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A variational approach to the determination of the dynamic strength of a material

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The problem of the determination of material strength properties through the Kolsky experimental technique is considered. Small size specimens of M1 copper alloy are tested on a split Hopkinson pressure bars equipment. The experimental data of tensile tests observed under both dynamic and quasi-static conditions are analysed within the framework of the incubation time criterion and the Sign-Perturbed Sums method. It is shown that the influence of a test performance error is considered in the data treatment procedure based on the developed method.

Keywords: Dynamic strength, incubation time criterion, Kolsky bar system, Sign-Perturbed Sums method.

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Introduction

Estimation of material strength properties under dynamic loading is a general practical problem. Experimental methods require thorough and well-arranged planning of each test stage from specimen preparation to data processing and interpretation in order to estimate all factors affecting the accuracy and repeatability of results. Deviation of a measured parameter from the true value can have both a constant systematic error and a random component. The goal of research is to develop procedures that allow for accounting and preventing the influence of various random interferences in estimation of material's strength characteristics.

A peculiarity of dynamic testing is the high variability of results depending on loading rate [1–7], as well as other factors affecting the material characteristics. Therefore, the conventional probabilistic methods for the case of a Gaussian distribution of random noises do not always ensure a reliable result, while insufficiency of experimental data in a wide range of change of external impact parameters prevent from applying the statistical analysis based on the central limit theorem. Thus, due to all the above-mentioned problems there is no standard unified engineering approach to estimation of materials dynamic strength properties.

The developed method of experimental-analytical analysis of material strength properties is based on the structural-time approach to prediction of the critical failure state under a random dynamic load [8,9]. The advantage of this approach is that material properties are determined not within one experiment, but based on an array of experimental points obtained with various speed conditions. The previous studies have shown high applicability of the structural-time approach to various problems of mechanics and physics [10]. Values of the model parameters within the framework of this study are determined indirectly using the SPS method — Sign-Perturbed Sums [11]. An estimate of the model parameters within the framework of this randomized approach is a certain confidence interval that contains the true parameter value with the given probability, which fully corresponds to the nature of experimental data.

The M1 copper alloy was chosen as the material under study. Experiments were carried out on a dynamic tension unit using split Hopkinson pressure bars. Expansion of experimental procedures for determination of materials– dynamic properties through implementation of variational analysis methods has allowed for applying a new technique for material testing within the framework of the structural-time approach.

1. Test procedure

Tension experiments in the region of the quasi-static loading conditions were performed on a Shimadzu AG-50kNX tensile tester. Dynamic tension experiments were performed on a Kolsky-method unit using split Hopkinson pressure bars [12,13]. The unit diagram is shown in Fig. 1. The unit consists of a pneumatic loading device with the gage of 27 mm, striker 400 mm long, split Hopkinson pressure bars with the diameter of 16 mm and loading bar length of 3000 mm, as well as a measuring bar — 1500 mm. The striker is accelerated by compressed air supplied by the compressor into the chamber, pressure is

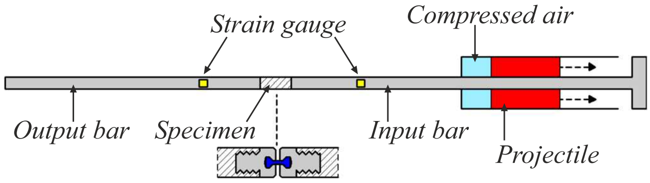

Figure 1. Experimental tension setup implementing the Kolsky method using split Hopkinson pressure bars.

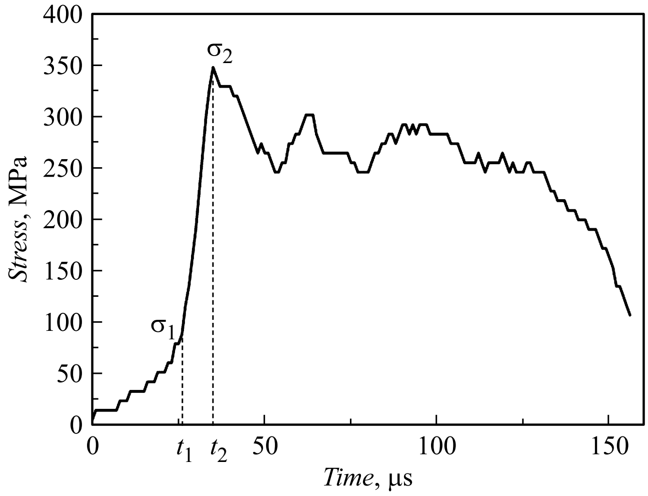

Figure 2. Tension stress curve of a specimen made of M1 copper alloy with the stress increase rate of 19 000 GPa/s.

monitored by a pressure gage. Valve opening time and signal synchronization is performed by a software package based on the Arduino microcomputer. After the valve opens, the striker accelerates and hits the anvil rigidly connected to the loading bar. A tension pulse is generated in the loading bar and is recorded by resistive strain sensors using a high-frequency amplifier. The pulse passing through the specimen is similarly recorded on the supporting bar.

Specimens of M1 copper were made in the shape of vanes, geometrical dimensions of the working part being 5 mm (length) and 2 mm (width). To reduce the error upon a change of experimental setups, identical retaining clamps were made for the dynamic unit and the static tensile tester. The experimental setup for dynamic tension of small specimens and justification of its use was discussed in [14] when comparing the test data for ISO-8256 standard specimens and small specimens on a tower-type drop hammer with an accelerator.

