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Effective Dielectric Permeability of a Composite with Matrix Ellipsoidal Inclusions

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A composite material based on ellipsoidal inclusions distributed in a bonding medium is considered. A formula is obtained for calculating the effective dielectric constant of such a material, taking into account the presence of a layer of a binder on the inclusions, which prevents their direct contact. It is shown that the calculated values of the effective dielectric constant correspond to the experimental values for a composite based on sections of conductive fibers.

Keywords: composite material, radar absorbing structure, diffraction of electromagnetic waves.

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Composite materials based on inclusions in the form of segments of conducting fibers, distributed in a dielectric matrix, are promising for use in Microwave equipment as radar-absorbing materials [1,2]. Interaction of such materials with an electromagnetic field depends on their effective dielectric constant (EDC). If inclusions are well-ordered, the EDC can be calculated based on the solution of the diffraction problem [3]. If inclusions are chaotic, the estimated EDC values, corresponding to the experiment, are provided when using the modified Odelevsky formula, and with a higher accuracy when using a generalized formula for the EDC of a composite based on ellipsoidal inclusions [4].

This paper suggests a method for calculating the EDC of a composite having ellipsoidal inclusions, taking into account the fact that they have a matrix shell that hinders direct electric contact between inclusions.

Polarizability α of a coated ellipsoid, being in a medium with dielectric constant ε_m in the external field, directed along the main axis, is expressed by the known formula [5]:

$$\alpha = v^{(e)} \frac{(\varepsilon^{(e)}-\varepsilon_m)\left[\varepsilon^{(e)}+(\varepsilon^{(i)}-\varepsilon^{(e)})(n^{(i)}-\eta n^{(e)})\right]+ +\eta\varepsilon^{(e)}(\varepsilon^{(i)}-\varepsilon^{(e)})}{\left[\varepsilon^{(e)}+(\varepsilon^{(i)}-\varepsilon^{(e)})(n^{(i)}-\eta n^{(e)})\right]\left[\varepsilon_m+(\varepsilon^{(e)}-\varepsilon_m)n^{(e)}\right]+ +\eta n^{(e)}\varepsilon^{(e)}(\varepsilon^{(i)}-\varepsilon^{(e)})}, \qquad (1)$$

where η is ratio of internal and external ellipsoids' volumes, $\eta = v^{(i)}/v^{(e)}$, $n^{(i)}$ and $n^{(e)}$ are depolarization coefficients (DC) along the main axis. The index (i) refers to the internal ellipsoid, the index (e) — to the external one. Formula (1) can be written as

$$\alpha = v^{(e)} \frac{\tilde{\varepsilon} - \varepsilon_m}{\varepsilon_m + n^{(e)}(\tilde{\varepsilon} - \varepsilon_m)}, \qquad (2)$$

with the designation

$$\tilde{\varepsilon} = \varepsilon^{(e)} + \eta\varepsilon^{(e)} \frac{\varepsilon^{(i)} - \varepsilon^{(e)}}{\varepsilon^{(e)} + (\varepsilon^{(i)} - \varepsilon^{(e)})(n^{(i)} - \eta n^{(e)})}. \qquad (3)$$

Formula (2) is the same as the expression for polarizability of an ellipsoid that has dielectric constant (3), volume and DC of which are the same as for the external ellipsoid [6]. This circumstance makes it possible to apply the formula for EDC of a composite with homogeneous ellipsoids-inclusions, obtained in [4] in the approximation of the effective medium theory, to a composite with „two-layer" ellipsoids that consist of inclusions as such and a matrix shell that coats them:

$$\frac{3(1-c)(\varepsilon_1 - \varepsilon)}{2\varepsilon + \varepsilon_1} + \frac{c(\varepsilon_2 - \varepsilon)}{(1-n)\varepsilon + n\varepsilon_2} = 0, \qquad (4)$$

where ε is EDC, ε_1, ε_2 are dielectric constants of the matrix and inclusions respectively, c is volumetric concentration of inclusions, n is depolarization coefficient for inclusions. Simple transformations reduce formula (4) to a quadratic equation in relation to ε.

Fig. 1 shows the coated ellipsoid included in the composite under study. As such, the ellipsoid-inclusion with dielectric constant ε_3, depolarization coefficient $n^{(i)}$ and volume $v^{(i)}$ is inside an ellipsoid made of the matrix material with DC $n^{(e)}$ and volume $v^{(e)}$. This „double" ellipsoid can be correlated to a homogeneous ellipsoid with dielectric constant $\tilde{\varepsilon}$, calculated using formula (3), where substitutions should be made

$$\varepsilon^{(i)} \to \varepsilon_3,\ \varepsilon^{(e)} \to \varepsilon_1,\ v^{(i)} = \frac{4}{3}\pi a_1 b_1 c_1,\ v^{(e)} = \frac{4}{3}\pi a_2 b_2 c_2, \qquad (5)$$

where a_1, b_1, c_1 are sizes of semi-axes of the internal ellipsoid along axes x, y, z; a_2, b_2, c_2 are sizes of semi-axes of the external ellipsoid; $n^{(i)}$ and $n^{(e)}$ are depolarization coefficients for the internal and external ellipsoid.

