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Boundary conditions for scattering problems of exchange spin waves in inhomogeneous magnetic structures

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A method for obtaining boundary conditions for magnetization in ferromagnetic structures with an inhomogeneous ground state — a relativistic and exchange spiral caused by the presence of competing exchange interactions — is proposed in this paper within the framework of a lattice model with a subsequent passage to the limit to the continuum. It is shown that such conditions are the equations of the dynamics of the magnetization of those boundary spins in which the symmetry is broken in comparison with the internal ones.

Keywords: scattering of exchange spin waves, exchange spiral, relativistic spiral, boundary conditions for magnetization.

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1. Introduction

Magnetic materials with heterogeneous ground state are promising media for observation of nonreciprocal propagation effect of exchange spin waves (ESW). Such effects constitute a physical basis of operation of magnon gates and other devices based on magnon logic which are described in detail in [1]. Occurrence of spin helicoid in such materials may be attributable to, in particular, Dzyaloshinski–Moriya relativistic interaction (DMI) [2]. Competition of exchange interactions of the first two nearest neighbours. Nonlocal nature of exchange interaction lead to a variety of possible types of exchange structures and wave modes in them. Modulated long-periodic magnetic structures and the related investigation using elastic neutron scattering are described in [3]. In particular, the monograph addresses magnetic materials where a spin helicoid may exist, types of long-periodic structures, ESW spectrum and magnetic phase transitions in ESW.

In view of this, the problem of ESW scattering by the interface of such structures and definition of the related coefficients is of importance. To solve the problem, appropriate boundary conditions (BC) are required for magnetization components and derivatives.

ESW scattering problems [4] and ESW generation problems [5] traditionally use the following BC

$$\mathbf{M}_+ \times \mathbf{M}_- = 0, \quad \frac{\alpha_+}{M_+^z}\mathbf{M}_+ \times \mathbf{M}'_+ = \frac{\alpha_-}{M_-^z}\mathbf{M}_- \times \mathbf{M}'_-, \quad (1)$$

where $\mathbf{M}_\pm$ and $\alpha_\pm$ are medium magnetizations and exchange constants („–“ corresponds to the medium to the left of the boundary, „+“ corresponds to the medium to the right of the boundary), z is the normal coordinate to the media interface, underscore designates z derivative. The first of them reflects an ideal close coupling between boundary spins, the second is derived by formal integration of Landau–Lifshitz equation (LLE) in a small neighborhood of the boundary.

However, for the magnetization dynamics equations containing also long-range order interactions beside the homogeneous exchange, a number of BCs corresponding to their order is required. In particular, the number generated by the volumetric energy density of unlimited magnetic medium

$$w = \frac{1}{2}\left(\sigma \mathbf{M}''^2 - \alpha \mathbf{M}'^2 + \beta M_z^2\right). \quad (2)$$

LLE is of fourth order by coordinate and requires four BCs for the problem to be solved. In equation (2) β is the uniaxial anisotropy constant, σ is the non-local exchange constant.

There is quite a number of publications devoted to determination of BCs for spin dynamics equations. In particular, in [6,7] they were derived within a continuum, and in [8], they were generalized for the terminal interlayer interaction constant and take into account the influence of the averaged interface parameters on the ESW scattering. The idea of BC derivation for two neighbour ferromagnets from a lattice model of a one-dimensional spin chain by limit transition to continuum was described in [9], where they are obtained by combining LLEs for boundary spins containing different orders of smallness of lattice constant. For this, lattice functions are addressed — dynamic magnetization components whose values coincide with the values of the related continuous functions in the lattice points. An approach borrowed from [9] is generalized herein for more complex structures — long-periodic structures with competing exchange interactions and with DMI.

