Interband two-photon linear-circular dichroism in semiconductors in the Kane approximation

© V.R. Rasulov, R.Ya. Rasulov¶, B.B. Akhmedov, I.A. Muminov

Fergana State University,
150100 Ferghana, Uzbekistan

¶ E-mail: r_rasulov51@mail.ru

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The dependences of the partial coefficients of interband two-photon absorption of light, which differ from each other by the types of optical transitions, are analyzed depending on the degree of polarization of the light, and a quantitative analysis of the coefficient of linear-circular dichroism of two-photon absorption of light is carried out. Expressions are obtained for the spectral dependence of the coefficient of interband two-photon absorption of light in narrow-gap semiconductors in the Kane model.

Keywords: initial, virtual and final states, interband two-photon absorption of light, Kane's approximation.

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1. Introduction

The first studies on two-photon interband transitions in crystals were made in early 60s of the last century shortly after introduction of lasers [1–3]. The theories of field perturbations of non-polarized electromagnetic wave [2,3], where two-band Kane model was applied, were used at calculation of matrix elements of two-photon transitions in crystals.

In studies [4–7] the linear-circular dichroism (LCD)[1] of two- and three-photon absorption of light in cubic symmetry crystals was examined both theoretically and experimentally.

Multi-photon light absorption in semiconductor with complex valence band, caused by direct optical transitions between sub-bands of heavy and light holes and dependent on light polarization degree, was examined in studies [8–17]. Non-linear interband single-photon absorption of polarized light in Weyl semimetals is examined in article [18]. In these studies it is assumed that non-linearity in dependence of coefficient of single-photon absorption on light intensity appears due to resonance absorption saturation [19]. This saturation both in interband [18], and intraband [9,10,16,17] light absorption is caused by photo-induced variation of function of charge carriers distribution in the area of a pulse space near surface, defined by the energy conservation principle and relaxation time, reverse value of which is equal to reverse values of relaxation time in terms of energy and pulse.

In studies [8,11,14] the multi-photon linear-circular dichroism (LCD) was examined in *p*-Ge in the developed non-linearity mode, when *n*-photon processes with $n = (1-5)$ make the consistent contribution to absorption. In studies [16,17] the four-photon processes in semiconductors, caused by optical transitions between subbands of valence band, were examined considering coherent saturation effect.

It should be noted, that in study [7] the theory of linear-circular dichroism of multi-photon interband absorption of various frequency and light polarization in semiconductors near Brillouin zone center in three-band approximation was made, when the following condition is met $\frac{2\pi e^2 I|\mathbf{ep}_{cv}|^2}{cn_\omega\omega^2 m_0^2(\hbar\omega)^2} \ll 1$, where $\mathbf{e}$ and I are the polarization vector and the light intensity, $p_{cv} = p_{c\mathbf{k},v\mathbf{k}} = \mathbf{ep}_{c\mathbf{k},v\mathbf{k}}$ is the interband matrix element of pulse operator, n_ω is the coefficient of medium light refraction on a frequency ω, m_0 is the free electron mass.

In this study, unlike with study [7], we made calculations of LCD of interband two-photon light absorption (TPLA), as well as spectral dependence of TPLA coefficient in semiconductors of InSb type in Kane model, where we consider the contributions to multi-quantum process of intermediate states in subbands of light and heavy holes and in the valence band, split out due to spin-orbit interaction, as well as in conductivity band considering coherent saturation effect. It should be noted that in semiconductors of InSb type the energy distance between the closest lower and upper conductivity bands ($\tilde{\Delta}$) is significantly larger that width of forbidden or spin-split band [20,21], that allows to perform the further studies in two-band approximation.

[1] Two-photon linear-circular dichroism, caused by interband optical transitions of electrons, was predicted by E.L. Ivchenko in study [4]

2. Classification of two-photon interband optical transitions

It is known that possibilities of single- or multi-photon optical transitions (OT) and the corresponding coefficients of LCD of light absorption are defined using the constituent matrix elements of the examined OT [4–17]. Therefore, further down the line we will perform the analysis of matrix elements related to two-photon OT, characterized with Feynman diagrams of + -type, which differ from each other by selection of initial states of charge carriers:

a) Let's assume that the initial states of charge carriers are located in the sub-band of heavy holes ($|V, \pm 3/2\rangle$) with energy E_{hh}, while virtual states — in the sub-band of heavy and light holes ($|V, \pm 3/2\rangle$) of the semiconductor valence band (Fig. 1, *a, b*). In this case, in Kane model the matrix elements of interband OT of $|V, \pm 3/2\rangle \to |m\rangle \to |c, \pm 1/2\rangle$, $|V, \pm 3/2\rangle \to |m\rangle \to |c, \mp 1/2\rangle$ type are defined by the matrix in order $(c, +1/2), (c, -1/2)$ and $(V, +3/2), (V, -3/2)$

$$\|M^{(2)}_{m',m}\| = \left(\frac{eA_0}{c\hbar}\right)^2 P_{cV} k \times \left\| \begin{array}{cc} \frac{2(A-B)e'_+e_{z'}}{-\hbar\omega} + \frac{e'^2_- B}{E_{lh}-E_{hh}-\hbar\omega} & \frac{\sqrt{2}Be'_z e'_-}{E_{lh}-E_{hh}-\hbar\omega} \\ \frac{i\sqrt{2}e'_z e'_+ B}{E_{lh}-E_{hh}-\hbar\omega} & -i\left(\frac{2(A-B)e'_-e_{z'}}{-\hbar\omega} + \frac{e'^2_+ B}{E_{lh}-E_{hh}-\hbar\omega}\right) \end{array} \right\|, \quad (1)$$

where P_{cV} is the Kane parameter [22,23], $E_{lh}(E_{hh})$ is the energy of light (heavy) holes, A, B are the crystal band parameters, $|c, \pm 1/2\rangle$ corresponds to electron states with energy E_c in the conductivity band, k is the wave vector of charge carriers in the end state, $e'_\pm = e'_x \pm ie'_y$, $e'_\alpha (\alpha = x, y, z)$ are the light polarization vector projections **e** on axis x', y', perpendicular to the wave vector **k**. The energy conservation principle, defined for the specified OT, is expressed using function $\delta(E_c - E_{hh} - 2\hbar\omega)$, E_{hh} is the heavy holes energy.

