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Thermoelectric heat transfer intensifiers

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The work of thermoelectric heat transfer intensifier is analyzed in the paper. Criteria for the efficiency of such intensifiers are given. The advantage of using a thermoelectric cooler is evaluated depending on the temperature difference on the heat sink when applied without a thermoelectric module.

It is shown that in many cases a thermoelectric cooler cannot intensify heat transfer. In case a thermoelectric cooler is used to reduce the object temperature, it is also necessary to reduce the heat sink thermal resistance.

Keywords: thermoelectric cooler, heat exchange, intensifier, efficiency.

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1. Introduction

To reduce temperature of heat-generating objects, sometimes it is recommended to use a thermoelectric cooler (TEC). In this case the object temperature decreases, and the heat sink temperature increases, which intensifies heat rejection into environment. Such devices are called heat exchange intensifiers [1–4]. It is clear that if the cooled object temperature is slightly higher than the ambient temperature, this idea is workable, but if the object temperature is noticeably higher than the ambient temperature, the advantages of thermoelectric heat exchange intensifiers over passive cooling are not evident. In this paper, we analyze the operation of such an intensifier, and give criteria for its efficiency.

2. Thermoelectric intesifier analysis

Let us consider two methods of cooling an object that generates heat with capacity Q_0. The first method (Figure 1, a) suggests that the heat from object *1* is rejected by heat sink *2* to the environment.

In the second case, TEC 3 is placed between the heat sink and the object (Figure 1, b). Let us find out whether TEC allows a more efficient lowering of the object temperature if its temperature even after thermoelectric cooling remains above the ambient temperature T_a, i. e. $T_0 > T_a$.

In the first case the difference between the object temperature and the ambient temperature ΔT_r equals

$$\Delta T_r = T_2 - T_a = R_r Q_0, \tag{1}$$

where R_r is the thermal resistance of heat sink *2*.

When using TEC, the heat balance equations for the TEC cold and hot sides are as follows:

$$\alpha I T_0 - \frac{1}{2} I^2 R - K\Delta T_1 = Q_0,$$

$$\alpha I T_1 + \frac{1}{2} I^2 R - K\Delta T_1 = Q_1, \tag{2}$$

where α is the total Seebeck coefficient of the TEC pellets, I is electric current, R is TEC electrical resistance, K is TEC thermal conductance, $\Delta T_1 = T_1 - T_0$ is the temperature difference between the TEC cold and hot sides, Q_1 is the heat rejected on the TEC hot side. The temperature behaviors of α, R, K are not taken into account.

In the case of Figure 1, b the temperature difference on the heat sink ΔT_{mr} can be written as:

$$\Delta T_{mr} = T_1 - T_a = R_r Q_1 = R_r \mu Q_0, \tag{3}$$

where $\mu = Q_1/Q_0$ is thermal coefficient.

If TEC works more efficiently than just a heat sink, it means that the object temperature in the case b) is lower than that in the case a) — see Figure 1, i.e.:

$$\Delta T_r > \Delta T_{mr} - \Delta T_1 = \mu \Delta T_r - \Delta T_1, \tag{4}$$

Remembering the ratio between coefficient of performance (COP) ε and thermal coefficient $\varepsilon = \frac{1}{\mu - 1}$ we obtain:

$$\Delta T_1 \varepsilon \geq \Delta T_r. \tag{5}$$

If inequality (5) is true, the use of TEC makes it possible to lower the object temperature compared to cooling by the heat sink only. Therefore, expression (5) is the main criterion for evaluating the efficiency of the thermoelectric heat exchange intensifiers. From (5) we also see that the growth of ε allows increasing the value of ΔT_r.