2. Boundary conditions for magnetic structures with DMI

Consider the following one-dimensional chain lattice Hamiltonian

$$
\begin{aligned}
-W =& A_- \sum_{n=-\infty}^{-1} \mathbf{S}_{n(-)}\mathbf{S}_{(n+1)(-)} \\
&+ A_+ \sum_{n=1}^{\infty} \mathbf{S}_{n(+)}\mathbf{S}_{(n+1)(+)} + J\mathbf{S}_{1(+)}\mathbf{S}_{1(-)} \\
&+ \mathbf{D}_- \cdot \sum_{n=-\infty}^{-1} \mathbf{S}_{n(-)} \times \mathbf{S}_{(n+1)(-)} \\
&+ \mathbf{D}_+ \cdot \sum_{n=1}^{\infty} \mathbf{S}_{n(+)} \times \mathbf{S}_{(n+1)(+)} + \mathbf{D} \cdot \mathbf{S}_{1(-)} \times \mathbf{S}_{1(+)} \\
&+ \frac{B_-}{2} \sum_{n=-\infty}^{-1} (\mathbf{S}_{n(-)}\mathbf{e}_-)^2 + \frac{B_+}{2} \sum_{n=1}^{\infty} (\mathbf{S}_{n(+)}\mathbf{e}_+)^2 \\
&+ \frac{\Sigma_-}{2} (\mathbf{S}_{1(-)}\mathbf{e}_{-s})^2 + \frac{\Sigma_+}{2} (\mathbf{S}_{1(+)}\mathbf{e}_{+s})^2, \qquad (3)
\end{aligned}
$$

where $A_\pm$ are the exchange integrals of neighbour media, J is the interlayer exchange integral, $\mathbf{D}_\pm$ are the Dzyaloshinski relativistic anisotropic exchange vectors, $\mathbf{D}$ is the Dzyaloshinski interlayer exchange vector, $B_\pm$ and $\Sigma_\pm$ are, respectively, volumetric and surface constants of single-ion uniaxial anisotropy with easy magnetization axis defined by single vectors $\mathbf{e}_\pm$ and $\mathbf{e}_{\pm \mathbf{s}}$. Enumeration of spins $\mathbf{S}_{k(\pm)}$ in each layer starts with 1 to ensure more symmetrical form of equations. In this model, the long-range order of spin interaction is not available. The Dzyaloshinski vectors in two media that determine the equilibrium magnetization rotation direction (chirality) are assumed as normal-directed to the boundary oriented along axis z.

Continuum limit is implemented by means of substitutions

$$
\mathbf{M} \to \frac{\mu_b}{sa}\mathbf{S}_n, \quad dz \to a, \quad A_\pm = \alpha_\pm \frac{\mu_b^2}{sa^2}, \quad J = G\frac{\mu_b^2}{sa^2},
$$

$$
D_\pm = d_\pm \frac{\mu_b^2}{sa^2}, \quad D = d\frac{\mu_b^2}{sa^2}, \quad B_\pm = \beta_\pm \frac{\mu_b^2}{sa},
$$

$$
\Sigma_\pm = \sigma_\pm \frac{\mu_b^2}{sa^2} \qquad (4)
$$

in the assumption that new constants are finite within wavelengths, there are many smaller lattice constants a. Here, μ_b is Bohr magneton, $\mathbf{S}_n$ is spin of n-th lattice point, G, d, $d_\pm$, $\beta_\pm$, $\sigma_\pm$ are specific exchange constants corresponding to introduced above, s is the area per one atom in the plane perpendicular to the chain direction.

Transform the exchange summands into (3):

$$
\ldots + \mathbf{S}_{n(\pm)} \frac{\mathbf{S}_{(\mathbf{n+1})(\pm)} + S_{(n-1)(\pm)}}{2} + \ldots
$$

$$
= \ldots + \mathbf{S}^2_{n(\pm)} + \frac{a^2}{z}\mathbf{S}_{n(\pm)}\mathbf{S}''_{n(\pm)} + \ldots,
$$

and summands with DMI

$$
\begin{aligned}
\ldots &+ \frac{\mathbf{D}_\pm}{2} \cdot \Big(\big[\mathbf{S}_{(n-1)(\pm)} \times \mathbf{S}_{n(\pm)}\big] + \big[\mathbf{S}_{n(\pm)} \times \mathbf{S}_{(n+1)(\pm)}\big] \Big) + \ldots \\
&= \ldots + \frac{\mathbf{D}_\pm}{2} \big[\mathbf{S}_{n(\pm)} \times (\mathbf{S}_{(n+1)(\pm)} - \mathbf{S}_{(n-1)(\pm)})\big] + \ldots \\
&= \ldots + a\mathbf{D}_\pm \big[\mathbf{S}_{n(\pm)} \times \mathbf{S}'_{\mathbf{n}(\pm)}\big] + \ldots .
\end{aligned}
$$