b) Let's assume that the initial states of charge carriers are located in the sub-band of light holes of the valence band, while virtual states — in sub-bands of heavy and light holes of the valence band (Fig. 2, *a, b*). In this case, in Kane model the matrix elements of interband OT of $|V, \pm 1/2\rangle \to |m\rangle \to |c, \pm 1/2\rangle$, $|V, \pm 1/2\rangle \to |m\rangle \to |c, \mp 1/2\rangle$ type are defined by the matrix in order $(c, +1/2), (c, -1/2)$ and $(V, +1/2), (V, -1/2)$, respectively and written the following way:

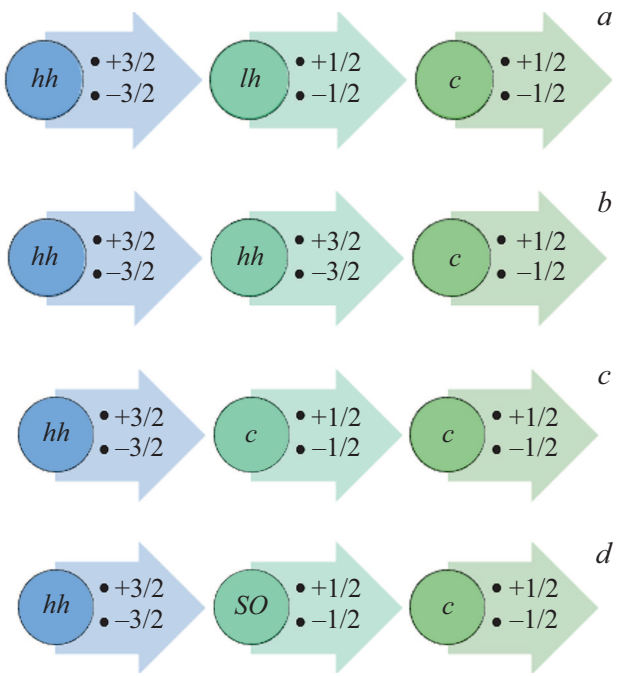

Figure 1. Schemes of two-photon OT, happened between sub-bands of heavy holes of the valence band and conductivity band, where *hh* (*lh*) is the sub-band of heavy (light) holes, *SO* is the spin-orbit splitting band.

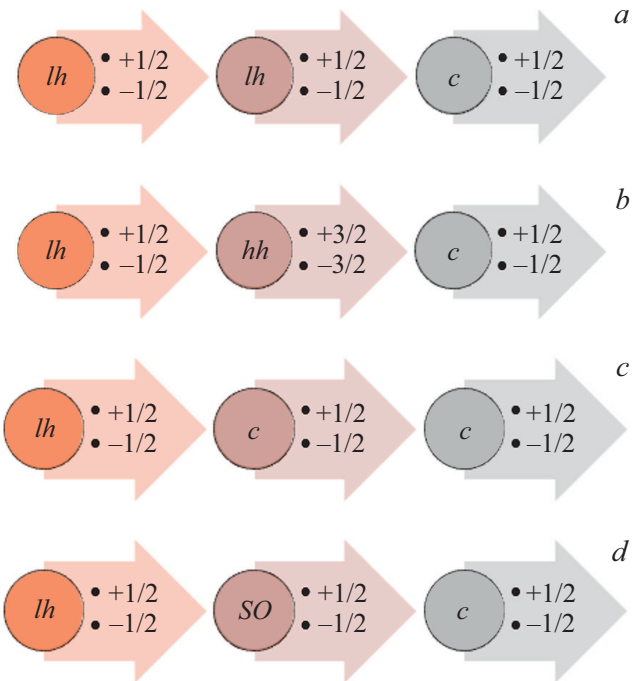

Figure 2. Schemes of two-photon OT, happened between sub-bands of light holes of the valence band and conductivity band.

$$\|M^{(2)}_{m',m}\| = \left(\frac{eA_0}{c\hbar}\right)^2 P_{cV} k \left\| \begin{array}{cc} \left(\frac{\sqrt{3}Be'^2_+}{E_{hh}-E_{lh}-\hbar\omega} - \frac{2}{\sqrt{3}}\frac{(A+B)e'_-e_{z'}}{\hbar\omega}\right) & 2\sqrt{\frac{2}{3}}\frac{(A+B)e'_-e'^2_z}{(-\hbar\omega)} \\ -i2\sqrt{\frac{2}{3}}\frac{(A+B)e'^2_z}{\hbar\omega} & -i\left(\frac{\sqrt{3}Be'^2_-}{E_{hh}-E_{lh}-\hbar\omega} - \frac{2}{\sqrt{3}}\frac{(A+B)e'_+e_{z'}}{\hbar\omega}\right) \end{array} \right\|, \quad (2)$$

The energy conservation principle for these OT is expressed using function $\delta(E_c - E_{lh} - 2\hbar\omega)$, E_{lh} is the light holes energy.