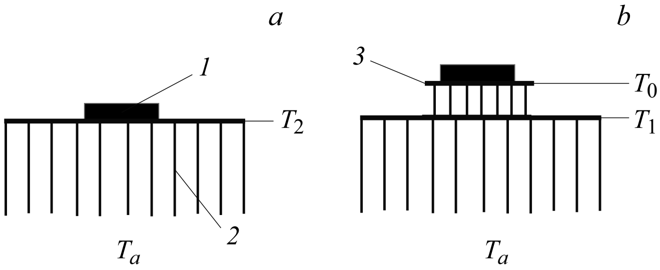

Figure 1. *a*) object *1* cooling by heat sink *2*; *b*) object *1* cooling by TEC 3 and by the same heat sink.

We assume the TEC operates in the maximum COP mode, which minimizes the heat rejected on the TEC hot side. For this mode the following formulae are known [3]:

$$I = \frac{\alpha \Delta T_1}{R(M-1)}, \tag{6}$$

where

$$M = \sqrt{1 + Z\frac{T_0 + T_1}{2}}, \tag{7}$$

and

$$Z = \frac{\alpha^2}{RK}. \tag{8}$$

In this mode the maximum cooling capacity is achieved when $\Delta T_1 \approx \Delta T_{\max}/2$ and $\varepsilon \approx 1$ is the TEC maximum temperature difference. From Eq. (5) we estimate $\Delta T_r \leq T_{\max}/2$ for the heat sink overheating at which a TEC application makes sense. A more detailed study of the problem is given below.

For the temperatures defined by Figure 1, *b*, we can write the following:

$$T_1 = T_a + \mu \Delta T_r, \tag{9}$$

and

$$T_0 = T_a + \mu \Delta T_r - \Delta T_1, \tag{10}$$

where $\mu = 1 + 1/\varepsilon$. From inequality (5) it follows that there is a maximal value $\Delta T_{r\,\max}$. If it is exceeded, the application of TEC only results in the deterioration of the heat transfer conditions. If ΔT_r equals $\Delta T_{r\,\max}$, the applications *a*) and *b*) are indifferent, i. e. this case corresponds to the equality of temperatures

$$T_2 = T_0, \tag{11}$$

which gives, in accordance with Eq. (5), the equality $\Delta T_1 = \Delta T_r/\varepsilon$. From the expression for ε in the maximum COP mode:

$$\varepsilon = \frac{MT_0 - T_1}{\Delta T_1(M+1)} \tag{12}$$

allowing for (9)–(11) we can obtain:

$$M_\varepsilon = \frac{T_a\varepsilon + (2\varepsilon + 1)\Delta T_r}{T_a\varepsilon}, \tag{13}$$

where

$$M_\varepsilon = \sqrt{1 + ZT_a + Z\Delta T_r\left(1 + \frac{1}{2\varepsilon}\right)}. \tag{14}$$

For real values of Z the following estimation is true: $Z\Delta T_r(1 + 1/2\varepsilon) \ll 1 + ZT_a$. Then for M_ε we can use the approximate expression:

$$M_\varepsilon \approx M_a\left(1 + \frac{Z\Delta T_r(2\varepsilon+1)}{4M_a^2\varepsilon}\right), \tag{15}$$

where

$$M_a = \sqrt{1 + ZT_a}. \tag{16}$$

Substituting (15) into (13) we can obtain the estimation:

$$\Delta T_{r\,\max} = \frac{\varepsilon T_a(M_a - 1)}{(2\varepsilon+1)\left[1 - \frac{ZT_a}{4M_a}\right]}, \tag{17}$$

Figure 2 shows an example of the dependence $\Delta T_{r\,\max}$ on Z at various values of COP and $T_a = 300\,\mathrm{K}$.

In Figure 2 we see that at small values of ε the limiting temperature difference on the heat sink is small (28–35 K at $\varepsilon = 0.5$). The higher the value of ε, the larger the value of $\Delta T_{r\,\max}$. At $\varepsilon = 10$ it achieves 57–64 K.

If $\varepsilon \to \infty$, which corresponds to $\Delta T_1 \to 0$ (see Eq. (9)), from (17) the limiting temperature difference on the heat sink can be written as:

$$\Delta T_r \to \frac{T_a(M_a - 1)}{2(1 - ZT_a/4M_a)}, \tag{18}$$

For these values TEC, regardless of an operating mode, only worsens the heat transfer. These results are shown in Table.

