions of Soffer [21] and examined the cases of a metal and a semiconductor with arbitrary degeneracy. There are some studies dedicated to taking into account the effect of dimensional quantization for solving the problems of interaction of the IR electromagnetic radiation with a super-thin metal film [22–26].The authors of these studies have restricted themselves in a case of the smooth surface and have not used the strict mathematical calculation based on the solution of the Liouville's quantum equation. Thus, the question on impact of the quantum electron transport on the electric and optical characteristics of the conductive nanolayers is still underinvestigated and requires additional investigation.

The present study has built a theoretical model of the interaction of the electromagnetic radiation with the semiconductor nanolayer. The problem was solved by the method similar to the studies [19,20], which takes into account the surface carrier scattering by the Soffer's boundary conditions imposed on the Liouville's equation.

2. Problem formulation

Let us consider the semiconductor nanolayer of the thickness a within the field of the flat monochromatic electromagnetic wave. Let us introduce the coordinate system so that the coordinate axes X and Y are directed in parallel to the plane of the nanolayer, so is the axis Z perpendicular thereto. It is suggested that the vector of the electric field strength is parallel to the axis X (the H-configuration). The nanolayer thickness can be comparable or less than the de Broglie wavelength of the charge carrier. In this case, the energy spectrum of the charge carrier will be perpendicularly discrete and longitudinally continuous. In case of the spherically-symmetrical energy band, the full

energy of the electron (hole) is expressed as follows:

$$\varepsilon_l = \frac{m}{2}(v_x^2 + v_y^2) + \varepsilon_{zl}, \tag{1}$$

where $\varepsilon_{zl} = (\pi\hbar l)^2/(2ma^2)$ — the eigenvalue of the charge carrier energy at the l-th subband, m — the effective mass of the electron (hole), $\hbar$ — the Planck constant.

The system of the charge carriers is characterized by the density operator [27]:

$$\hat{\rho}(z, \mathbf{k}_\parallel, t) = \sum_l W_l |\psi_l(z, \mathbf{k}_\parallel, t)\rangle\langle\psi_l(z, \mathbf{k}_\parallel, t)|, \tag{2}$$

which complies with the Liouville's quantum equation.

$$i\hbar\frac{\partial\hat{\rho}}{\partial t} = [\hat{H}, \hat{\rho}]. \tag{3}$$

Here, ψ_l — the wave function of the system of the charge carriers, W_l — the statistical weight characterizing the probability of the system being the state ψ_l, $\hat{H}$ — the Hamiltonian of the system, $\mathbf{k}_\parallel$ — the longitudinal component of the wave vector.

The present study suggests a small deviation of the system of the charge carriers from the equilibrium state. The density operator will be written as a sum of the equilibrium operator $\hat{\rho}^{(0)}$ and the non-equilibrium correction $\hat{\rho}^{(1)}$:

$$\hat{\rho}(z, \mathbf{k}_\parallel, t) = \hat{\rho}^{(0)} + \hat{\rho}^{(1)}(z, \mathbf{k}_\parallel)\exp(-i\omega t). \tag{4}$$

A case of the semiconductor with arbitrary degeneracy is examined. The operator of the density of the equilibrium system of the charge carriers will be written as

$$\hat{\rho}^{(0)} = \frac{1}{1 + \exp\big((\hat{H}_0 - \mu)/k_0T\big)}, \tag{5}$$

where $\hat{H}_0$ — the Hamiltonian of the equilibrium system, μ — the chemical potential, k_0 — the Boltzmann's constant, T — the temperature.

Using (4) and allowing the elastic carrier scattering, the Liouville equation (3) can be reduced to the form (6) by the method similar to the studies [19,20]:

$$-i\omega f_l^{(1)} + v_{zl}\frac{\partial f_l^{(1)}}{\partial z} + \frac{e\mathbf{E}}{\hbar}\frac{\partial f_l^{(0)}}{\partial \mathbf{k}_\parallel} = -\frac{f_l^{(1)}}{\tau}, \tag{6}$$

where τ — the relaxation time, f_l — the function of distribution of the charge carriers on the l-th subband, which acts as a diagonal element of the density matrix ρ_{ll}. For the function f_l, the expansion similar to the density operator is true:

$$f_l(z, \mathbf{k}_\parallel, t) = f_l^{(0)} + f_l^{(1)}(z, \mathbf{k}_\parallel)\exp(-i\omega t), \tag{7}$$

$$f_l^{(0)} = \frac{1}{1 + \exp\big((\varepsilon_l - \mu)/k_0T\big)}. \tag{8}$$

The boundary conditions are described by the Soffer model, which takes into account the dependence of the reflectorizing coefficients of the surfaces of the $q_{1,2}$ nanolayer on the $g_{1,2}$ roughness parameters and the charge carrier incidence to the internal surface of the ϑ nanolayer:

$$\begin{cases} f_l^{(1)+} = q_1(g_1, \vartheta) f_l^{(1)-} & \text{atz} = 0, \\ f_l^{(1)-} = q_2(g_2, \vartheta) f_l^{(1)+} & \text{atz} = a, \end{cases} \tag{9}$$

$$q_{1,2}(g_{1,2}, \vartheta) = \exp\big(-(4\pi g_{1,2}\cos\vartheta)^2\big), \tag{10}$$

$$g_{1,2} = \frac{g_{s1,2}}{\lambda_{\mathbf{B}}}, \tag{11}$$

where $f_l^{(1)\pm}$ — the functions of the distribution of the electrons (holes) with a positive and negative projection of the wave vector on the axis Z, respectively, $g_{s1,2}$ — the mean square height of the surface relief of the lower and upper surface, respectively, λ_{B} — the de Broglie wavelength of the charge carrier.

The found function of distribution allows calculating the density of the current j induced by the electromagnetic wave and the integral conductivity σ_a [19,20] by the formulas (12), (13):

$$j = 2e\left(\frac{m}{h}\right)^3 v_1 \sum_l \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v_x(f_l^{(1)+} + f_l^{(1)-})dv_x dv_y, \tag{12}$$

$$\sigma_a = \int_0^a \frac{j}{E_x}dz, \tag{13}$$

where v_1 — the projection of the velocity of the electron (hole) at the first subband to the axis Z.

The present study suggests that the range of the electromagnetic-radiation frequencies is limited from above the frequency of the plasma resonance. The electromagnetic wave is weak, so the effects related with the quantum nature of the electromagnetic radiation are not taken into account. The behavior of the electromagnetic wave can be described by the Maxwell's equations [28]:

$$\begin{cases} \dfrac{\partial E_x}{\partial z} = ikH_y, \\ \dfrac{\partial H_y}{\partial z} - ik(1 - \sin^2\theta)E_x = -\dfrac{4\pi}{c}j. \end{cases} \tag{14}$$

Here k — the module of the wave vector, θ — the incidence of the electromagnetic wave, c — the speed of light in vacuum.

By solving the system (14) of the equations, we can determine the relation between the components of the strength of the electric and magnetic field and the current density induced by the electromagnetic wave (the integral conductivity). This relation allows finding the surface impedances and the optical coefficients.

3. Mathematical calculations

Let us note that the problem is solved by the method similar to the studies [19,20]. By solving the equation (6) taking into account the boundary conditions (9), we obtain the following expressions for the functions $f_l^{(1)\pm}$:

$$f_l^{(1)+}(z) = -\frac{ev_x E_x}{\nu}\frac{\partial f_l^{(0)}}{\partial \varepsilon}(1-\phi_l^+ e^{-\Omega_l \xi}), \quad (15)$$

$$f_l^{(1)-}(z) = -\frac{ev_x E_x}{\nu}\frac{\partial f_l^{(0)}}{\partial \varepsilon}(1-\phi_l^- e^{-\Omega_l (1-\xi)}), \quad (16)$$

$$\phi_l^+ = \frac{(1-q_1)+q_1(1-q_2)e^{-\Omega_l}}{1-q_1q_2e^{-2\Omega_l}}, \quad (17)$$

$$\phi_l^- = \frac{(1-q_2)+q_2(1-q_1)e^{\Omega_l}}{1-q_1q_2e^{2\Omega_l}}, \quad (18)$$

$$\Omega_l = a\nu/v_{zl}, \quad \xi = z/a. \quad (19)$$

Here $\nu = \tau^{-1} - i\omega$ — the complex frequency of the carrier scattering. By substituting (15), (16) in the expression for the current density (12) and the integral conductivity (13), we obtain

$$\sigma_a = \frac{2\pi e^2 \nu_1 a}{\nu k_0 T}\left(\frac{m}{h}\right)^3\left(\frac{2k_0T}{m}\right)^2 \times \sum_{l=1}^{\infty}\ln\left(\exp\left(\frac{\mu-\varepsilon_{zl}}{k_0T}\right)+1\right)(1-\chi(\Omega_l)), \quad (20)$$

$$\chi(\Omega_l) = \frac{1}{2\Omega_l}(1-e^{-\Omega_l}) \times \frac{2-q_1-q_2+(q_1+q_2-2q_1q_2)e^{-\Omega_l}}{1-q_1q_2e^{-2\Omega_l}}. \quad (21)$$