Having calculated the value $\tilde{\varepsilon}$, the following substitutions should be made in formula (4) for EDC of a composite with inclusions of the form shown in Fig. 1:

$$n \to n^{(e)}, \qquad \varepsilon_2 \to \tilde{\varepsilon}. \qquad (6)$$

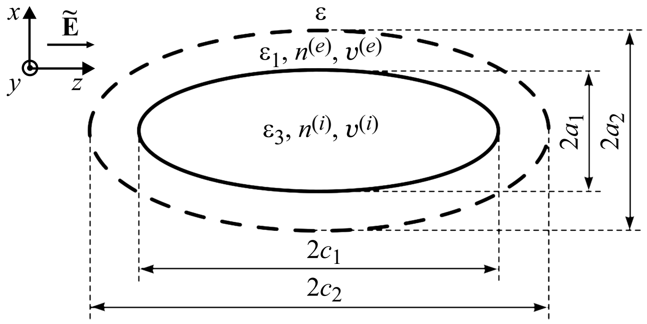

Figure 1. The ellipsoid-inclusion with a matrix shell, located in a medium with effective dielectric constant in the external field directed along the z axis.

Let us designate volumetric concentration of uncoated ellipsoids-inclusions in the composite as p. Then the composite where inclusions are coated ellipsoids, corresponds to the volumetric concentration of inclusions c,

$$c = p\frac{v^{(e)}}{v^{(i)}} = \frac{a_2 b_2 c_2}{a_1 b_1 c_1}p. \tag{7}$$

An inclusion model for the considered composite based on segments of carbon fibers-dipoles, length of which is by several orders greater than their thickness, is an oblong ellipsoid of revolution the volume of which is equal to the inclusion volume [2]. Such an ellipsoid efficiently interacts only with the electric field component along its long axis. This makes it possible to correlate a composite with thin inclusions, chaotically oriented in the plane, to a composite with inclusions similarly oriented in the field direction and to apply formula (4), making a substitution in (7)

$$p \to Kp, \tag{8}$$

where K is orientation factor, $K = 1$ if the long ellipsoid axes are oriented along the field, $K = 1/3$ if inclusions are chaotically oriented in space and $K = 1/2$ in case of chaotic orientation in the plane parallel to the middle field in the composite [2].

The EDC for a composite based on cylindrical dipoles, chaotically oriented in the plane, was calculated on the basis of dipole correlation to equivoluminar ellipsoids using the substitution (8). Dipole length is $2h = 10\,\mathrm{mm}$, their radius is $r = 4\,\mu\mathrm{m}$, specific conductivity is $\sigma = 71\,400\,\mathrm{S/m}$, dielectric constant of the matrix is $\varepsilon_1 = 1.8$, dipole volumetric concentration is $p = 0.05\%$. These parameter values correspond to the carbon fiber-based composite experimentally studied in [2]. The experimental values of EDC components for the given composite at different values of frequency f are given in Fig. 2, *a*, *b* (experimental data *1*).

The large semi-axis of the ellipsoid, correlated to a cylindrical dipole, is $c_1 = h$, the small semi-axis $a_1 = b_1 = \sqrt{3/2}r$ was determined based on the condition of equality of dipole and ellipsoid volumes. Depolarization coefficients were calculated using the known formula [6]. The dielectric constant for an uncoated ellipsoid was calculated using the formula $\varepsilon_3 = (i\sigma/\omega)/\varepsilon_0$, where i is the imaginary unit, ω is cyclic frequency, ε_0 is dielectric constant of vacuum.

The estimated frequency dependences of EDC components, obtained using the above-mentioned formulas, are shown in Fig. 2, *a*, *b* (curves *2*). The calculations assumed $c_2 = 1.2c_1 = 1.2h$, $a_2 = 10a_1 = 10\sqrt{3/2}r$, $b_2 = a_2$.

Fig. 2, *a*, *b* (curves *3*) gives the results of EDC calculation using the modified Odelevsky formula [7]:

$$\varepsilon = \varepsilon_1\left(1 + \frac{Kp}{(1 - Kp/p_c)n + \varepsilon_1/(\varepsilon_3 - \varepsilon_1)}\right), \tag{9}$$

where p_c is the parameter interpreted as the percolation threshold. Varying of the parameter p_c has shown that its value equal to 0.0029 ensures the optimal matching between the estimated values of ε and the experimental ones.