c) Let's assume that the initial states of charge carriers are located in the sub-band of light holes of the valence band, while virtual states — in the conductivity band (Fig. 2, *c*). In this case, in Kane model the matrix elements of interband OT of $|V, \pm 1/2\rangle \to |m\rangle \to |c, \pm 1/2\rangle$, $|V, \pm 1/2\rangle \to |m\rangle \to |c, \mp 1/2\rangle$ type are defined by the matrix in order $(c, +1/2), (c, -1/2)$ and $(V, +1/2), (V, -1/2)$, respectively and written the following way:

$$\|M^{(2)}_{m',m}\| = \frac{1}{\sqrt{3}} \left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{\hbar\omega} \frac{\hbar^2 k}{m_c} P_{cV} k \left\| \begin{matrix} e'_+ & \sqrt{2}e'_z \\ -i\sqrt{2}e'_z & ie'_- \end{matrix} \right\|, \tag{3}$$

where $|m\rangle$ — $|c, \pm 1/2\rangle$, E_g is the forbidden band width, while the energy conservation principle is expressed using function $\delta(E_c - E_{lh} - 2\hbar\omega)$.

d) Let's assume that the initial states of charge carriers are located in the sub-band of light holes of the valence band, while virtual states — in the spin-orbit splitting band (Fig. 2, *d*). In this case, in Kane model the matrix elements of interband OT of $|V, \pm 1/2\rangle \to |m\rangle \to |c, \pm 1/2\rangle$, $|V, \pm 1/2\rangle \to |m\rangle \to |c, \mp 1/2\rangle$ type are defined by the matrix in order $(c, +1/2), (c, -1/2)$ and $(V, +1/2), (V, -1/2)$, respectively and written the following way:

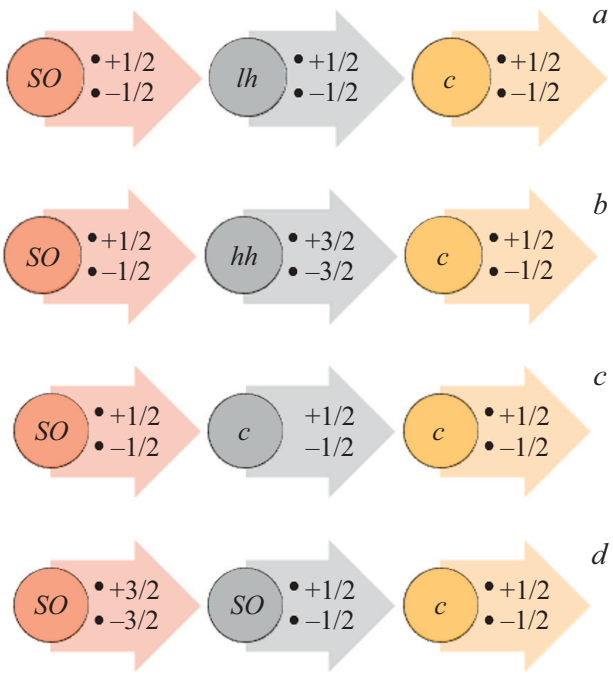

Figure 3. Schemes of two-photon OT, happened between the spin-orbit splitting band and conductivity band.

$$(\widehat{M})_{c,lh} = \left(\frac{eA_0}{c\hbar}\right)^2 \frac{P_{cV} Bk}{2\sqrt{3}(E_{hh} - E_{lh} - \hbar\omega)} \begin{bmatrix} 6e'_z e'^2_- + (E_{lh} + \hbar\omega) \frac{e'_+(4e'^2_z + e'^2_\perp)}{\hbar\omega} & -2\sqrt{2}(E_{lh} + \hbar\omega) \frac{e'_z(4e'^2_z + e'^2_\perp)}{\hbar\omega} + i3e'^3_+ \\ 3e'^3_- - i2\sqrt{2}(E_{lh} + \hbar\omega) \frac{e'_+(4e'^2_z + e'^2_\perp)}{\hbar\omega} & 6ie'_z e'^2_+ + i(E_{lh} + \hbar\omega) \frac{e'_+(4e'^2_z + e'^2_\perp)}{\hbar\omega} \end{bmatrix}, \tag{4}$$

where $|m\rangle = |SO, \pm 1/2\rangle$. The energy conservation principle for that case is described with function $\delta(E_c - E_{lh} - 2\hbar\omega)$.

e) Let's assume that the initial states of charge carriers are located in the sub-band of heavy holes of the valence band, while virtual states — in the spin-orbit splitting band (Fig. 1, *c*). In this case, in Kane model the matrix elements of interband OT of $|V, \pm 3/2\rangle \to |m\rangle \to |c, \pm 1/2\rangle$, $|V, \pm 3/2\rangle \to |m\rangle \to |c, \mp 1/2\rangle$ type are defined by the matrix in order $(c, +1/2), (c, -1/2)$ and $(V, +3/2), (V, -3/2)$, respectively and written the following way:

$$\|M^{(2)}_{m',m}\| = \frac{1}{\sqrt{6}} \left(\frac{eA_0}{c\hbar}\right)^2 \frac{P_{cV}}{E_\Delta - E_{hh} - \hbar\omega} \times \left\| \begin{matrix} -\sqrt{3}e'_+ H^{*\prime} - ie'_z(G' - F') & -ie'_-(G' - F') + e'_z H^{*\prime} \\ ie'_z H' - e'_+(G' - F') & e'_z(G' - F') + i\sqrt{3}e' H' \end{matrix} \right\|, \tag{5}$$

where E_Δ is the energy spectrum of charge carriers in the spin-orbit splitting band, G', F', H' are the first derivative in terms of wave vector of charge carries from values G, F, H, which are defined using the formula (24.20) from study [22], $(^*)$ is the complex conjugation sign. The energy conservation principle is expressed using the function $\delta(E_c - E_{hh} - 2\hbar\omega)$.

