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Numerical simulation of supersonic gas flow in a conical nozzle with local plasma heating

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The results of calculations and experiments on plasma heating of the supersonic flow of gases in the conical nozzle by an external high-frequency inductor are given. The simulation was carried out in order to increase the specific impulse and efficiency of the liquid rocket engine. The results of numerical simulations are qualitatively consistent with the experiments conducted in a water-cooled quartz reactor.

Keywords: Low-temperature plasma, specific high-frequency power, skin layer.

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Heating of supersonic gas flow by low-temperature plasma (LTP) in a conical nozzle by means of a high-frequency (HF) inductor field is interesting for local heating of gas flow generated during fuel combustion in the combustion chamber (CC) of a liquid rocket engine, for increasing its thrust and in other cases. It is assumed that the electromagnetic HF field heats electrons to a temperature T_e much higher than the temperature of atoms, molecules, ions T_a, according to the two-temperature model [1]. We are not aware of any work on plasma heating of supersonic multi-atomic gas flows. In the work of Abramovich [2, v. 1] the problem of external heating of gas flow in a cylindrical tube was considered, where local heating led to a decrease of gas density, but during the motion, the gases flowed into the space occupied by denser gases and their speed sharply decreased. In a conical plasma nozzle, a local increase in temperature greatly reduces the average density of gases, increasing the thermal speed of atoms (ions) $V_{at} = (8k_\mathrm{B}T_a/\pi m_a)^{1/2}$, where k_B — Boltzmann constant, m_a — mass of atom (ion). The enthalpy Σh_i in the heated region and velocity V_{at} are related to pressure, but not by adiabat or polytrope, but by the non-linear relation characteristic of plasma, so the pressure of „hot" gases will exceed that in the unheated region, and the gas density will significantly decrease, which follows from the first law of thermodynamics.

A simple model was used, where the plasma heating zone was limited to the volume strictly below the inductor coils and the gas flow was treated as chemically equilibrium in each calculated volume. Supersonic gas flows were assumed to be laminar with boundary velocity value $V_0 = 1553$ m/s at the nozzle inlet, while heat transfer by radiation for LTP was not considered due to low calculated temperatures for electrons ($T_e < 3750$ K) and molecules, atoms (ions) ($T_a < 3200$ K). Temperature boundary conditions: $T_a = 2528$ K for the plasma generation region at the inductor inlet, which is separated from the inductor coils by a conical ceramic nozzle with surface temperature $T_c = 1000$ K and operating frequency $F = 27.12$ MHz, consisting of two turns of copper tube diameter 5 mm with a gap between turns 0.5 mm and inner diameter of 30 mm. The temperature value at the nozzle outlet was obtained by repeated iterations of solving the equations. The plasma parameters at the inductor inlet depend on the gas conductivity $\sigma_s < 30\,(\Omega \cdot \mathrm{m})^{-1}$ for temperatures $T_a = 2528-2674$ K at pressures 1−15 MPa in CC [3]. This value of σ_s is not sufficient to effectively absorb the electromagnetic HF field and heat the gas flow in the inductor. If you increase the ionization degree of the gases in the CC in some way, you can increase the conductivity to values of $\sigma_s \geqslant 200\,(\Omega \cdot \mathrm{m})^{-1}$. The conducting gas will then be the load, and the electrons will form a short-circuited coil of a virtual transformer with a high current density. When HF field energy penetrates to a skin layer depth Δ_p commensurated with the input radius of the inductor R_{in}, at this depth ∼ 87% of the energy is absorbed, heating the gas stream most effectively, otherwise heating will be negligible. The calculations were carried out for a conical nozzle geometry with an inlet radius of $R_{in} = 5$ mm, an outlet radius of $R_{out} = 30$ mm and a nozzle length of 150 mm. Dielectric nozzle wall thickness is 8 mm. CC heat output ∼ 26 kW at 1 MPa pressure for volume $5 \cdot 10^{-4}\,\mathrm{m}^3$, fuel consumption $G = 0.052$ kg/s for kerosene with nitric acid [2] was equal to the product of current cross section by calculated density and gas flow rate. Based on the known from the literature data dependences of the heat capacity of gases plasma from temperature [4–6] the calculation was made of enthalpy $\Sigma h_i = 11.7$ MJ/kg for temperatures $\Delta T = 2500-3750$ K considering the dissociation of H_2O and CO_2 by work [7], nozzle pressures ∼ 0.1−0.25 MPa and fractions of k_i for gases at the inlet to the heating zone: $N_2 = 17.3\%$, $H_2O = 43\%$, $CO_2 = 22\%$, other ∼ 17.7%. The energy of the electrons $\sim k_\mathrm{B}T_e \cdot 3/2$ corresponded to a temperature $T_e = 3750$ K in the nozzle volume, and at the nozzle surface $T_e \approx T_a$ was taken. The specific HF power $P_{sp} = 1.2\,\mathrm{kW/cm^3}$ was significantly higher than that of the CC ($52\,\mathrm{W/cm^3}$), to compensate for gas dissociation losses ($\sim 14.2\,\mathrm{W/cm^3}$), internal friction ($\sim 1.3\,\mathrm{W/sm^3}$) and nozzle wall heat loss ($\sim 16.5\,\mathrm{W/sm^3}$).