The obtained restrictions for the heat sink temperature, given in Figure 2 and Table, show that for large heat sink temperature difference, TEC does not allow lowering the object temperature, which significantly reduces the application scope of thermoelectric heat exchange intensifiers.

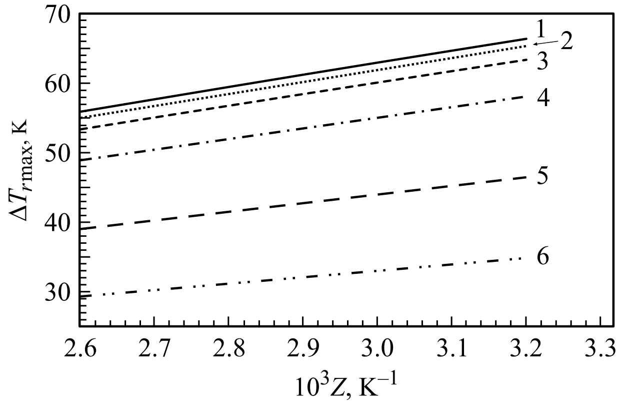

Figure 2. Behavior $\Delta T_{r\,\max}$ vs Z for different values of COP ε (0.5–10) at $T_a = 300\,\mathrm{K}$.

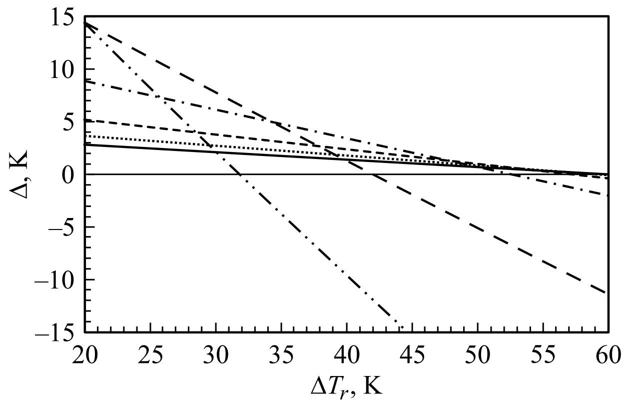

Figure 3. The object temperature drop Δ depending on the heat sink temperature difference ΔT_r at various values of ε and $T_a = 300$ K for *a*) $Z = 0.0026$ 1/K, *b*) $Z = 0.0032$ 1/K. The negative values of Δ mean that the use of TEC increases the object temperature.

The use of thermoelectric intensifiers should lead to a drop Δ of the object temperature:

$$\Delta = T_2 - T_0 = \Delta T_1 - \frac{\Delta T_r}{\varepsilon}. \quad (19)$$

From Eq. (7) it follows that

$$M = \sqrt{1 + ZT_1 - Z\,\frac{\Delta T_1}{2}} \approx M_1 - Z\,\frac{\Delta T_1}{4M_1^2}, \quad (20)$$

where

$$M_1 = \sqrt{1 + ZT_1}. \quad (21)$$

From Eq. (12) using Eq. (20) we can obtain:

$$x^2 ZT_1(\varepsilon + 1) - x[ZT_1 + 4M_1^2(\varepsilon + 1) + 4M_1\varepsilon] + 4M_1(M_1 - 1) = 0, \quad (22)$$

where

$$x = \frac{\Delta T_1}{T_1}. \quad (23)$$

Having solved Eq. (22) allowing for Eq. (9) we can find the relation between ΔT_1 and T_1 for various values of Z.

Figures 3, *a, b* present the object temperature drop Δ depending on the heat sink temperature difference for two values of Z.