In the latter expression, the following may be written

$$
\begin{aligned}
\mathbf{D}_\pm \big[\mathbf{S}_{n(\pm)} \times \mathbf{S}'_{n(\pm)}\big] &= D_\pm \cdot e_z e_{zjk} S_{n(\pm)j} S'_{n(\pm)k} \\
&= -D_\pm S_{n(\pm)j} e_{jzk} S'_{n(\pm)k} = -D_\pm \mathbf{S}_{n(\pm)} \big[\boldsymbol{\nabla} \times \mathbf{S}_{n(\pm)}\big], \quad (5)
\end{aligned}
$$

if $\mathbf{D}$ is oriented along the normal to the boundary. Here, $e_z = 1$ is a single vector projection on direction z.

Taking into account (5), the surface energy is reduced to

$$
\begin{aligned}
w &= \frac{W}{s} \\
&= \int_{-\infty}^{0} \Big(-\frac{\alpha_-}{2}\mathbf{M}_-\Delta\mathbf{M}_- + d_-\mathbf{M}_-[\boldsymbol{\nabla}\times\mathbf{M}_-] - \frac{\beta_-}{2}(\mathbf{M}_-\mathbf{e}_-)^2 \Big) dz \\
&+ \int_{0}^{\infty} \Big(-\frac{\alpha_+}{2}\mathbf{M}_+\Delta\mathbf{M}_+ + d_+\mathbf{M}_+[\boldsymbol{\nabla}\times\mathbf{M}_+] - \frac{\beta_+}{2}(\mathbf{M}_+\mathbf{e}_+)^2 \Big) dz \\
&- G\mathbf{M}_-(0)\mathbf{M}_+(0) - \mathbf{D} \cdot \mathbf{M}_-(0) \times \mathbf{M}_+(0) \\
&- \frac{\sigma_-}{2}(\mathbf{M}_-(0)\mathbf{e}_{-s})^2 - \frac{\sigma_+}{2}(\mathbf{M}_+(0)\mathbf{e}_{+s})^2. \qquad (6)
\end{aligned}
$$

Write LLE for boundary spins in each medium

$$
\hbar S_0 \dot{\mathbf{S}}_n = -\left[\mathbf{S}_n \times \frac{\partial W}{\partial \mathbf{S}_n}\right], \qquad (7)
$$

where S_0 is the spin magnitude. Using Hamiltonian (3), we obtain (7) in the form of

$$
\begin{aligned}
&S_{0(-)}\hbar\dot{\mathbf{S}}_{-1(-)} \\
&= \big[\mathbf{S}_{-1(-)} \times (A_-\mathbf{S}_{-2(-)} + \mathbf{D}_- \times \mathbf{S}_{-2(-)} + J\mathbf{S}_{1(+)} \\
&\quad - \mathbf{D} \times \mathbf{S}_{1(+)} + B_-(\mathbf{S}_{-1(-)}\mathbf{e}_-)\mathbf{e}_- + \sigma_-(\mathbf{S}_{-1(-)}\mathbf{e}_-)\mathbf{e}_-)\big], \\
&S_{0(+)}\hbar\dot{\mathbf{S}}_{1(+)} \\
&= \big[\mathbf{S}_{1(+)} \times (A_+\mathbf{S}_{2(+)} - \mathbf{D}_+ \times \mathbf{S}_{2(+)} + J\mathbf{S}_{-1(-)} \\
&\quad + \mathbf{D} \times \mathbf{S}_{-1(-)} + B_+(\mathbf{S}_{1(+)}\mathbf{e}_+)\mathbf{e}_+\sigma_+(\mathbf{S}_{1(+)}\mathbf{e}_+)\mathbf{e}_+)\big]. \qquad (8)
\end{aligned}
$$

Expand the spins adjacent to the boundary ones in a series up to the values of the first order of lattice constant smallness. It should be noted that taking into account (4), the left-hand side of expressions (8) has a higher order of smallness of a than the remaining summands. Then

$$\mathbf{S}_{\pm 2(\pm)} = \mathbf{S}_{\pm 1(\pm)} \pm a\mathbf{S}'_{\pm 1(\pm)}. \tag{9}$$