Let's introduce the dimensionless parameters:

$$u_{zl} = \frac{\varepsilon_{zl}}{k_0T}, \quad u_\mu = \frac{\mu}{k_0T}, \quad (22)$$

$$x_0 = \frac{a}{\lambda_{B0}}, \quad x_\lambda = \frac{\Lambda}{\lambda_{B0}}, \quad y_0 = \omega\tau_v, \quad (23)$$

$$z_0 = \nu\tau_v = \frac{\tau_v}{\tau} - i\omega\tau_v = \kappa - iy_0, \quad (24)$$

where Λ — the length of the free path of the charge carriers taking into account the volume scattering. The present study assumes that Λ is determined by thermal oscillations of the crystal lattice and the presence of an impurity, i.e. it does not depend on the thickness. The parameters u_{zl} and u_μ characterize respectively a discrete components of the full energy of the charge carrier and the chemical potential, as rated to k_0T. y_0 — the product of the frequency of the electromagnetic wave by the time of the free path of the charge carriers in the macroscopic sample τ_v. The parameters x_0 and x_λ are a thickness of the nanolayer and a length of the free path of the charge carriers rated to the de Broglie wavelength of the charge carrier λ_{B0}, moving at some characteristic speed v_{0v}. Since for parameter rating, it is necessary to use the values independent on the nanolayer thickness, the for v_{0v} one accepts the mean square speed of motion of the charge carrier without quantization of the energy spectrum [20]:

$$n_v v_{0v}^2 = 2\left(\frac{m}{h}\right)^3\frac{5}{3}\iiint v^2 f^{(0)} d^3v, \quad (25)$$

where n_v — the concentration of the charge carriers in the macroscopic sample, $f^{(0)}$ — the equilibrium function of distribution of the charge carriers in a classic case.

Using the standard expression for the concentration of the charge carries [20] in case of the arbitrary degeneracy and integrating the expression (25), we obtain

$$v_{0v} = \left(\frac{10}{3}\frac{k_0T}{m}\frac{I_{3/2}}{I_{1/2}}\right)^{1/2}, \quad (26)$$

$$I_s = \int_0^\infty \frac{u^s du}{\exp(u-u_\mu)+1}. \quad (27)$$

The z_0 parameter includes the ratio of the times of the free path in the quantum and classical cases κ. From the condition of independence of Λ on the thickness, we obtain the following expression for κ:

$$\kappa = \frac{\tau_v}{\tau} = \frac{v_0}{v_{0v}}, \quad (28)$$

where v_0 — the characteristic speed proportional to the mean square speed of the charge carrier taking into account the quantization of the energy spectrum, which is determined as follows [19,20]:

$$nv_0^2 = 4\left(\frac{m}{h}\right)^3 v_1 \frac{5}{3}\sum_{l=1}^{\infty}\iint (v_\parallel^2 + v_{zl}^2) f_l^{(0)} d^2v, \quad (29)$$

where $v_\parallel$ — the longitudinal component of the speed in the nanolayer, n — the concentration of the charge carriers in the nanolayer determined by the expression [19,20]

$$n = 4\left(\frac{m}{h}\right)^3 v_1 \sum_{l=1}^{\infty}\iint f_l^{(0)} d^2v. \quad (30)$$

Taking into account (29), (30), the expression for the speed v_0 takes the following form:

$$v_0 = \left(\frac{10}{3}\frac{k_0T}{m}\frac{K}{P}\right)^{1/2}, \quad (31)$$

$$K = \sum_{l=1}^{\infty}\int_{u_{zl}}^{\infty}\frac{u du}{\exp(u-u_\mu)+1}, \quad (32)$$

$$P = \sum_{l=1}^{\infty}\ln\left(\exp(u_\mu - u_{zl})+1\right). \quad (33)$$

By substituting the dimensionless parameters in the expression for the conductivity, we obtain:

$$\sigma_a = \sigma_0 a \Sigma; \tag{34}$$

$$\sigma_0 = \frac{n_v e^2 \tau_v}{m}, \tag{35}$$

$$\Sigma = \frac{\sqrt{u_{0v}}}{2x_0 I_{1/2} z_0} \sum_{l=1}^{\infty} \ln\left(\exp(u_\mu - u_{zl}) + 1\right)\left(1 - \chi\left(\frac{2x_0^2 z_0}{l x_\lambda}\right)\right), \tag{36}$$

$$q_{1,2}(g_{1,2}, \vartheta) = \exp\left(-(2\pi g_{1,2} l / x_0)^2\right), \tag{37}$$

$$u_{0v} = \frac{m v_{0v}^2}{2k_0 T}. \tag{38}$$

Here σ_0 — the classical conductivity of the macroscopic sample.