It can be seen from the figure that for large values of ε the effect of temperature reduction due to the use of thermoelectric heat exchange intensifier is very insignificant (does not exceed 5 K). The greatest effect of the intensifier ($\Delta = 12{-}18$ K) is for comparatively small ε ($\varepsilon = 1 \div 2.5$), but in the area where the object temperature is rather low ($\Delta T_r = 20{-}30$ K).

Maximal heat sink temperature difference $\Delta T_{r\,\max}$, above which the use of TECs only results in the object temperature growth at $T_a = 300$ K

Figure-of-Merit	Z, 1/K			
	0.0026	0.0028	0.003	0.0032
$T_{r\,\max}$, K	58.7	62.4	66.1	69.7

3. Discussion of results

The above study shows that to reduce the cooled object temperature at large values $\Delta T_r > 40$ K, the TEC should work in the modes of large COP ($\varepsilon = 5 \div 10$). However, the effect of temperature lowering is very small: depending on the value of Z it does not exceed 5 K, i. e. a heat exchange intensifier for high COP is inefficient. Besides, operational modes with large values of ε involve both lower TEC temperature difference of ΔT_1 and lower TEC electric current — see Eq. (6). It should be accompanied by a decrease of TEC cooling capacity.

One of the main energy characteristic of TEC is the maximal cooling capacity at zero temperature difference on the TEC $Q_{\max}$:

$$Q_{\max} = \frac{KZ(T_1 - \Delta T_{\max})^2}{2}. \quad (24)$$

For small values of ΔT_1, neglecting second-order terms of $\frac{T_1}{T_0}$, we can obtain from Eq. (2) for cooling capacity in the maximum COP mode:

$$Q_0 = K\Delta T_1\left(\frac{ZT_0}{M - 1} - 1\right). \quad (25)$$

Finding K from Eq. (25) and substituting it into Eq. (24) we obtain

$$Q_{\max} = Q_0\,\frac{T_0}{\Delta T_1}\,\frac{ZT_0}{2\left(\frac{ZT_0}{M-1} - 1\right)}\,\frac{(T_1 - \Delta T_{\max})^2}{T_0^2}. \quad (26)$$

From Eq. (26) it follows that for small values of $\frac{\Delta T_1}{T_0}$ the maximal cooling capacity $Q_{\max}$ grows proportionally to $\frac{T_0}{\Delta T_1}$, and this increases both the dimensions (at a constant pellet height) and the cost of the TEC. As a result of a small thermal and economic efficiency, it is most unpromising for a thermoelectric heat exchange intensifier to work with small temperature differences, resulting from a large value of ε.

On the other hand, TEC operation at $\varepsilon < 1$ is not efficient for heat exchange intensifying either. Such modes improve heat transfer only up to the heat sink temperature difference $\Delta T_r < 30 \div 35$ K, and to reduce the temperature of the object by 10 K, the value of ΔT_r should not exceed $22 \div 28$ K.

Thus, in many cases, TEC can intensify heat transfer quite poorly. Operational modes with $\varepsilon = 1 \div 2.5$ providing temperature drop by 5 K at $\Delta T_r = 35 \div 40$ K have also very small prospects.

To reduce the temperature of the object by TEC, it is necessary to reduce the heat sink thermal resistance simultaneously. But if the heat sink is replaced, it is obvious that it can be chosen in such a way that additional cooling by TEC will not be needed. This is especially true for the application of intensifiers in electronic equipment [5], where the maximum operating temperature of silicon active elements can be up to 80−90°C. For such temperatures, the real Z values of industrial TECs are in the range $(2.6 \times 2.8) \cdot 10^{-3}\,\mathrm{K}^{-1}$, which makes the use of thermoelectric heat exchange intensifiers inefficient, according to Table.

The idea that a thermoelectric intensifier is welcome when the heat sink does not cope with its task follows from the method of simulating such intensifiers. In this method the heat sink is always optimized for TEC, therefore it is possible to find an optimal solution. In this paper we answered a different question: is TEC needed for a given heat sink or not.

Conflict of interest

The authors declare that they have no conflict of interest.

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