Atoms, ions and electrons enthalpy equation for the fraction k_i in the local volume

$$(n_a + n_i)k_\mathrm{B}T_a \cdot 5/2 + n_e k_\mathrm{B} T_e \cdot 3/2 = k_i \Sigma h_i \rho_i,$$

where n_a, n_i, n_e — concentrations of atoms, ions and electrons [$\mathrm{m^{-3}}$], ρ_i — local gas density [$\mathrm{kg/m^3}$]. The Navier–Stokes equations were taken as the basis for the velocity of the expanding gas flows, taking into account the internal friction and Lorentz forces in the magnetic field of the inductor by work [8]. The energy balance equation of gases in differential form [4] was also used, where the source of internal heat is joule heating by electrons determined by electron and ion conductivity. The concentration of electrons (ions) $n(T)$ was calculated from the system of Saha–Eggert equations for the LTP components, taking into account the decrease of atom ionization energies with increasing electron concentration and temperature using the Ecker–Weitzel formula [6]. Ten components of k_i atoms (ions) were calculated: O, H, OH, H_2O, CO, CO_2, N_2, etc. — chemically equilibrium gas flow for the kerosene/nitric acid mixture based on [3, v. 5, p. 79]. The skin layer size is $\Delta_p = \sqrt{2/(\mu_v \sigma_s(T) 2\pi F)}$, where μ_v — the magnetic constant of the vacuum. The plasma conductivity $\sigma_s(T)$ including electron drift in a strong electromagnetic HF field corresponds to

$$\sigma_s(T) = qn(T)[\mu_0(T) + \mu_{dr}(T)], \tag{1}$$

, where

$$\mu_0(T) = (q/m_e)[R_\mathrm{D}/V_{et}(T)],$$

$$\mu_{dr}(T) = (q/m_e)[L_{ea}/V_{et}(T)],$$

q — electron charge, $R_D = \sqrt{\varepsilon k_\mathrm{B} T/q^2 n(T)}$ — Debye length, $V_{et} = (8k_\mathrm{B}T_e/\pi m_e)^{1/2}$ — average electron speed, $\mu_0(T)\,[\mathrm{m^2/(V\cdot s)}]$ — electron mobility in a weak electric field, $\mu_{dr}(T)\,[\mathrm{m^2/(V\cdot s)}]$ — electron mobility in strong electric field, $L_{ea} = 1/Q_{ea}N_a$ [m] — average free path of electrons in electric field before interaction with atoms, $Q_{ea} = 16 \cdot 10^{-20}\,\mathrm{m^2}$ — average Ramsauer interaction cross section of electrons with H, O, C, N atoms [6], N_a [$\mathrm{m^{-3}}$] — average concentration of atoms in calculated volume. The calculation by formula (1) for $\sigma_s(T)$ in the heating section gives a value of $\langle\sigma_s\rangle = 193{-}909\,(\Omega\cdot\mathrm{m})^{-1}$ and the degree of gas ionization $ion(T) = 3.2\cdot 10^{-4}{-}1.7\cdot 10^{-3}$ in the temperature range $T_a = 2528{-}3087\,\mathrm{K}$ when the average values of viscosity and thermal conductivity at the nozzle inlet are known [3, v. 5, p. 79], and their values change linearly and by less than 14–38% during heating.

The compulsory calculation of electric and magnetic fields was carried out using Maxwell equations, which can be reduced to one equation for the tangential component of the complex electric field $\dot{E}_\theta$ with magnetic permeability of the plasma equal to one, in the framework of axial symmetry without considering bias currents

$$\frac{\partial^2}{\partial z^2}\dot{E}_\theta + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r} r\dot{E}_\theta\right) = -j\omega\mu_v\sigma_s\dot{E}_\theta, \tag{2}$$

where j is the imaginary unit, ω is the angular frequency.