The problem of interaction of the electromagnetic wave of the H-configuration with the thin metal layer was solved in the study [28] in the quasi-classical approximation. Using the system (14) of the equations and taking into account that the wavelength of the electromagnetic radiation is much bigger than the nanolayer thickness, the authors of the study [28] have obtained the relation between the reflectances R, the coefficients of transmission T, absorption A and the integral conductivity σ_a:

$$R = \frac{B^2}{|1 + B|^2}, \tag{39}$$

$$T = \frac{1}{|1 + B|^2}, \tag{40}$$

$$A = 1 - R - T = \frac{2\mathrm{Re}(B)}{|1 + B|^2}, \tag{41}$$

$$B = \frac{2\pi\sigma_a}{c \cos\theta}. \tag{42}$$

The function B included in the expressions (39)–(41), is proportional to the nanolayer conductivity and the mobility of the charge carriers. It characterizes the responsiveness of the charge carriers to the external electromagnetic radiation. If this function is zero, then the reflectances and the coefficients of absorption will be zero, so will the coefficient of transmission unity. In this case, the electromagnetic wave is passing through the nanolayer: the system of the charge carriers will neither absorb the radiation nor participate in the formation of the reflected wave. By substituting the expression for the integral conductivity (34) in the function B (42), we obtain

$$B = \frac{1}{2} \frac{\Sigma \rho s^2}{\cos\theta} \frac{x_0}{x_\lambda}. \tag{43}$$

Here, the additional dimensionless parameters are introduced:

$$\rho = \frac{v_{0v}}{c}, \quad s = \omega_p \tau_v, \tag{44}$$

where c — the speed of light in vacuum, ω_p — the plasma frequency.

Let us note that the above-described theoretical model is built for the case of the semiconductor with the arbitrary degeneracy. Such parameters, as the band gap E_g, the concentration of the donor N_D and acceptor N_A impurity will affect the degeneracy degree of the conductor. With the increase in the band gap, the valence band and the conductivity band are widened: the valence band ceiling is shifted downward in relation to the Fermi level, so is a bottom of the conductivity band upward. Therefore, with the increase in E_g for the electron and hole gases the chemical potential will go into the negative range of the values. This results in the decrease of the degree of the degeneracy of the electron and hole gas. The increase in the concentration of the donor impurity shifts the Fermi level upward, thereby resulting in the degeneracy degree of the electron gas and the decrease in the degeneracy of the hole gas. And vice versa, the acceptor impurity increases the degeneracy degree of the hole gas and decreases the degeneracy degree of the electron gas.

3.1. Limit cases

Let us examine the case of the degenerate electron gas corresponding to the condition $u_\mu \gg 1$. This case is contributed by the high concentration, the small effective mass of the charge carriers and the low temperature. The equilibrium function of distribution takes the form of the stepped approximation:

$$f_l^{(0)}(\varepsilon_l) = \begin{cases} 1, & 0 < \varepsilon_l < \varepsilon_\mathrm{F}, \\ 0, & \varepsilon_l > \varepsilon_\mathrm{F}, \end{cases} \tag{45}$$

where ε_F — the Fermi energy.

In this case, the exponent in the expression (36) is much higher than unity. It can be written:

$$\ln\left(\exp\left(u_\mu - \frac{u_{0v} l^2}{4x_0^2}\right) + 1\right) \approx u_\mu - \frac{u_{0v} l^2}{4x_0^2}. \tag{46}$$

From the expression for the distribution function (45) it follows that the charge carriers occupy a limited number of the subbands, whose number is found as the ration of the wave number of the charge carrier at the highest subband k_N to the wave number of the charge carrier at the first subband k_1:

$$N = \frac{k_N}{k_1} = \left[\frac{k_\mathrm{F}}{k_1}\right] = [2x_0], \tag{47}$$

where k_F — the wave vector of the charge carrier with the Fermi energy. It is followed from the above said that the upper limit of summing in the expression (36) will be the number N (47). The reflectances, the coefficients of transmission, absorption will be determined by the expressions (39)–(41), which include the function B to be determined as follows:

$$B = \frac{3\rho s^2}{8 x_\lambda z_0 \cos\theta} \sum_{l=1}^{N} \left(1 - \frac{l^2}{4x_0^2}\right)\left(1 - \chi\left(\frac{2x_0^2 z_0}{x_\lambda l}\right)\right). \tag{48}$$

Let us proceed to the case of the nondegenerate electron gas, which corresponds to the condition $u_\mu \to -\infty$. This case is characterized by the low concentration, the big effective mass of the charge carriers and the high temperature. The equilibrium function of distribution takes the form of the classical distribution of the Maxwell-Boltzmann.

$$f_l^{(0)}(\varepsilon_l) = \exp\big((\mu - \varepsilon_l)/k_0T\big). \tag{49}$$

In contrast to the previous limit case, the exponent in the expression (35) is a small magnitude. Expanding the logarithm into a Taylor series, we obtain

$$\ln\left(\exp\left(u_\mu - \frac{u_{0v}l^2}{4x_0^2}\right) + 1\right) \approx \exp\left(u_\mu - \frac{u_{0v}l^2}{4x_0^2}\right) + \ldots \tag{50}$$

