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Method for measuring periods of waveguide diffractive optical elements

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An original method of the periods of waveguide diffractive optical elements (DOEs) measuring is proposed. In contrast to standard techniques, the proposed method provides measurements of waveguide gratings periods while the diffracted light is propagating in the substrate material. The method is simple and has a small error

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One of the most demanded holographic optical elements (HOEs) at present are waveguide holographic gratings, which form the basis of many augmented reality displays, such as HoloLens, WaveOptics, DigiLens, Magic Leap and the like [1,2] . For optimal operation of such displays, it is necessary to provide compensation for chromatic aberrations of waveguide holograms, and, consequently, the control of the periods and orientation of the gratings is extremely critical. However, the diffracted beams of waveguide diffractive optical elements (DOEs) or HOEs are located in the waveguide, and direct measurement of these parameters is difficult.

Figure 1, *a* shows the photograph of typical waveguide HOE on a glass fiber (refraction index $n_D \approx 1.51$) 2 mm thick and 200 mm long. Surface-relief diffraction grating (DG) with a small aperture is made by holographic method on the thin layer of photoresist. Its period is less than $0.5\,\mu$m, so the diffracted orders for light with wavelength of $640\,\mu$m propagate exclusively in the waveguide material. To measure the DG period by conventional methods [3,4], you can use additional output prisms or make oblique polished facets of the light fiber (waveguide), however, these methods are laborious and require accurate consideration of the geometry and refractive properties of the waveguide. The use of microscopy and micro-profilometry methods makes it possible to quickly measure the period, however, with low precision. In practice, due to the rotation of the DOE, it is possible to change the angle of incidence on the DG and remove one of the diffracted orders from the waveguide mode. In this work the simple and efficient technique for measuring the parameters of waveguide DOEs by rotating the DG by the angle of least deviation, is proposed.

The method consists in the appropriate spatial setting of the DOE and measurement of the smallest deflection angle θ' of the first diffraction order $k = \pm 1$. The desired grating period b for the used wavelength (in vacuum) λ is found by the formula

$$b = \frac{\lambda}{2\sin\frac{\theta'}{2}}. \qquad (1)$$

Under the condition of positive sign for the angles measured from the normal to the DG in a clockwise direction, the deflection angle is equal to $\theta = \alpha - \varphi$ for the transmission grating and $\theta = \varphi - \alpha$ for the reflective grating, where α and φ are the angle of incidence and the angle of diffraction, respectively (Fig. 1, *b*). Both of these angles are related by the basic equation of the diffraction grating

$$b(\sin\varphi - \sin\alpha) = k\lambda. \qquad (2)$$

When the angle of incidence α changes, the diffraction angle φ will also change, while there is such a value $\alpha = \varphi$ at which the deflection angle will be minimal, i.e. $\theta = \theta'$. The specified DG setting acquired its name as autocollimating one [5,6]. The fact that the diffracted order returns back to the source in the DG setting makes it relatively easy to perform the required measurement of the b period.

In Fig. 1, *b* the beam A incidenting at the angle α diffracts in the -1-th order at an angle φ in the forward direction from the source (beam B'). Due to the Fresnel reflection, the -1-th order also arises in the opposite direction, towards the radiation source (beam B). The beam diffracted in the $+1'$-th order, exceeding the critical angle (not indicated), propagates in the substrate material and represents the waveguide or lightguide mode. Secondarily hitting on the DG, it diffracts at the same angle α towards the source. This beam, denoted by the letter C, will always be distanced in space by some interval and parallel to the specularly reflected ray D.

By rotating the DG relative to the normal to the Figure, one can achieve equal angles $\alpha = \varphi$, while the beam B will return to the source, and θ will correspond to the smallest deflection angle θ' [7, 8]. In order to verify the validity of the $\alpha = \varphi$ condition, let us consider the beam B' propagating forward after diffraction (Fig. 1, *b*). Since the measurement of angles is performed in air, the index of the waveguide material does not affect the result, then, by substituting the diffraction equation (2) for the first order into the deflection