The equations for the relations between the magnetic field vector potential $\dot{H}_m$ and the electric field strength $\dot{E}_\theta$ in complex form by work [9] were also used:

$$\frac{\partial}{\partial r}(\dot{E}_\theta) + \frac{\dot{E}_\theta}{r} = -j\omega\mu_v\dot{H}_m, \quad -\frac{d\dot{H}_m}{dr} = \sigma_s\dot{E}_\theta. \tag{3}$$

The boundary conditions for equations (2), (3) were similar to those used in [10]: at $r = 0$ $E_\theta = 0$, at $r = R_c$ $\frac{1}{r}\frac{d}{dr}(r\dot{E}_\theta) = j\omega\mu_v\dot{H}_m$, where $\dot{H}_m = H_{me}(z)$ — the magnetic field amplitude at the inductor surface along the conical nozzle. The relative dimensionless coordinate $m = (R/\Delta_p)\sqrt{2}$, where R — the radius of the conical nozzle [m], is introduced. Solutions to equations (2), (3) for the moduli H_m and E_θ and for the complex electron current density amplitude $\dot{\delta}_m$ and its module δ_m are obtained in the Bessel functions ber(m) and bei(m) in [9]:

$$\dot{H}_m = H_{me}\frac{\mathrm{ber}(m) + j\,\mathrm{bei}(m)}{\mathrm{ber}(m_2) + j\,\mathrm{bei}(m_2)},$$

$$H_m = H_{me}\sqrt{\frac{\mathrm{ber}^2(m) + \mathrm{bei}^2(m)}{\mathrm{ber}^2(m_2) + \mathrm{bei}^2(m_2)}}, \tag{4}$$

where $H_{me} = W_0 I_\mathrm{I}\sqrt{2}/L_\mathrm{I}$ [(A · coil)/m], W_0 — number of turns, I_I — inductor current [A], $L_\mathrm{I} = 0.01\,\mathrm{m}$ — inductor length, $m_2 = (R_{in}/\Delta_p)\sqrt{2}$, R_{in} — inlet radius of calculated nozzle volume [m].

$$\dot{\delta}_m = \delta_{me}\frac{\mathrm{ber}'(m) + j\,\mathrm{bei}'(m)}{\mathrm{ber}(m_2) + j\,\mathrm{bei}(m_2)},$$

$$\delta_m = \delta_{me}\sqrt{\frac{[(\mathrm{ber}'(m)]^2 + [\mathrm{bei}'(m)]^2}{\mathrm{ber}^2(m_2) + \mathrm{bei}^2(m_2)}}, \tag{5}$$

where $\delta_{me} = \sqrt{2}H_{me}/\Delta_p$ — electron current density at the nozzle surface [$\mathrm{A/m^2}$], functions ber$'(m)$ and bei$'(m)$ — derivatives on R of corresponding functions. Figure 1, *a* shows the results of calculating the dimensionless functions $\Psi_{Hme1}(m) = H_m/H_{me1}$, $\Psi_{Hme2}(m) = H_m/H_{me2}$ by formula (4), describing the magnetic field change and equal to the amplitude ratio, depending on m.

The relations between the modulus value E_m and the current density modulus δ_m are determined by Ohm's law: $\delta_m = \sigma_s E_m$ and is zero at $m = 0$ ($R = 0$) at the nozzle axis. The electric field E_m changes as m changes with a dependence close to linear for parameters $m_2 < 3.2$ and values $m = 0{-}4$. It was noted above that at the optimum specific conductivity σ_s, the skin layer value was $\Delta_p \approx R_{in}$. Then, the optimum design value is $m_2 = 1.6{-}3.2$ and is related to the average plasma conductivity σ_{si} in