Based on the above, we obtain the following expression for the function B:

$$B = \frac{\rho s^2}{2\cos\theta x_\lambda z_0}\sqrt{\frac{5}{2\pi}}\sum_{l=1}^{\infty}\exp\left(-\frac{5l^2}{8x_0^2}\right)\left(1 - \chi\left(\frac{2x_0^2 z_0}{x_\lambda l}\right)\right). \tag{51}$$

Let us examine the quasi-classical case, in which the thickness of the nanolayer is much bigger than the de Broglie wavelength of the charge carries, but it is still comparable with their length of free path ($a \gg \lambda_B$, $a \lesssim \Lambda$). In this case, we can come from summing by the number of the subband l to the integration by the z-component of the velocity vector v_z. As a result, we obtain the following expression:

$$B = \frac{\rho s^2}{2\cos\theta}\frac{x_0}{x_\lambda z_0}\left\{1 - \frac{1}{2I_{1/2}}\int_0^{\infty}\frac{1}{\sqrt{u_z}}\ln\big(\exp(u_\mu - u_z) + 1\big)\right.$$

$$\left.\times\,\chi\left(\frac{\sqrt{u_{0v}}x_0 z_0}{x_\lambda\sqrt{u_z}}\right)du_z\right\}. \tag{52}$$

In the case of the degenerate electron gas, the expression (52) agrees with the result of the study [28].

4. Analysis of results

Figures 1–3 show the plotted dependences of the reflectances, the coefficients of transmission and absorption on the dimensionless thickness of the nanolayer. It follows from the figures that with the decrease in the thickness the reflectance is decreasing, and the coefficient of transmission is increasing. This effect can occur due to the decrease in the concentration of the free charge carriers, creating the secondary reflected wave, which is due to the reduction of the number of the allowed energy states. The coefficient of absorption is increasing with the decrease in the thickness, and when $x_0 < 0.75$ it is decreasing. At the value $x_0 \approx 0.5$ (the thickness of the nanolayer is equal to the half of the de Broglie of the charge carrier), the reflectance is close to zero, so is the coefficient of transmission to unity: the electromagnetic radiation is almost fully passing through the nanolayer.

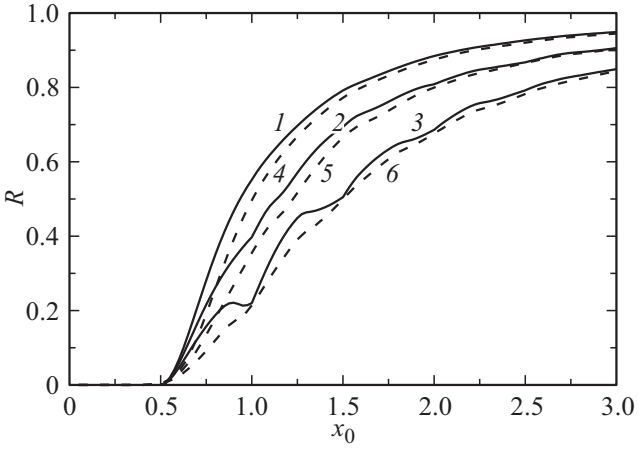

Figure 1. Dependences of the reflectance R on the dimensionless thickness x_0 at the values $g_1 = g_2 = 0.15$, $x_\lambda = 8$, $\theta = 66^\circ$, $\rho = 0.005$, $s = 200$. *1, 4* — $y_0 = 10$; *2, 5* — $y_0 = 20$; *3, 6* — $y_0 = 30$. Solid curves *1–3* are built for the case of the degenerate electron gas, so are the dashed curves *4–6* — for the non-degenerate electron gas.

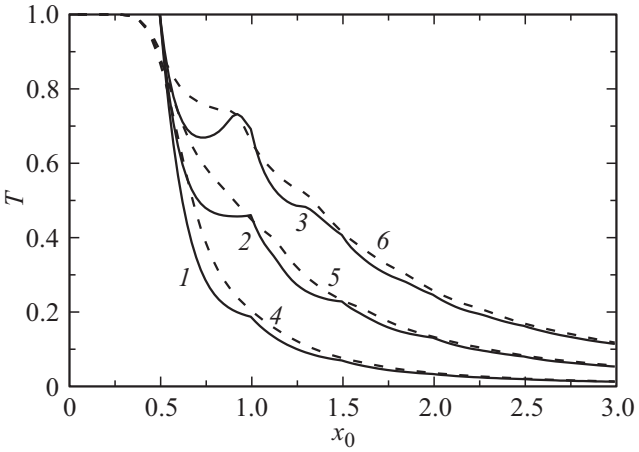

Figure 2. Dependences of the coefficient of transmission T on the dimensionless thickness x_0 at the values $g_1 = g_2 = 0.15$, $x_\lambda = 8$, $\theta = 66^\circ$, $\rho = 0.005$, $s = 200$. *1, 4* — $y_0 = 10$; *2, 5* — $y_0 = 20$; *3, 6* — $y_0 = 30$. Solid curves *1–3* are built for the case of the degenerate electron gas, so are the dashed curves *4–6* — for the non-degenerate electron gas.