Applying (9) to (8), we obtain the following system

$$\begin{aligned}
&-aA_-\left[\mathbf{S}_{-1(-)} \times \mathbf{S}'_{-1(-)}\right] + \left[\mathbf{S}_{-1(-)} \times (\mathbf{D}_- \times \mathbf{S}_{-1(-)})\right] \\
&\quad + J\left[\mathbf{S}_{-1(-)} \times \mathbf{S}_{1(+)}\right] - \left[\mathbf{S}_{-1(-)} \times (\mathbf{D} \times \mathbf{S}_{1(+)})\right] \\
&\quad + B_-(\mathbf{S}_{-1(-)}\mathbf{e}_-)[\mathbf{S}_{-1(-)} \times \mathbf{e}_-] \\
&\quad + \Sigma_-(\mathbf{S}_{-1(-)}\mathbf{e}_{s-})[\mathbf{S}_{-1(-)} \times \mathbf{e}_{s-}] = 0, \\
&aA_+\left[\mathbf{S}_{1(+)} \times \mathbf{S}'_{1(+)}\right] - \left[\mathbf{S}_{1(+)} \times (\mathbf{D}_+ \times \mathbf{S}_{1(+)})\right] \\
&\quad + J\left[\mathbf{S}_{1(+)} \times \mathbf{S}_{-1(-)}\right] + \left[\mathbf{S}_{1(+)} \times (\mathbf{D} \times \mathbf{S}_{-1(-)})\right] \\
&\quad + B_+(\mathbf{S}_{1(+)}\mathbf{e}_+)[\mathbf{S}_{1(+)} \times \mathbf{e}_+] \\
&\quad + \Sigma_+(\mathbf{S}_{1(+)}\mathbf{e}_{s+})[\mathbf{S}_{1(+)} \times \mathbf{e}_{s+}] = 0.
\end{aligned} \tag{10}$$

It should be noted that constants $B_\pm$ at $a \to 0$ grow slower that the rest ones and, therefore, they may be neglected in the continuum model. For direct integration of dynamic equations, this is indicated by the zero limit of an integral of the continuous magnetization function as the integration limits tend to zero.

Rewrite BC in continuum model

$$\begin{aligned}
&-\alpha_-\left[\mathbf{M}_- \times (\mathbf{M}'_- - [\mathbf{K}_- \times \mathbf{M}_-])\right] + G[\mathbf{M}_- \times \mathbf{M}_+] \\
&\quad - \left[\mathbf{M}_- \times [\mathbf{d} \times \mathbf{M}_+]\right] + \sigma_-(\mathbf{M}_-\mathbf{e}_{s-})[\mathbf{M}_- \times \mathbf{e}_{s-}] = 0, \\
&\alpha_+\left[\mathbf{M}_+ \times (\mathbf{M}'_+ - [\mathbf{K}_+ \times \mathbf{M}_+])\right] + G\left[\mathbf{M}_+ \times \mathbf{M}_-\right] \\
&\quad + \left[\mathbf{M}_+ \times [\mathbf{d} \times \mathbf{M}_-]\right] + \sigma_+(\mathbf{M}_+\mathbf{e}_{s+})[\mathbf{M}_+ \times \mathbf{e}_{s+}] = 0,
\end{aligned} \tag{11}$$

where $\mathbf{K} = \frac{\mathbf{d}}{\alpha}$ id the helicoid wave vector in each medium. It should be noted that summand $\mathbf{K}_\pm \times \mathbf{M}_\pm$ may be excluded by gauge transformation of transition to the helicoid coordinate system.

3. BCs taking into account long-range magnetic order in ferromagnets

In the presence of a long-range order of the exchange interaction, the one-dimensional chain Hamiltonian is as follows

$$W = -A\Sigma_n \mathbf{S}_n\mathbf{S}_{n+1} + B\Sigma_n \mathbf{S}_n\mathbf{S}_{n+2},$$

$A > 0$, $B > 0$. Rewrite it in the form of

$$W = \frac{1}{2}\left(-A\sum_n \mathbf{S}_n(\mathbf{S}_{n-1} + \mathbf{S}_{n+1}) + B\sum_n \mathbf{S}_n(\mathbf{S}_{n-2} + \mathbf{S}_{n+2})\right). \tag{12}$$

ferromagnetic limit in this model corresponds to $B = 0$. The first summand in (12) is responsible for ferromagnetic ordering of neighbour spins, and the second summand is responsible for antiferromagnetic ordering of spins located at a distance equal to double lattice constant. Thus, competition of two exchange interactions occurs and may result in the appearance of an exchange helix.