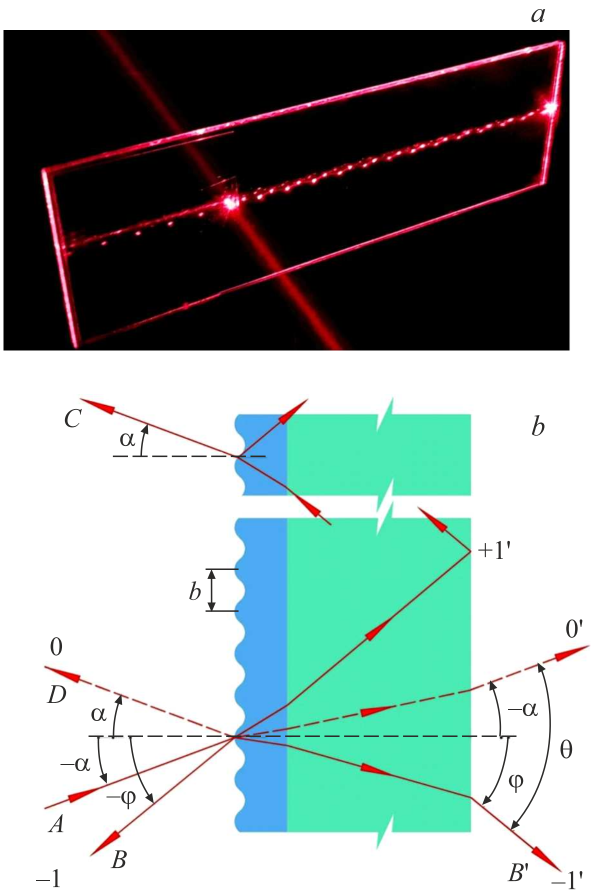

Figure 1. *a* is mode of HOE propagating in the substrate material; *b* is diffraction pattern on the waveguide DOE.

angle of the transmissive DG

$$|\theta| = |\alpha - \varphi|, \tag{3}$$

differentiating with respect to the angle α and equating the resulting expression to zero, we obtain the extremum conditions

$$\frac{d\theta}{d\alpha} = 1 - \frac{\cos\alpha}{\sqrt{1 - \left(\frac{\lambda}{b} + \sin\alpha\right)^2}} = 0. \tag{4}$$

Solving it, we obtain an expression for the angle of incidence α corresponding to the smallest deflection angle θ':

$$\alpha = \arccos\frac{\sqrt{4b^2 - \lambda^2}}{2b}. \tag{5}$$

To prove $\alpha = \varphi$, it suffices, using (5), to write down the sine of the angle

$$\sin\alpha = \sqrt{1 - \frac{4b^2 - \lambda^2}{4b^2}} = \frac{\lambda}{2b}, \tag{6}$$

to substitute it into formula (2) and deduce

$$\sin\varphi = \frac{\lambda}{b} - \frac{\lambda}{2b} = \frac{\lambda}{2b}. \tag{7}$$

To estimate the error of the method, we denote the half-angle $\theta/2 = \beta \equiv (\alpha + \varphi)/2$ in (1) and differentiate it with respect to β. It should be noted that β is the angle between the normal to the DG and the angle bisector formed by the rays A and B in the autocollimation $\beta = \alpha = \varphi = \theta'/2$. After replacing the differentials with finite increments $db \to \Delta$ and $d\beta \to \delta$ we have

$$\Delta = -\frac{\lambda\cos\beta}{2\sin^2\beta}\delta = -\frac{b}{\tan\beta}\delta. \tag{8}$$

Here Δ is the absolute error of the period b, and δ is the error in determining the angle β. The minus sign means that with positive δ errors, the period value calculated by formula (1) will decrease. Dividing (8) by the period b, we obtain an expression for the relative error

$$\varepsilon = \frac{\Delta}{b} = -\frac{\delta}{\tan\beta}. \tag{9}$$

The formula for this function is shown in Fig. 2. In the region of small diffraction angles and when using a standard turntable with accuracy of $\pm 0.1^\circ$, the proposed method is not optimal, while at angles $\beta = \theta'/2 \geqslant 10^\circ$ error no longer exceeds 1