There are evidently oscillations of the dependences of the optical coefficients on the thickness. In case of the degenerate electron gas the oscillations are more pronounced than in the case of the non-degenerate gas. Let us note that among all the optical coefficients the strongest oscillation is in the coefficient of absorption, while the ratio between the first maximum and the first minimum of absorption at the frequency of the incident radiation $y_0 = 30$ is equal to 4. The possible reason of occurrence of the oscillations is the following one. At the thicknesses comparable or less than the de Broglie wavelength of the charge carriers, the perpendicular component of the velocity

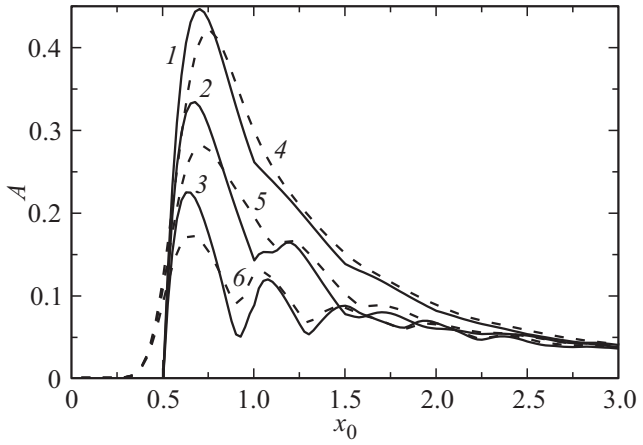

Figure 3. Dependences of the coefficient of absorption A on the dimensionless thickness x_0 at the values $g_1 = g_2 = 0.15$, $x_\lambda = 8$, $\rho = 66^\circ$, $\theta = 0.005$, $s = 200$. *1, 4* — $y_0 = 10$; *2, 5* — $y_0 = 20$; *3, 6* — $y_0 = 30$. Solid curves *1–3* are built for the case of the degenerate electron gas, so are the dashed curves *4–6* — for the non-degenerate electron gas.

vector takes a discrete number of the values. Therefore, the discrete magnitude will be the frequency of the surface scattering of the charge carrier. Provided that $\tau_s = nT$ (τ_s — the time of motion of the charge carrier from one surface of the nanolayer to another, T — the period of oscillations of the electric field strength, n — the positive integer number), the part of the charge carriers on one subband is subjected to the surface scattering when the orientation of the electric field strength is changing (when the field strength is zero). Therefore, at some values x_0 and y_0 the surface of the nanolayer will weakly affect the optical characteristics: there are the minimums of the coefficient of absorption and the maximums of that of transmission. The smooth change of the thickness results in the continuous change of the parameter τ_s, and periodically this parameter will satisfy the condition of occurrence of the above-mentioned effect $\tau_s = nT$, creating the oscillations of the dependences of the optical coefficients on the thickness. With the increase in the frequency, the oscillation maximums (minimums) are shifted towards the lesser thicknesses, and the oscillation period is decreasing. At the relatively low frequencies ($y_0 < 10$) the above-said oscillation effect is disappearing. In case of the degenerate electron gas, there are small oscillations of the coefficients of absorption and transmission, which are caused by the surges of the density states, with the period equal to the half de Broglie wavelength of the charge carriers.

Figures 4–6 show the dependences of the reflectances, the coefficients of transmission and absorption on the dimensionless thickness of the incident electromagnetic wave. It is clear from the figures that with the increase in the frequency, the reflectances and the coefficients of absorption are decreasing, while the coefficient of transmission is increasing. This behavior is related to the fact that the carriers fail to response to the high-frequency oscillations of the electric field strength. Therefore, with the increase in the frequency, the transfer of the energy of the electromagnetic wave by the charge carrier is decreasing. There are evidently the oscillations of the optical spectra. The cause of the occurrence of the oscillations is similar to Fig. 1–3, but here the period of the oscillations of the electric field strength is changing. With the change of the frequency y_0, the parameter T will periodically satisfy the condition $\tau_s = nT$ of occurrence of the minimums of absorption and the maximums of transmission. It results in the oscillating dependence of the optical spectra. We note that in the