The similar way it is possible to present the expressions for optical transitions, where the initial states of charge carriers are located in the sub-band of heavy holes, while the virtual states — in conductivity band and spin-orbit splitting band (Fig. 1, *c, d*), and expressions for optical transitions, where the initial states of charge carriers are located in spin-orbit splitting band, while the virtual states — in the sub-band of light and heavy holes, as well as in conductivity band and spin-orbit splitting band (Fig. 3, *a*–*d*), which are not presented for brevity's sake, but are considered at calculations of spectral dependence of light absorption coefficient and its linear-circular dichroism.

It should be noted that the wave spectrum in the end state of electrons, participating in interband OT, is defined using the expression $k^{(2\omega)}_{c,L} = \sqrt{\frac{2\mu^{(c,L)}_+}{\hbar^2}(2\hbar\omega - E_g)}$, where m_c is the electrons effective mass in conductivity band, m_L is the holes effective mass in sub-band L, $L = lh$ $(L = hh)$ — for light (heavy) holes, $\mu^{(c,L)}_\perp = \frac{m_c m_L}{m_c + m_L}$ is the reduced effective

mass of charge carriers. Then for energy of light and heavy holes the following relations are valid:

a) if OT is happened from the sub-band of heavy holes, then

$$E_{L=hh}(k^{(2\omega)}_{c,L=hh}) = \frac{m_c}{m_c + m_{hh}}(2\hbar\omega - E_g),$$

$$E_{lh}(k^{(2\omega)}_{c,L=hh}) = \frac{m_c\, m_{hh}}{m_{lh}(m_c + m_{hh})}(2\hbar\omega - E_g);$$

b) if OT is happened from the sub-band of light holes, then

$$E_{L=hh}(k^{(2\omega)}_{c,L=lh}) = \frac{m_c\, m_{lh}}{m_{hh}(m_c + m_{lh})}(2\hbar\omega - E_g),$$

$$E_{lh}(k^{(2\omega)}_{c,L=lh}) = \frac{m_c}{m_c + m_{lh}}(2\hbar\omega - E_g).$$

Matrix elements of two-photon transitions, happened from spin-split band to conductivity band, where virtual states of charge carriers are located in the sub-bands of valence band, in the conductivity band, as well as in spin-orbit splitting band of semiconductor, which are presented in Fig. 3 and defined the similar way as in the abovementioned cases.

Thus, interband two-photon OT were classified in narrow-gap crystal and expressions were observed for matrix elements depending on band parameters, polarization degree and light frequency.

3. Interband two-photon absorption of polarized light and its linear-circular dichroism

In this section we observe the expression for spectral dependence of the coefficient of interband TPLA in narrow-gap semiconductors in Kane model. In the further calculations we use the calculation method proposed in studies [4,7,10,18]).

It should be noted that the coefficient of multi-photon light absorption consists of partial constituents, which naturally depend on the band, in which charge carriers are located both in initial and virtual state.

In the further (intermediate) calculations instead of $\sum_{\mathbf{k}}(f_L - f_{\text{cond}})\delta(E_{\text{cond}} - E_L - 2\hbar\omega)F(k)$ we use the expression $\frac{1}{(2\pi)^3}F(k_{c,L})k^2_{c,L}$, where $k_{c,L}$ is the wave vector, defined from the energy conservation principle: $E_c - E_L - 2\hbar\omega = 0$. Particularly, in spherical approximation in the energy spectrum of charge carriers, i.e. in case of $E_L = E_L^{(0)} + \frac{\hbar^2k^2}{2m_L}$, the wave vector of charge carriers, participating in interband OT, is defined as $k^2_{c,L} = \frac{2\mu_+^{(c,L)}}{\hbar^2}(2\hbar\omega - E_g)$, where $\mu_+^{(c,lh)} = \frac{m_c m_L}{m_c + m_L}$ is the reduced effective mass, m_L is the charge carriers effective mass in the band (or sub-band) with number L. Particularly, $L = c$ for the conductivity band, then $E_L^{(0)} = E_g$, while $L = lh(hh)$ for sub-band of light (heavy) holes of the valence band, then $E_L^{(0)} = 0$.

It should be noted that the frequency dependence of denominators in the matrix elements is defined with the energy conservation principle, type of examined OT and virtual states. For instance, if virtual states are in the valence band, and initial state is in the heavy holes sub-band, then the denominator in the matrix element of this period is defined with the expression

$$E_{hh} - E_{lh} - \hbar\omega = \frac{m_c}{m_{hh}}\frac{m_{hh} - m_{lh}}{m_c + m_{lh}}(2\hbar\omega - E_g) + \hbar\omega,$$

if this transition is happened from the light holes sub-band, then the denominator in the matrix element of this transition is defined as

$$E_{lh} - E_{hh} - \hbar\omega = \frac{m_c}{m_{lh}}\frac{m_{hh} - m_{lh}}{m_{hh} + m_c}(2\hbar\omega - E_g),$$

where the following relations are considered:

$$A - B = \frac{\hbar^2}{2m_{hh}}, \quad A + B = \frac{\hbar^2}{2m_{lh}}.$$

Then we calculate the partial coefficients of two-photon absorption (e.g. see the formula (1) from study [16]), that are differ from each other by OT type, i.e. from initial, intermediate and virtual states:

a) if initial state is in the heavy holes sub-band of the valence band, then, according to [16,18,19], the coefficient of interband two-photon light absorption can be defined with the expression[2]

$$K^{(2)}_{C,\pm1/2;V,\pm3/2} = \frac{2\pi}{\hbar}\,2\hbar\omega\,\frac{1}{I}\,\Xi^{(2)}_{C,hh}\left(\frac{eA_0}{c\hbar}\right)^2 P^2_{cV}k^2$$