Figure 3 shows the experimental dependence of the half-angle of deflection $\theta/2$ on the angle of incidence α, performed for the case of beam propagation in the forward direction according to the diagram shown in Fig. 1, *b* (beam B'). The Figure demonstrates the presence of extremum of the angle $\theta = \theta'$ at $\alpha = \varphi$. To measure this dependence, the DOE was mounted on the goniometer providing an accuracy of $\pm 5''$, and the laser with wavelength of $\lambda = 532$ nm was used as a light source. Based on the measured angle of least deviation $\theta' = 72^\circ 03' 08'' \pm 5''$, the period calculated by formula (1) will be $b_{gon} = 452.26 \mp 0.02$ nm. In the case of autocollimation measurement (using beam B in

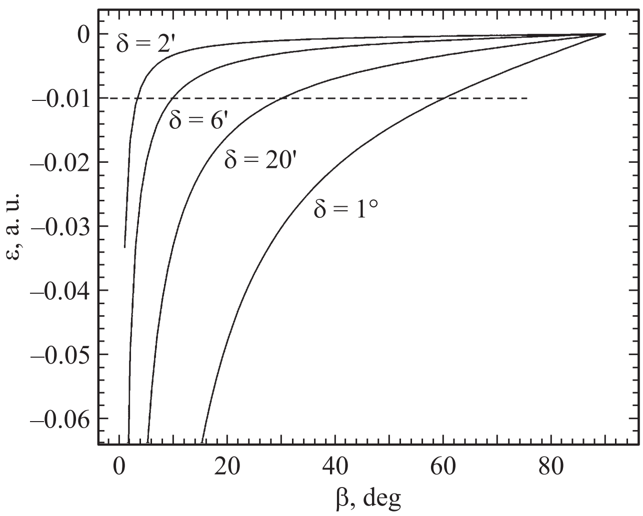

Figure 2. Error $\varepsilon = \Delta/b$ when determining the value of the period in the autocollimation mode depending on the angle $\beta = \theta'/2$.

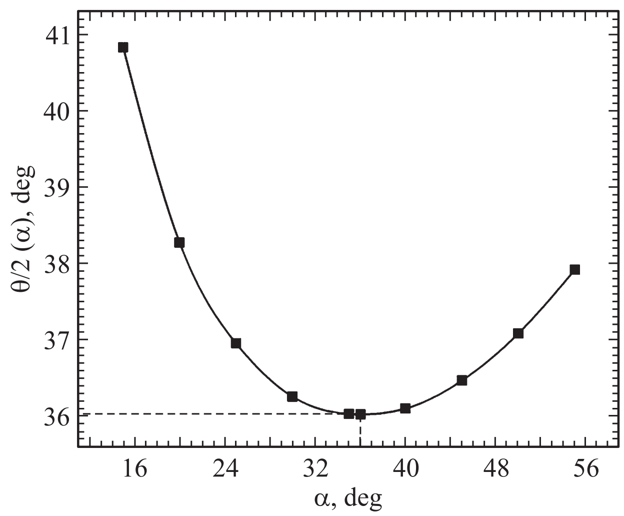

Figure 3. Experimental dependence of the half-angle $\theta/2$ on the angle of incidence α for a grating with a period $b = 452.26 \mp 0.02$ nm and the wavelength of monochromatic light used 532 nm.

Fig. 1, b) and the turntable with guaranteed accuracy of $1'$, the measured angle was $\beta = \theta/2 = 36.033 \pm 0.017°$, the period corresponding to it $b_{auto} = 452.19 \mp 0.18$ nm. The ratio of the difference between the obtained period values to the more accurate value is $\frac{b_{gon}-b_{auto}}{b_{gon}} \approx 1.5 \cdot 10^{-4}$, which in absolute comparison does not exceed 0.27 nm, taking into account an unfavorable combination of errors

In conclusion, we note a practically important case „of a multilayer" HOE used to form color images in augmented reality displays. Each layer of the HOE contains a waveguide and DP designed for specific wavelength (usually red, green and blue). It is difficult to measure such DOEs (DOEs) by conventional methods, and the proposed method makes it possible to determine the DP periods for each layer relatively simply and with high accuracy in practice, without requiring information about the waveguide material and geometry. Currently, the development of augmented (mixed) reality displays is moving in the direction of complicating the optic schemes of waveguide holograms. The development of new methods for studying waveguide DOEs under these conditions is an urgent and demanded task.

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Conflict of interest

The authors declare that they have no conflict of interest.

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