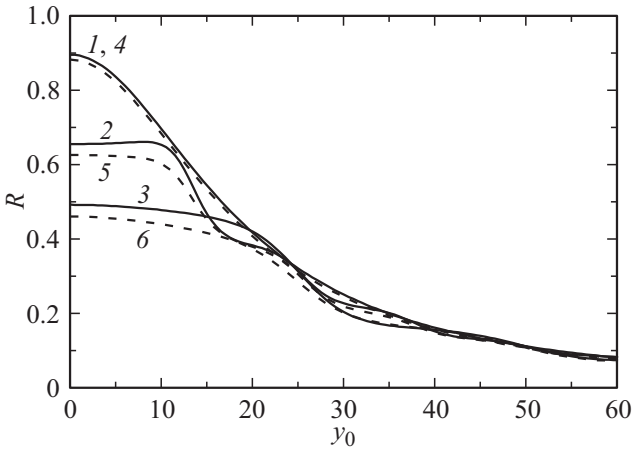

Figure 4. Dependences of the reflectance R on the dimensionless frequency of the electromagnetic wave y_0 at the values $x_0 = 1$, $x_\lambda = 8$, $\theta = 66^\circ$, $\rho = 0.005$, $s = 200$. *1, 4* — $g_1 = g_2 = 0$; *2, 5* — $g_1 = 0$, $g_2 = 0.25$; *3, 6* — $g_1 = g_2 = 0.25$. Solid curves *1–3* are built for the case of the degenerate electron gas, so are the dashed curves *4-6* — for the non-degenerate electron gas.

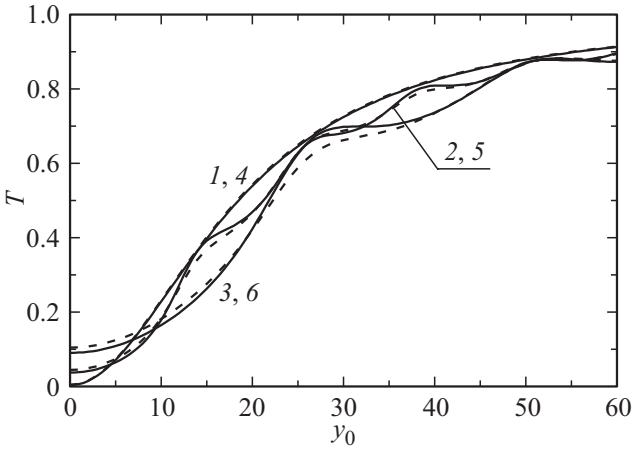

Figure 5. Dependences of the transmittance T on the dimensionless frequency of the electromagnetic wave y_0 at the values $x_0 = 1$, $x_\lambda = 8$, $\theta = 66^\circ$, $\rho = 0.005$, $s = 200$. *1, 4* — $g_1 = g_2 = 0$; *2, 5* — $g_1 = 0$, $g_2 = 0.25$; *3, 6* — $g_1 = g_2 = 0.25$. Solid curves *1–3* are built for the case of the degenerate electron gas, so are the dashed curves *4–6* — for the non-degenerate electron gas.

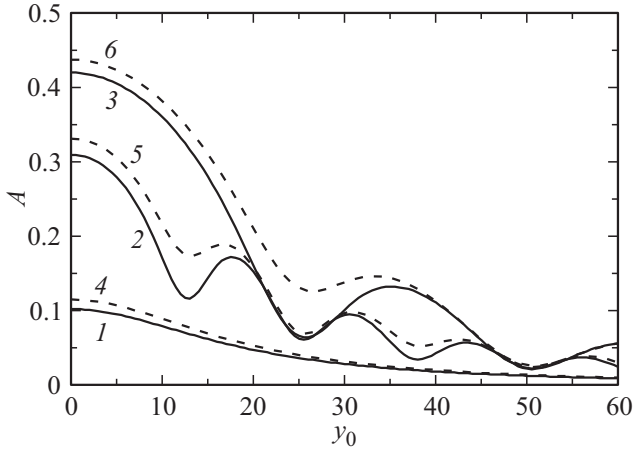

Figure 6. Dependences of the coefficient of absorption A on the dimensionless frequency of the electromagnetic wave y_0 at the values $x_0 = 1$, $x_\lambda = 8$, $\theta = 66°$, $\rho = 0.005$, $s = 200$. *1, 4* — $g_1 = g_2 = 0$; *2, 5* — $g_1 = 0$, $g_2 = 0.25$; *3, 6* — $g_1 = g_2 = 0.25$. Solid curves *1–3* are built for the case of the degenerate electron gas, so are the dashed curves *4–6* — for the non-degenerate electron gas.

case of one mirror surface and another rough surface (the curves *2* and *5*), the period of the oscillations is in two times less than in the case when both the surfaces are rough (the curves *3* and *6*).