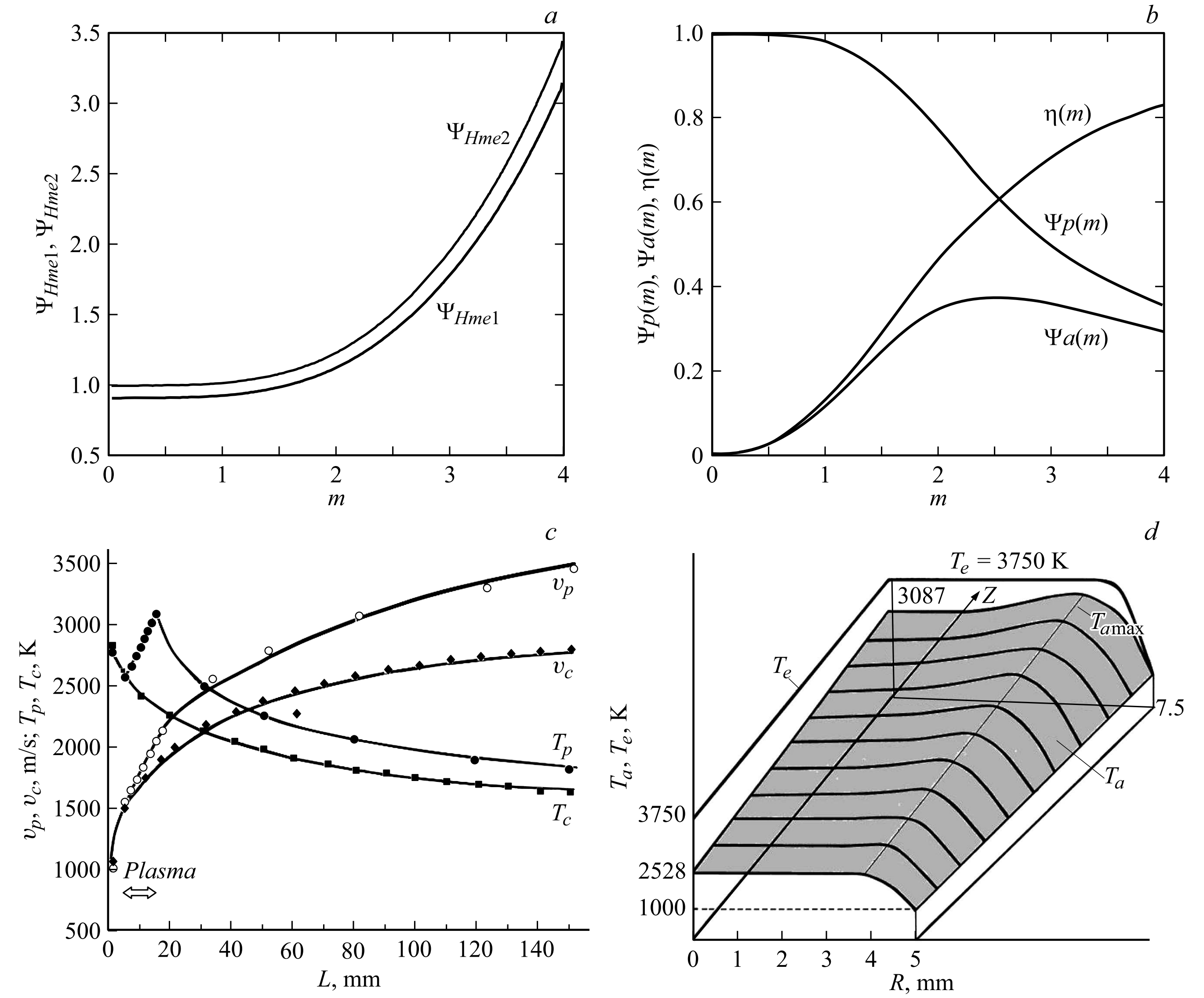


Figure 1. a — dependences of dimensionless functions Ψ_{Hme1} and Ψ_{Hme2} on dimensionless coordinate m for parameters $m_2 = 1.6202$ and 0.8101, respectively; b — dependences of dimensionless functions for the active Ψa and reactive Ψp components of the electromagnetic HF field and the relation $\eta(m) = \Psi a/\Psi p$ from the dimensionless coordinate m; c — temperature T_c and average gas velocity v_c along the nozzle axis without plasma heating, temperature T_p and velocity v_p with plasma heating at a length from 5 to 15 mm; d — 3D plot of plasma heated cone nozzle section with radius from 5 to 7.5 mm for gas temperature T_a and electron temperature T_e.