As in the previous case, assume that the magnetizations change negligibly within the lattice constant, which makes it possible to expand then in the Taylor's series:

$$\mathbf{S}_{n\pm k} = \mathbf{S}_n \pm \mathbf{S}'_n ka + \frac{1}{2}\mathbf{S}''_n(ka)^2 \pm \frac{1}{6}\mathbf{S}'''_n(ka)^3 + \frac{1}{24}\mathbf{S}''''_n(ka)^4 \tag{13}$$

and to write the initial Hamiltonian (12) as follows

$$W = \frac{1}{2}\left((4B - A)a\Sigma_n\mathbf{S}_n\mathbf{S}''_n a + \frac{1}{12}(16B - A)a^3\Sigma_n\mathbf{S}_n\mathbf{S}''''_n a\right). \tag{14}$$

Continuum transition is implemented using the substitutions $na \to x$, $a \to dz$, $\mathbf{S}_n \to as\mathbf{M}(z)$ under the summation symbol. To represent Hamiltonian (14) in the form summands containing coordinate derivatives, the quantities

$$(4B - A)a^3 s = \alpha > 0, \quad \frac{1}{12}(16B - A)a^5 s = \sigma > 0 \tag{15}$$

shall be finite. From (15), we have

$$A = \frac{1}{s}\frac{4}{a^5}\left(\sigma - \frac{\alpha}{3}\alpha^2\right), \quad B = \frac{1}{s}\frac{1}{a^5}\left(\sigma - \frac{\alpha}{12}\alpha^2\right). \tag{16}$$

The volumetric energy density of unlimited structure within a continuum limit corresponding to (14) is as follows [3]:

$$w = \frac{1}{2}\mathbf{M}(\alpha\mathbf{M}'' + \sigma\mathbf{M}''''). \tag{17}$$

Consider the interface of two semi-infinite one-dimensional magnetic ion chains with Hamiltonian (14). Enumerate them with positive indices to the depth of each medium from the interface. Neglecting the long-range order at the interface indicated by summand $-J_2(\mathbf{S}_{1(+)}\mathbf{S}_{2(-)} + \mathbf{S}_{2(+)}\mathbf{S}_{1(-)})$, the one-dimensional lattice structure Hamiltonian is as follows

$$\begin{aligned}
W = &-A_-\sum_{n\geq 1}\mathbf{S}_{n(-)}\mathbf{S}_{(n+1)(-)} - A_+\sum_{n\geq 1}\mathbf{S}_{n(+)}\mathbf{S}_{(n+1)(+)} \\
&+ B_-\sum_{n\geq 1}\mathbf{S}_{n(-)}\mathbf{S}_{(n+2)(-)} + B_+\sum_{n\geq 1}\mathbf{S}_{n(+)}\mathbf{S}_{(n+2)(+)} \\
&- J\mathbf{S}_{1(+)}\mathbf{S}_{1(-)}.
\end{aligned} \tag{18}$$

The last summand in (18) describes the interlayer interaction.

Write LLE for two boundary and sub-boundary spins

$$\hbar S_1\dot{\mathbf{S}}_{1(-)} = \left[\mathbf{S}_{1(-)} \times (-A_-\mathbf{S}_{2(-)} + B_-\mathbf{S}_{3(-)} - J\mathbf{S}_{1(+)})\right],$$
$$\hbar S_1\dot{\mathbf{S}}_{1(+)} = \left[\mathbf{S}_{1(+)} \times (-A_+\mathbf{S}_{2(+)} + B_+\mathbf{S}_{3(+)} - J\mathbf{S}_{1(-)})\right],$$
$$\hbar S_2\dot{\mathbf{S}}_{2(-)} = \left[\mathbf{S}_{2(-)} \times (-A_-(\mathbf{S}_{1(-)} + \mathbf{S}_{3(-)}) + B_-\mathbf{S}_{4(-)})\right],$$
$$\hbar S_2\dot{\mathbf{S}}_{2(+)} = \left[\mathbf{S}_{2(+)} \times (-A_+(\mathbf{S}_{1(+)} + \mathbf{S}_{3(+)}) + B_+\mathbf{S}_{4(+)})\right]. \quad (19)$$