It can be seen from the figures that the estimated values both of the real and imaginary parts of EDC for the model

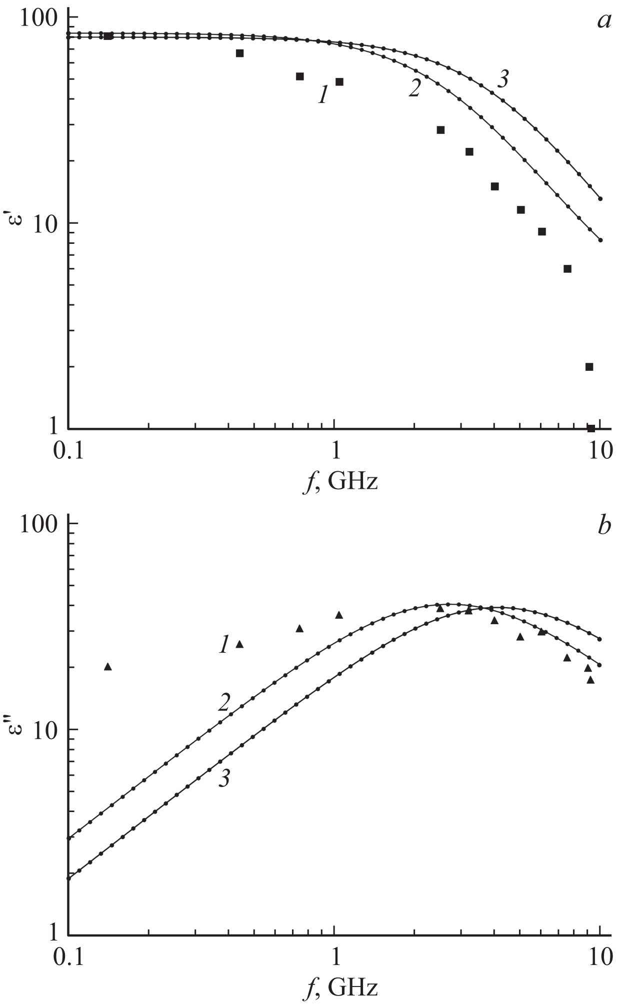

Figure 2. Dependences of the real (*a*) and imaginary (*b*) parts of effective dielectric constant for a carbon-fiber based composite on frequency. *1* — experimental data, *2* — calculation using the obtained formulas, *3* — calculation using the modified Odelevsky formula.

of „two-layer" inclusions are closer to the experimental values than those obtained during EDC calculation using formula (9). Comparison of the curves *2* with the EDC calculation results for the same composite using the generalized formula, which are given in [4], shows their closeness in the larger part of the frequency range.

EDC formulas have been obtained for the composite with ellipsoidal inclusions coated with a matrix layer. Matching between the estimated EDC values and the experimental values for a composite based on segments of conductive fibers in case of appropriate selection of matrix shell sizes is shown. A comparison of EDC calculation using the obtained formulas by the known methods is given.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] A.N. Lagarkov, A.K. Sarychev, Y.R. Smychkovich, A.P. Vinogradov, J. Electromagn. Waves Appl., **6** (9), 1159 (1992).

[2] K.N. Rozanov, *Chastotno-zavisimye magnitnye i dielektricheskiye svoistva kompozitnykh materialov dlya shirokopolosnykh SVCh primeneniy*, dokt. diss. (ITPE RAN, M., 2018) (in Russian).

[3] V.I. Ponomarenko, I.M. Lagunov, Journal of Communications Technology and Electronics, **64** (5), 440 (2019). DOI: 10.1134/S0033849419050103

[4] V.I. Ponomarenko, I.M. Lagunov, Journal of Communications Technology and Electronics, **66** (4), 345 (2021). DOI: 10.31857/S0033849421040094

[5] C. Bohren, D. Huffman, *Pogloshcheniye i rasseyaniye sveta malymi chastitsami* (Mir, M., 1986) (in Russian).

[6] B.Ya. Balagurov, *Elektrofizicheskiye svoistva kompozitov. Makroskopicheskaya teoriya* (LENAND, M., 2018) (in Russian).

[7] S.N. Starostenko, K.N. Rozanov, A.O. Shiryayev, A.N. Lagarkov, A.N. Shalygin,Physics of the Solid State **59** (11), 2183 (2017). DOI: 10.21883/FTT.2017.11.45058.20k