$$\times\left(\left\langle \frac{\left|\frac{2(A-B)e'_+e_{z'}}{-\hbar\omega} + \frac{e'^2_-B}{(E_{lh}-E_{hh}-\hbar\omega)}\right|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\left(\frac{eA_0}{c\hbar}\right)^2 P_{cV}k\right]^2\left|\frac{2(A-B)e'_+e_{z'}}{-\hbar\omega}+\frac{e'^2_-B}{(E_{lh}-E_{hh}-\hbar\omega)}\right|^2}}\right\rangle\right.$$

$$\left.+\left\langle \frac{\left|\sqrt{2}B\,\frac{e'_ze'_-}{(E_{lh}-E_{hh}-\hbar\omega)}\right|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\left(\frac{eA_0}{c\hbar}\right)^2 P_{cV}k\right]^2\left|\sqrt{2}B\,\frac{e'_ze'_-}{(E_{lh}-E_{hh}-\hbar\omega)}\right|^2}}\right\rangle\right), \tag{6}$$

from where, after averaging over solid angles of the wave vector of charge carriers ($\langle\ldots\rangle$) and not considering the

[2] Both here and later we examine the area of low light intensity, when the perturbation theory is applied.

coherent saturation, we get

$$K_{C,\pm1/2;V,\pm3/2}=\frac{8\pi^2}{\hbar}\,\hbar\omega\,\frac{1}{I}\,\frac{(\mu_+^{c,hh})^{3/2}}{(2\pi)^3\hbar^3}\sqrt{2}\sqrt{2\hbar\omega-E_g}\,f_{hh}$$

$$\times\left[\frac{m_c}{m_c+m_{hh}}\,(2\hbar\omega-E_g)\right]\left[\left(\frac{eA_0}{c\hbar}\right)^2 P_{cV}k\right]^2\Re^{(2)}_{C,\pm1/2;V,\pm3/2}. \quad (7)$$

Here, $\Xi^{(2)}_{C,L}=\sum\limits_{\mathbf{k}}(f_L-f_{\rm cond})\delta(E_{\rm cond}-E_L-2\hbar\omega)$,

$$\Re^{(2)}_{C,\pm1/2;V,\pm3/2}=\frac{1}{4\pi}$$

$$\times\left\langle\left|\frac{2(A-B)e'_+e_{z'}}{(-\hbar\omega)}+\frac{e'^2_-B}{(E_{lh}-E_{hh}-\hbar\omega)}\right|^2\right\rangle$$

$$+\left\langle\left|\sqrt{2}B\,\frac{e'_ze'_-}{(E_{lh}-E_{hh}-\hbar\omega)}\right|^2\right\rangle$$

$$=\frac{B^2}{15(\hbar\omega)^2}\left[\left(2\,\frac{A-B}{B}\right)^2 a^{(2)}_{C,\pm1/2;V,\pm3/2}\right.$$

$$\left.+\left(\frac{\hbar\omega}{E_{lh}-E_{hh}-\hbar\omega}\right)^2 b^{(2)}_{C,\pm1/2;V,\pm3/2}\right], \quad (8)$$

that in spherical approximation in energy spectrum of charge carriers is written as

$$\Re^{(2,sfer)}_{C,\pm1/2;V,\pm3/2}=\frac{B^2}{15(\hbar\omega)^2}\left[\frac{16m_{lh}^2}{(m_{hh}-m_{lh})^2}\,a^{(2)}_{C,\pm1/2;V,\pm3/2}\right.$$

$$\left.+\left(\frac{\hbar\omega(m_{hh}+m_c)m_{lh}}{m_c(m_{hh}-m_{lh})(2\hbar\omega-E_g)}\right)^2 b^{(2)}_{C,\pm1/2;V,\pm3/2}\right], \quad (9)$$

where for linear (circular)-polarized light $a^{(2)}_{C,\pm1/2;V,\pm3/2}=2\,(a^{(2)}_{C,\pm1/2;V,\pm3/2}=9)$, $a^{(2)}_{C,\pm1/2;V,\pm3/2}=3$ $(b^{(2)}_{C,\pm1/2;V,\pm3/2}=13)$, $b^{(2)}_{C,\pm1/2;V,\pm3/2}=3\,(b^{(2)}_{C,\pm1/2;V,\pm3/2}=13)$. In this case the coefficient of LCD for these OT depends on light frequency and band parameters;

b) if the initial state is in the light holes sub-band, then we get

$$K^{(2)}_{C,\pm1/2;V,\pm3/2}=\frac{2\pi}{\hbar}\,2\hbar\omega\,\frac{1}{I}\,\Xi^{(2)}_{c,lh}\left(\frac{eA_0}{c\hbar}\right)^4\frac{P_{cV}^2k^2}{3}$$

$$\times\left(\left\langle\frac{\left|\frac{3Be'^2_+}{(E_{hh}-E_{lh}-\hbar\omega)}+2\,\frac{(A+B)e'_-e_{z'}}{(-\hbar\omega)}\right|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\left(\frac{eA_0}{c\hbar}\right)^2\frac{P_{cV}k}{\sqrt{3}}\right]^2\left|\frac{3Be'^2_+}{(E_{hh}-E_{lh}-\hbar\omega)}+2\,\frac{(A+B)e'_-e_{z'}}{(-\hbar\omega)}\right|^2}}\right\rangle\right.$$

$$\left.+\left\langle\frac{\left|2\sqrt{2}(A+B)\,\frac{e'^2_z}{(-\hbar\omega)}\right|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\left(\frac{eA_0}{c\hbar}\right)^2\frac{pk}{\sqrt{3}}\right]^2\left|2\sqrt{2}(A+B)\,\frac{e'^2_z}{(-\hbar\omega)}\right|^2}}\right\rangle\right), \quad (10)$$