each design volume. An electromagnetic HF wave travels along the normal from the inductor surface with energy $\dot{S}_h = -\{(H_{me})^2 m_2/(2\sqrt{2}\sigma_s\Delta_p)\}[\Psi a + j\Psi p]$, equal to the vector product of the complex conjugate electric $\dot{E}_m$ and magnetic $\dot{H}_m$ fields, where Ψa, Ψp — are functions of the argument m_2, determining the active and reactive powers, respectively:

$$\Psi a = \frac{2}{m_2}\frac{\mathrm{ber}'(m_2)\mathrm{ber}(m_2) + \mathrm{bei}'(m_2)\mathrm{bei}(m_2)}{\mathrm{ber}^2(m_2) + \mathrm{bei}^2(m_2)},$$

$$\Psi p = \frac{2}{m_2}\frac{\mathrm{bei}'(m_2)\mathrm{ber}(m_2) - \mathrm{ber}'(m_2)\mathrm{bei}(m_2)}{\mathrm{ber}^2(m_2) + \mathrm{bei}^2(m_2)}. \quad (6)$$

Figure 1, b shows the dimensionless functions Ψa and Ψp, calculated according to formula (6), defining the vector $\dot{S}_h$, and the relations $\eta(m) = \Psi a/\Psi p$ — of plasma heating efficiency of gases. It can be seen that $\eta(m) < 70\%$ for the optimal value of m_2. Vector $\dot{S}_h$ contains longitudinal and transverse components. The longitudinal vector coincides with the gas flow and is the source that accelerates the flow. The transverse vector deflects the gas flow away from the walls towards the center. The calculation for modulus $\dot{S}_h$ gives a small longitudinal pressure $P_L \approx 30$ Pa at inductor current $I_{\mathrm{I}} = 71$ A. The specific power of the electromagnetic HF field per unit volume ΨP_i was determined by the dependence $\Psi P_i = (\delta_m)^2/\langle\sigma_s(T)\rangle = 1.2 \cdot 10^3$ W/cm^3 at an average value of $\langle\sigma_s(T)\rangle \approx 235\,(\Omega\cdot\text{m})^{-1}$, $\Delta_p \approx 10\,\text{mm}$, $H_{me} = 9563\,\text{A/m}$, $I_\text{I} = 71\,\text{A}$, $m_2 = 3$.

The recombination time of ions in the plasma $\langle\tau_r\rangle$ should be longer than the transit time of the heated region $\tau_{in} \sim 8\,\mu\text{s}$, as confirmed by theory [11] and experiments [12].

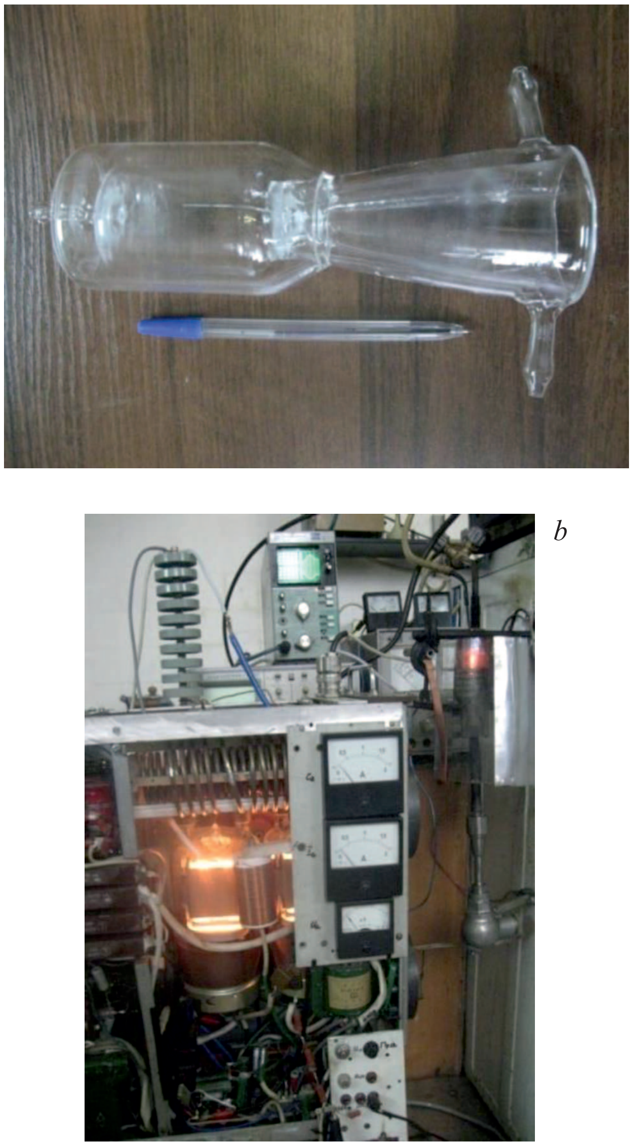
The simulation determines the gas dynamic relations for supersonic flows in a cone nozzle without external heating, which are based on work [2, v. 1], where formulas for relative velocity $\lambda = v(z)/v_{cr}$, pressure $P(\lambda)$, temperature $T(\lambda)$ and gas density $\rho(\lambda)$ along the nozzle are given. Here, $v(z)$ — gas flow rate along the nozzle axis, $v_{cr} = 996\,\text{m/s}$ — gas flow rate, $P_{cr} = 581\,\text{kPa}$ — pressure, $T_{cr} = 2816\,\text{K}$ — temperature, and $\rho_{cr} = 0.658\,\text{kg/m}^3$ — gas density at the critical section. Then, at 5 mm length, a heated section consisting of ten volumes (1 mm pitch) was introduced to the design cross-section with a pressure equal to 0.1 MPa. The calculations were based on the work control method [4,5], which provides detailed calculation programs for MathCAD. Gas and plasma were considered ideal. The calculation results are shown in Fig. 1, *c* and are close to the data of [5, v. 5, p. 79], where incomplete fuel combustion in the CC is taken into account, and calculation of changes in the chemical composition of gases along the whole nozzle was also made, which was the basis of our work.