If the the boundary-like defect were absent (which corresponds to a homogeneous medium), the right-hand side would have the same order od smallness as the left-hand side and we would have the magnetic moment dynamics equations in unlimited medium. While the difference between the interlayer exchange constant and intralayer exchange constant reduces the order of a by 1 in the right-hand side and as a result the left-hand side (dynamic) becomes negligibly small compared with the summands in the right-hand side within the continuum limit. The rest summands in the right-hand side in four equations shall be reduced to a combination of two quantities $\mathbf{S}_{1(-)}$ and $\mathbf{S}_{1(+)}$ by means of Taylor's expansions, and then the sought-for BCs will be obtained. Taking into account the direction of axis z,

$$\mathbf{S}_{(1+k)(\pm)} = \mathbf{S}_{1(\pm)} \pm \mathbf{S}'_{1(\pm)}k\alpha + \frac{1}{2}\mathbf{S}''_{a(\pm)}(k\alpha)^2 \pm \frac{1}{6}\mathbf{S}'''_{1(\pm)}(k\alpha)^3. \quad (20)$$

Apply the expansions (19) to (18). Taking into account the expressions for A and B (16), introduce magnetizations instead of spins and designation $J\alpha^5 = G$ similar to (16). Then with an accuracy up to the terms of the third order of smallness of lattice constant, the equations (19) will be as follows

$$G[\mathbf{M}_{1(-)} \times \mathbf{M}_{1(+)}] - 2a\sigma_-[\mathbf{M}_{1(-)} \times \mathbf{M}'_{1(-)}]$$
$$+ a^3\left(\frac{2}{3}\sigma_-[\mathbf{M}_{1(-)} \times \mathbf{M}'''_{1(-)}] + \frac{7}{6}\alpha_-[\mathbf{M}_{1(-)} \times \mathbf{M}'_{1(-)}]\right) = 0,$$
$$- G[\mathbf{M}_{1(+)} \times \mathbf{M}_{1(-)}] - 2a\sigma_+[\mathbf{M}_{1(+)} \times \mathbf{M}'_{1(+)}]$$
$$+ a^3\left(\frac{2}{3}\sigma_-[\mathbf{M}_{1(+)} \times \mathbf{M}'''_{1(+)}] + \frac{7}{6}\alpha_+[\mathbf{M}_{1(+)} \times \mathbf{M}'_{1(+)}]\right) = 0,$$
$$2a\sigma_-[\mathbf{M}_{2(-)} \times \mathbf{M}'_{2(-)}] + 2a^2\sigma_-[\mathbf{M}_{2(-)} \times \mathbf{M}''_{2(-)}]$$
$$+ a^3\left[\mathbf{M}_{2(-)} \times \left(\frac{4\sigma_-}{3}\mathbf{M}'''_{2(-)} - \frac{\alpha_-}{6}\mathbf{M}'_{2(-)}\right)\right] = 0,$$
$$2a\sigma_+[\mathbf{M}_{2(+)} \times \mathbf{M}'_{2(+)}] - 2a^2\sigma_+[\mathbf{M}_{2(+)} \times \mathbf{M}''_{2(+)}]$$
$$+ a^3\left[\mathbf{M}_{2(+)} \times \left(\frac{4\sigma_+}{3}\mathbf{M}'''_{2(+)} - \frac{\alpha_+}{6}\mathbf{M}'_{2(+)}\right)\right] = 0. \quad (21)$$

Within the continuum limit, assume $\mathbf{M}_{1(\pm)} \to \mathbf{M}_\pm$ in the first two equations (21) and $\mathbf{M}_{2(\pm)} \to \mathbf{M}_\pm$ in the other equations. By subtracting the second equation from the first one (21), we obtain in zero approximation

$$[\mathbf{M}_- \times \mathbf{M}_+] = 0, \quad (22)$$

i.e. the collinear condition of neighbour magnetic moments of two media with absolutely close exchange coupling between them.