or

$$K^{(2)}_{C,\pm1/2;V,\pm3/2}=\frac{32\pi}{\hbar}\,\hbar\omega\,\frac{1}{I}\,\Xi_{c,lh}\left(\frac{eA_0}{c\hbar}\right)^2\frac{P_{cV}^2k^2}{3}$$

$$\times\,\Re^{(2)}_{C,\pm1/2;V,\pm1/2}. \quad (11)$$

Here,

$$\Re^{(2)}_{C,\pm1/2;V,\pm1/2}=\frac{1}{4\pi}$$

$$\times\left(\left\langle\left|\frac{3Be'^2_+}{(E_{hh}-E_{lh}-\hbar\omega)}+2\,\frac{(A+B)e'_-e_{z'}}{(-\hbar\omega)}\right|^2\right\rangle\right.$$

$$\left.+\left\langle\left|2\sqrt{2}(A+B)\,\frac{e'^2_z}{(-\hbar\omega)}\right|^2\right\rangle\right)=\frac{B^2}{15(\hbar\omega)^2}$$

$$\times\left[4\left(\frac{A+B}{B}\right)^2+\left(\frac{3\hbar\omega}{E_{hh}-E_{lh}-\hbar\omega}\right)^2\right]a^{(2)}_{C,\pm1/2;V,\pm1/2}, \quad (12)$$

that in spherical approximation of the energy spectrum of charge carriers is written as

$$\Re^{(2,sfer)}_{C,\pm1/2;V,\pm1/2}=\frac{\hbar^4(m_{hh}-m_{lh})^2}{15(4\hbar\omega m_{hh}m_{lh})^2}\left[4\left(\frac{2m_{hh}}{m_{hh}-m_{lh}}\right)^2\right.$$

$$\left.+\left(\frac{3\hbar\omega}{\frac{m_c}{m_{hh}}\,\frac{m_{hh}-m_{lh}}{m_c+m_{lh}}\,(2\hbar\omega-E_g)+\hbar\omega}\right)^2\right]a^{(2)}_{C,\pm1/2;V,\pm1/2}, \quad (13)$$

where for linear (circular)-polarized light $a^{(2)}_{C,\pm1/2;V,\pm1/2}=8$ $(a^{(2)}_{C,\pm1/2;V,\pm1/2}=7)$, the coefficient of LCD for these OT does not depend on light frequency and is equal to 8/7.

Now let's assume that the virtual states of charge carriers are in the conductivity band. In this case:

a) if the initial state is in the heavy holes sub-band of the valence band, then

$$K^{(2)}_{C,\pm1/2;V,\pm3/2}=\frac{2\pi}{\hbar}\,2\hbar\omega\,\frac{1}{I}\,\Xi_{c,hh}\left(\frac{eA_0}{c\hbar}\right)^4\left(\frac{P_{cV}k}{\hbar\omega}\,\frac{\hbar^2}{m_c}\right)^2$$

$$\times\left\langle\frac{|e'_ze'_-|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\left(\frac{eA_0}{c\hbar}\right)^2\frac{P_{cV}k}{\hbar\omega}\,\frac{\hbar^2}{m_c}\right]^2|e'_ze'_-|^2}}\right\rangle, \quad (14)$$

from which, not considering the contribution of the coherent saturation effect to $K^{(2)}_{C,\pm1/2;V,\pm3/2}$, we get

$$K^{(2)}_{C,\pm1/2;V,\pm3/2}=\frac{2\pi}{\hbar}\,2\hbar\omega\,\frac{1}{I}\,\Xi_{c,hh}\left(\frac{eA_0}{c\hbar}\right)^4$$

$$\times\left(\frac{P_{cV}k}{\hbar\omega}\,\frac{\hbar^2}{m_c}\right)^2\frac{1}{15}\,a^{(2)}_{C,\pm1/2;V,\pm3/2}, \quad (15)$$

where for linear (circular)-polarized light $a^{(2)}_{C,\pm1/2;V,\pm3/2}=2$ $(a^{(2)}_{C,\pm1/2;V,\pm3/2}=3)$, the coefficient of LCD for these OT is constant and equal to 2/3;

b) if the initial state is in the light holes sub-band of the valence band, then

$$K^{(2)}_{C,\pm1/2;V,\pm1/2} = \frac{2\pi}{\hbar}\, 2\hbar\omega\, \frac{1}{I}\, \Xi_{c,lh} \left(\frac{eA_0}{c\hbar}\right)^4 \left(\frac{P_{cV}k}{\hbar\omega}\, \frac{\hbar^2}{m_c}\right)^2 \times \Im^{(2)}_{C,\pm1/2;V,\pm1/2}, \tag{16}$$

$$\Im^{(2)}_{C,\pm1/2;V,\pm1/2} = \left\langle \frac{|e'_z e'_+|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\frac{1}{\sqrt{3}}\left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{\hbar\omega}\,\frac{\hbar^2}{m_c}\,P_{cV}k\right]^2 |e'_z e'_+|^2}} \right\rangle + \left\langle \frac{|\sqrt{2}e'^2_z|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\frac{1}{\sqrt{3}}\left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{\hbar\omega}\,\frac{\hbar^2}{m_c}\,P_{cV}k\right]^2 |\sqrt{2}e'^2_z|^2}} \right\rangle, \tag{17}$$

from which, not considering the contribution of the coherent saturation effect to $K^{(2)}_{C,\pm1/2;V,\pm1/2}$, we get, that for a light with linear (circular) polarization $\Im^{(2)}_{C,\pm1/2;V,\pm1/2} = 8/15$ ($\Im^{(2)}_{C,\pm1/2;V,\pm1/2} = 7/15$), and the coefficient of LCD is equal to 7/8.