Calculations in AutoCAD2001 Professional were performed as long as the pressure value did not exceed the previous value by more than 1 kPa. The λ values were calculated to at least 10^{-6} decimal place, then the error of the functions $f_c(\lambda)$, $P(\lambda)$, $T(\lambda)$, $\rho(\lambda)$ was 10^{-4}. Calculations after the plasma heating section were carried out considering that the value $L = 16\,\text{mm}$ was considered as the new critical section with radius $R_{cr2} = 7.67\,\text{mm}$, for which the pressure $\sim 0.1\,\text{MPa}$, so the calculated thrust at sea level is ~ 108 N for gas flow with velocity $v_{cr2} = 2094\,\text{m/s}$, and the total thrust is $\sim 185\,\text{N}$ for altitude $\sim 16\,\text{km}$. It can be seen from Fig. 1, *c* that the gases heating in the conical nozzle leads to a temperature rise of 1.33 times at the end of the heating zone, and the velocity v_p and temperature T_p is greater than the velocity v_c and temperature T_c without heating. Figure 1, *d* shows calculation results for temperatures T_a and T_e in the plasma heated gas section in longitudinal and radial direction.

The calculated increase in calculated thrust at the sea level was 6.4% for a plasma enthalpy of $\sim 11.7\,\text{MJ/kg}$ and the increase in total thrust at 16 km altitude was as high as 19%. At higher altitudes, with increasing nozzle length and inductor current, the total thrust can increase even more due to the longitudinal component of the magnetic pressure of the HF field.

The experiments were carried out in a water-cooled quartz reactor (Fig. 2, *a*) with dimensions close to the design and in a plasma/gas-flame stand (Fig. 2, *b*). The enthalpy of the gas flows was measured with a massive tungsten−rhenium thermocouple TVR-251 with a molybdenum casing. Experiments were carried out with gas, liquid and combined combustible mixtures at mass flow rates up to 0.01 kg/s. Based on calculations, physical tests and experiments, it was concluded necessary to modify the kerosene fuel with the self-ignition property and make a new type of special oxidizer. This oxidizer with the optimal composition was made and tested in a quartz reactor and showed sufficient stability of physical properties for one year. The measuring stand with a HF generator (1.5 kW), based on GU-81M tubes operating at 13.56 MHz, allowed to determine the average conductivity of supersonic gas flow at the exit of the conical nozzle. Experiments with plasma heating of gas flows at temperature $\sim 2500\,\text{K}$ and pressure $\leqslant 0.2\,\text{MPa}$ in CC showed the following: 1) the temperature at the nozzle outlet is additionally increased up to 90 K; 2) the average conductivity of the gas stream significantly increases; 3) the enthalpy of the whole gas stream increases; 4) the efficiency of plasma heating is 8−10%; 5) the peak

a



b

Figure 2. *a* — quartz reactor; *b* — plasma/gas flame stand.

thermal power of the Quartz reactor CC briefly reached 4 kW.

Thus, calculations and experiments show the possibility of plasma heating of supersonic gas flow in a conical nozzle, which will increase the specific impulse of a liquid rocket engine due to the increase of gas velocity in the calculated cross section.

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Conflict of interest

The author declares that he has no conflict of interest.

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