5. Conclusion

The present study has obtained the analytical expressions for the optical coefficients of the semiconductor nanolayer taking into account the quantum theory of the transfer phenomena. It is established that the dependences of the optical coefficients on the thickness of the nanolayer and the frequency of the electromagnetic wave are of an oscillating nature. The most significant oscillations are for the coefficient of absorption, while for the degenerate electron gas the maximum value of the coefficient of absorption exceeds in 4 times the minimum value at the dimensionless frequency $y_0 = 30$. It is established that the frequency of the electromagnetic wave affects the oscillation period: with the increase in the frequency, the period is decreasing. For the degenerate electron gas the dependences of the optical coefficients on the thickness and the frequency have a more pronounced nature in comparison with the case of the non-degenerate gas.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] V.S. Kalinovskiy, E.V. Kontrosh, E.A. Grebenschikova, V.M. Andreev. ZhTF, **91** (7), 1067 (2021) (in Russian).
[2] I.E. Panajotti, E.I. Terukov, I.S. Shakhraj. Pis'ma ZhTF, **46** (17), 3 (2020) (in Russian).
[3] V.S. Kalinovskiy, E.V. Kontrosh, A.V. Andreeva, V.M. Andreev, V.V. Malyutina-Bronskaya, V.B. Zalesskii, A.M. Lemeshevskaya, V.I. Kuzoro, V.I. Khalimanovitch, M.K. Zaitseva. Pis'ma ZhTF, **45** (16), 52 (2019) (in Russian).
[4] A. Valavanis, L. Lever, C.A. Evans, Z. Ikonic, R.W. Kelsall. Phys. Rev. B, **78**, 035420 (2008).
[5] A.N. Yablonkiy, R.Kh. Zhukavin, N.A. Bekin, A.V. Novikov, D.V. Yurasov, M.V. Shaleev. FTP, **52** (12), 1629 (2016) (in Russian).
[6] S.A. Afanas'ev, D.I. Sementsov, I.V. Fedorova. ZhTF, **87**,(12), 1849 (2017) (in Russian).
[7] L.K. Orlov, N.S. Volkova, N.L. Ivina, M.L. Orlov. FTP, **52** (9), 1006 (2018)/ (in Russian).
[8] A.V. Bogatskaya, N.V. Klenov, P.M. Nikiphorova, A.M. Popov, A.E. Schegolev. Opt. i spektr., **130** (4), 481 (2022) (in Russian).
[9] B.L. Tavger, V.Ya. Demikhovskiy. UFN, **96** (1), 61 (1968) (in Russian).
[10] V.N. Lutskii. Phys. Status Solidi A, **1**, 199 (1970).
[11] M.I. Elinson, V.A. Volkov, V.N. Lutskij, T.N. Pinsker. Thin Sol. Films, **12**, 383 (1972).
[12] T. Ando, A.B. Fowler, F. Stern. Rev. Mod. Phys., **54**, 437 (1982).
[13] L. Sheng, D.Y. Xing, Z.D. Wang. Phys. Rev. B, **51**, 7325 (1995).
[14] D. Ketenoglu, B. Ünal. Physica A: Statistical Mech. and its Appl., **391**, 3828 (2012).
[15] R.C. Munoz, C. Arenas. Appl. Phys. Rev., **4**, 011102 (2017).
[16] A.E. Meyerovich, I.V. Ponomarev. Phys. Rev. B, **65**, 155413 (2002).
[17] S. Chatterjee, A.E. Meyerovich. Phys. Rev. B, **84**, 165432 (2011).
[18] R.I. Bihun, Z.V. Stasyuk, O.A. Balitskii. Physica B: Condens. Matter, **487**, 73 (2019).
[19] I.A. Kuznetzova, O.V. Savenko, D.N. Romanov. FTP, **55** (9), 789 (2021) (in Russian).
[20] I.A. Kuznetsova, O.V. Savenko, D.N. Romanov. Phys. Lett. A, **427**, 127933 (2022).
[21] S.B. Soffer. J. Appl. Phys., **38** (4), 1710 (1967).
[22] M. Xiao, R. Villagomez, L. Alvarez. J. Phys.: Condens. Matter, **12**, 2925 (2000).
[23] M. Xiao, N. Rakov. Int. J. Mod. Phys. B, **20** (7), 841 (2006).
[24] W.B. Su, C.S. Chang, T.T. Tsong. J. Phys. D: Appl. Phys., **43**, 013001 (2010).
[25] R. Villagómez, M. Xiao. Optik, **127**, 5920 (2016).
[26] V.V. Starostenko, V.B. Orlenson, A.S. Mazinov, L.N. Akhramovitch. Pis'ma ZhTF, **46** (9), 43 (2020) (in Russian).
[27] K. Blum. *Teoriya matritsy plotnosti i ee prilozheniya* (M., Mir, 1983) (in Russian).
[28] A.I. Utkin, A.A. Yushkanov. Opt. i spektr., **117** (4), 650 (2014) (in Russian).