Now let's assume that the virtual states of charge carriers are in the spin-orbit extended band. In this case:

a) if the initial state is in the heavy holes sub-band of the valence band, then we get that

$$K^{(2)}_{C,\pm1/2;V,\pm3/2} = \frac{2\pi}{\hbar}\, 2\hbar\omega\, \frac{1}{I}\, \Xi_{c,hh} \times \left[\left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{\sqrt{2}}\, \frac{BkP_{cV}}{(E_\Delta - E_{hh} - \hbar\omega)}\right]^2 \Phi^{(2)}_{C,\pm1/2;V,\pm3/2}. \tag{18}$$

Here,

$$\Phi^{(2)}_{C,\pm1/2;V,\pm3/2} = \left\langle \frac{|e'_z e'_-|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{\sqrt{2}}\,\frac{BkP_{cV}}{(E_\Delta - E_{hh} - \hbar\omega)}\right]^2 |e'_z e'_-|^2}} \right\rangle + \left\langle \frac{|e'^2_\perp|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{\sqrt{2}}\,\frac{BkP_{cV}}{(E_\Delta - E_{hh} - \hbar\omega)}\right]^2 |e'^2_\perp|^2}} \right\rangle, \tag{19}$$

from which, not considering the contribution of the coherent saturation effect to $K^{(2)}_{C,\pm1/2;V,\pm3/2}$, we get, that for a light with linear (circular) polarization the coefficient of LCD is equal to 2/3;

b) if the initial state is in the light holes sub-band of the valence band, then the coefficient of two-photon polarized light absorption is defined as

$$K^{(2)}_{C,\pm1/2;V,\pm1/2} = \frac{2\pi}{\hbar}\, 2\hbar\omega\, \frac{1}{I}\, \Xi_{c,lh} \times \left[\left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{\sqrt{6}}\, \frac{BkP_{cV}}{(E_\Delta - E_{hh} - \hbar\omega)}\right]^2 \Phi^{(2)}_{C,\pm1/2;V,\pm1/2}. \tag{20}$$

$$\Phi^{(2)}_{C,\pm1/2;V,\pm1/2} = \left\langle \frac{|3e'^2_\pm + 4e^2_{z'}|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{\sqrt{6}}\,\frac{BkP_{cV}}{(E_\Delta - E_{hh} - \hbar\omega)}\right]^2 |3e'^2_\pm + 4e^2_{z'}|^2}} \right\rangle + \left\langle \frac{|e'_z e'_+|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left[\frac{1}{\sqrt{3}}\left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{\hbar\omega}\,\frac{\hbar^2}{m_c}\,P_{cV}k\right]^2 |e'_z e'_+|^2}} \right\rangle, \tag{21}$$

from which, not considering the contribution of the coherent saturation effect to $K^{(2)}_{C,\pm1/2;V,\pm3/2}$, we get, that the coefficient of LCD is equal to 3/2.

It should be noted that the aggregate coefficient of TPLA ($K^{(2)}_{c,V}$) is defined by the sum of the abovementioned partial coefficients of TPLA. Thus, the main contribution to LCD of TPLA is made by OT, coming from the light holes sub-band to the conductivity band.

Then we calculate the spectral dependence of the aggregate coefficient of TPLA in Kane model and use the following expressions for energy spectra of charge carriers in parabolical approximation:

$$E_c(\mathbf{k}) = E_g + \frac{\hbar^2k^2}{2m_0} + \frac{k^2P^2_{cV}\left(E_g + \frac{2}{3}\Delta\right)}{E_g(E_g+\Delta)},$$
$$E_{hh}(\mathbf{k}) = \frac{\hbar^2k^2}{2m_0}, \quad E_{lh}(\mathbf{k}) = \frac{\hbar^2k^2}{2m_0} - \frac{2k^2P^2_{cV}}{3E_g},$$
$$E_{SO}(\mathbf{k}) = -\Delta + \frac{\hbar^2k^2}{2m_0} - \frac{k^2P^2_{cV}}{3(\Delta+E_g)}, \tag{22}$$

where $E_g(\Delta)$ is the forbidden (spin-orbit) band width, P_{cV} is the Kane parameter [23]. Then the spectral dependence of the coefficient of two-photon absorption of linearly polarized light in the area of low values of the wave vector of charge carriers is written as

$$K^{(2)}_{c,V}(\omega) = K^{(0)}_{c,V}\Im^{(2,l)}_{c,V}\left(\frac{2\hbar\omega}{E_g}\right), \tag{23}$$

where $K^{(0)}_{c,V} = \frac{4\pi e^2P_{cV}}{\hbar c^2 n^2_\omega E^3_g}$, $E_g \ll E_{SO}$ for case $l=1$, $E_g \gg E_{SO}$ for case $l=2$,

$$\Im^{(2,1)}_{c,V}(\xi) = \frac{4\xi^{3/2}}{15\sqrt{6}(\xi+1)^3}\left[480\,\frac{(\xi+1)^{1/2}}{(3\xi+1)^2} + \frac{(\xi+2)^{3/2}}{(\xi+1)^5} \times \Big(9(\xi+1)^4 + 40(\xi+1)^2 + 96\Big)\right], \tag{24}$$