By addition of the first two equations (21), we obtain

$$2a\left(\sigma_-[\mathbf{M}_- \times \mathbf{M}'_-] - \sigma_+[\mathbf{M}_+ \times \mathbf{M}'_+]\right)$$
$$- a^3\left(\frac{2}{3}\left(\sigma_-[\mathbf{M}_- \times \mathbf{M}'''_-] - \sigma_+[\mathbf{M}_+ \times \mathbf{M}'''_+]\right)\right.$$
$$\left. + \frac{7}{6}\left(\alpha_-[\mathbf{M}_- \times \mathbf{M}'_-] - \alpha_+[\mathbf{M}_+ \times \mathbf{M}'_+]\right)\right) = 0. \quad (23)$$

Then put together the third and fourth equations (21)

$$2a\left(\sigma_+[\mathbf{M}_+ \times \mathbf{M}'_+] - \sigma_-[\mathbf{M}_- \times \mathbf{M}'_-]\right)$$
$$- 2a^2\left(\sigma_+[\mathbf{M}_+ \times \mathbf{M}''_+] + \sigma_-[\mathbf{M}_- \times \mathbf{M}''_-]\right)$$
$$+ a^3\left(\frac{4}{3}\left(\sigma_+[\mathbf{M}_+ \times \mathbf{M}'''_+] - \sigma_-[\mathbf{M}_- \times \mathbf{M}'''_-]\right)\right.$$
$$\left. - \frac{1}{6}\left(\alpha_+[\mathbf{M}_+ \times \mathbf{M}'_+] - \alpha_-[\mathbf{M}_- \times \mathbf{M}'_-]\right)\right) = 0. \quad (24)$$

And finally, by combining the equations (23) and (24), we find

$$-2a^2\left(\sigma_+[\mathbf{M}_+ \times \mathbf{M}''_+] + \sigma_-[\mathbf{M}_- \times \mathbf{M}''_-]\right)$$
$$+ a^3\left(\left(\sigma_+[\mathbf{M}_+ \times \mathbf{M}'''_+] + \alpha_+[\mathbf{M}_+ \times \mathbf{M}'_+]\right)\right.$$
$$\left. - \left(\sigma_-[\mathbf{M}_- \times \mathbf{M}'''_-] + \alpha_-[\mathbf{M}_- \times \mathbf{M}'_-]\right)\right) = 0. \quad (25)$$

In first approximation, the equation (25) gives

$$\sigma_-[\mathbf{M}_- \times \mathbf{M}'_-] = \sigma_+[\mathbf{M}_+ \times \mathbf{M}'_+], \quad (26)$$

and from (24) in the second and third approximations, we obtain

$$\sigma_+[\mathbf{M}_+ \times \mathbf{M}''_+] = -\sigma_-[\mathbf{M}_- \times \mathbf{M}''_-], \quad (27)$$

$$\sigma_+[\mathbf{M}_+ \times \mathbf{M}'''_+] + \alpha_+[\mathbf{M}_+ \times \mathbf{M}'_+]$$
$$= \sigma_-[\mathbf{M}_- \times \mathbf{M}'''_-] + \alpha_-[\mathbf{M}_- \times \mathbf{M}'_-]. \quad (28)$$

The obtained relations (22), (26)−(28) are a BC system for the exchange helix magnetization dynamics. It shall be noted that the last condition (28) may be formally obtained by integration of the corresponding LLE in the small neighborhood of the boundary.

4. Conclusion

The presence of non-local exchange in a ferromagnet may lead not only to the occurrence of a heterogeneous ground state, but also to the increase in the number of normal modes. Therefore for proper formulation of the scattering problem, the number of boundary conditions corresponding to the order of the equation is required. This paper offers a method to derive the boundary condition in a continuum model for structures with Dzyaloshinski relativistic exchange or long-range order of exchange interaction. It is shown that sub-boundary spin dynamics equations whose symmetry is distorted compared with the internal spin serve as such BCs. Such symmetry distortion in the equations leads to reduction of the order of smallness of non-local summands of lattice constant.

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Conflict of interest

The author declares that he has no conflict of interest.

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