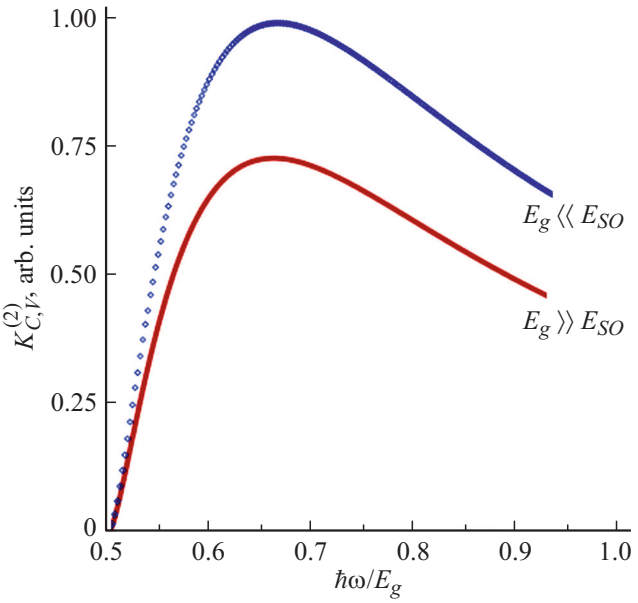

Figure 4. Spectral dependence of the coefficient of interband two-photon absorption of the linearly polarized light in InSb crystal, corresponding to cases $E_g \gg E_{SO}$ and $E_g \ll E_{SO}$.

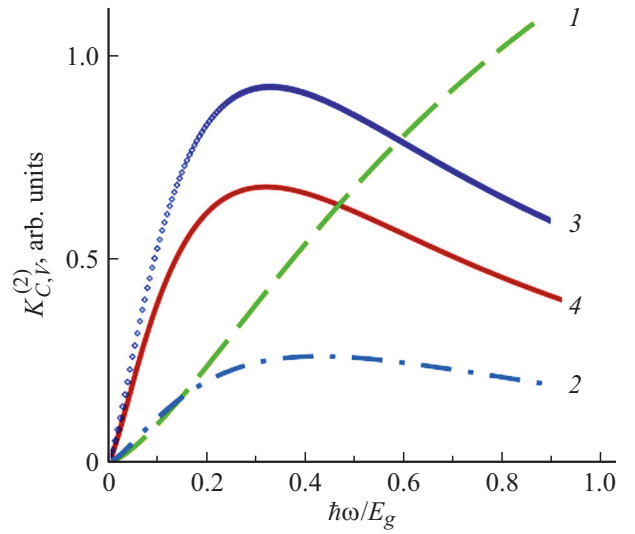

Figure 5. Spectral dependence of the coefficient of interband two-photon absorption of the linearly polarized light in InSb crystal, corresponding to cases $E_g \ll E_{SO}$ (curves *1* and *3*) and $E_g \gg E_{SO}$ (curves *2* and *4*). Curves *1* and *2* correspond to parabolical, and curves *3* and *4* — non-parabolical approximation in the energy spectrum of charge carriers.

$$\Im^{(2,2)}_{c,V}(\xi) = \frac{32\xi^{3/2}}{15(\xi+1)^3}\left\{36\,\frac{(\xi+1)^{1/2}}{(3\xi+1)^2} + \frac{(\xi+2)^{3/2}}{(\xi+1)^5} \times \left((\xi+1)^4 + 2(\xi+1)^2 + 6\right)\right\}, \qquad (25)$$

$\xi = (2\cdot\hbar\omega - E_g)/E_g$.

Figures 4 and 5 show the dependence of $K^{(2)}_{c,V}(\omega)$ for InSb in two cases: $E_g \ll E_{SO}$ and $E_g \gg E_{SO}$. It was assumed in the calculations that the initial states of charge carriers are fully occupied, while the end states — completely empty. These results show that at InSb lighting with linearly polarized light, both in case of $E_g \ll E_{SO}$ and $E_g \gg E_{SO}$, the spectral dependence $K^{(2)}_{c,V}(\omega)$ increases with frequency rise, reaches maximum and then decreases (Figs. 4 and 5, curves *3, 4*). This is explained by complexity of the band structure of semiconductor in Kane model, that is expressed in matrix elements and energy spectra. Because of it, the complex dependencies of state density and energy of both initial and end states of photoexcited charge carriers on light frequency appear. If we limit ourselves to spherical approximation in the energy spectrum, then $K^{(2)}_{c,V}(\omega)$ will increase with frequency rise if $E_g \ll E_{SO}$ (Fig. 4).

It should be noted that if considering the anisotropy of the energy spectrum of electrons in the valence band in two-band approximation and in the area of low values of the wave vector of charge carriers, our results on polarization dependence of the coefficient of two-photon light absorption in narrow-gap semiconductors coincide with the results of study [7]. Quantitative values of band parameters were taken from study [24].

4. Conclusion

In conclusion let's summarize the main results and findings of the performed study.

1. Matrix elements of interband two-photon OT in narrow-gap semiconductor are classified depending on light polarization vector component.

2. In Kane approximation, both considering and not considering the coherent saturation effect, the polarization and spectral dependencies of partial coefficients of TPLA and LCD, that differ from each other by OT type, are calculated.

3. Based on the golden rule of quantum mechanics it is shown, that at InSb lighting with linearly polarized light, bot in case of $E_g \ll E_{SO}$ and $E_g \gg E_{SO}$, the spectral dependence $K^{(2)}_{c,V}(\omega)$ increases with frequency rise, reaches maximum, and then decreases, and this case is explained with complexity of dependencies of states density and energy of both initial and end states of photoexcited charge carriers on light frequency, which are related to peculiarities of semiconductor band structure in Kane model. If we limit ourselves to spherical approximation in the energy spectrum, then $K^{(2)}_{c,V}(\omega)$ will increase with frequency rise if $E_g \ll E_{SO}$ (Fig. 5, curve *1*).

4. Theory of LCD, related to interband two-photon OT in narrow-gap semiconductors in Kane approximation, is developed.

Conflict of interest

The authors declare that they have no conflict of interest.

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