The threshold quantity to characterize the specimen failure in this paper is the dependence of the maximum breaking stress on stress increase rate. It should be noted that an amplifier with the gain factor of 100× was used upon reception of a signal from tension of a small low-strength specimen, which significantly restricts the frequency range of its operation. The signal is amplified at frequencies up to 200 kHz without distortion of the original. As frequency increases, reliability of the recorded signal decreases.

Another aspect during handling of a numeric data array is the need to minimize the random error related to manual (expert) data analysis. For the purposes of the present paper, the target value for each single experiment is a pair consisting of tensile strength and stress increase rate. Material strength value depends on method for original signal filtration and smoothing, and no questions arise when determining it. The situation is different when estimating the stress increase rate, therefore, an automated algorithm will be used in each experiment to minimize the error of its determination. Let us exemplify the operation of the suggested algorithm by the tension stress curves for a specimen made of M1 copper alloy, stress increase rate being 19 000 GPa/s (Fig. 2). The signal profile, corresponding to the first pass of a tension wave through the specimen and the measuring bar, is automatically distinguished from the data array recorded by the oscillograph. The quantity $\sigma_{UTS} = \sigma_2$ can be taken as the ultimate tensile strength in the expert analysis of the plot, and $\dot{\sigma} = (\sigma_2 - \sigma_1)/(t_2 - t_1)$ can be taken as the loading rate. However, tension curves in time can differ significantly from the curve in Fig. 2 due to random noises, a different in contact interaction at the initial stage of tension of the specimen and clamps, primary digitization of the common signal etc. The following algorithm was used in the automated setup:

1. Time (t_2), corresponding to the maximum stress value ($\sigma_{UTS} = \sigma_2$), as well as the total experiment time (t_{full}) are determined in the time cycle.

2. The obtained value is used to record three intervals with the magnitude of $dt_1 = 0.3 \cdot t_2$, $dt_2 = 0.4 \cdot dt_2$, $dt_3 = 0.5 \cdot t_2$.

3. $\dot{\sigma}_j = \max_i(\sigma(t_i + dt_j) - \sigma(t_i))/dt_j$, where $j = 1 \ldots 3$, and $t_i = 0 \ldots (t_{\text{full}} - dt_j)$, is determined in the cycle for each value of the interval.

4. The stress increase rate for each experiment is the average value of $\dot{\sigma} = (\dot{\sigma}_1 + \dot{\sigma}_2 + \dot{\sigma}_3)/3$.

The suggested approach has been tried on a large amount of experimental data with a different tensile stress profile and allows for automated determination of the stress increase rate with a high stability.

The moving average procedure for the obtained signal profile with variation of averaging segment width can be considered as primary processing of experimental data arrays that precedes the above-mentioned algorithm. The segments 2, 4, 5, 8 and 10 μs wide were considered, which corresponds to the respective sampling rates of 500, 250, 200, 125 and 100 kHz. The dependence of the maximum tensile strength of M1 copper on stress increase rate, determined, according to the suggested algorithm, at variation of the original pulse averaging parameters is shown in Fig. 3. Each test corresponds to five points in the plot, which provides an additional data array in the spot experiment conditions. It should be noted that the moving average procedure at variation of averaging segment width

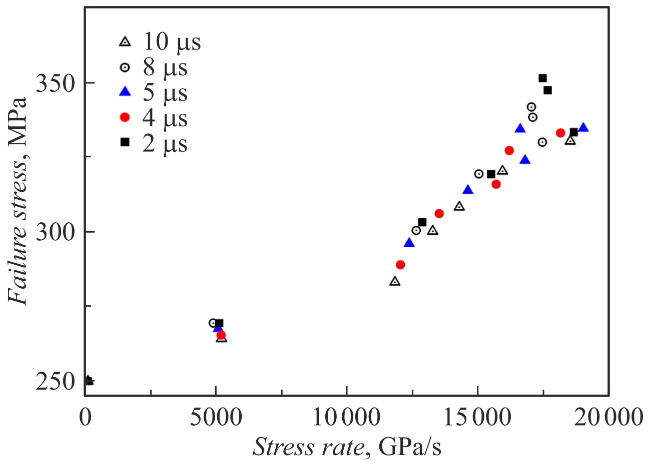

Figure 3. Dependence of the maximum tensile strength of M1 copper on stress increase rate at a variation in the original pulse averaging parameters.

in time adjusts the position of the point in the curve both along the axis of abscissas and along the axis of ordinates.

Analysis of the experimental data in Fig. 3 reveals a dependence of the maximum tensile strength of copper specimens on stress increase rate. As the rate increases, the peak values increase from 250 MPa under quasi-static loading to 350 MPa when the stress increase rate is about 20000 GPa/s. It should be noted that the spread in determination of the extreme stress values increases at higher stress increase rates.

2. Analysis of experimental data

The well-proven incubation time criterion [10] will be taken as a criterion of material tension failure in order to analyze the material strength characteristics in a wide range of variation of the loading pulse parameters:

$$\frac{1}{\tau}\int\limits_{t-\tau}^{t}\frac{\sigma(s)}{\sigma_{UTS}}ds \leq 1, \tag{1}$$

where t — time, σ — dependence of breaking stress on time, σ_{UTS} — ultimate tensile strength under quasi-static loading, τ — incubation time of failure accountable for material dynamic strength. On the assumption of linear stress increase at a constant rate $\dot{\sigma}$ in a specimen till failure or the start of irreversible deformation, the rate dependence of the critical stress level can be analytically calculated using the following formula:

$$\sigma_*(\dot{\sigma}) = \varphi(\tau, \dot{\sigma}) = \begin{cases} \sigma_{UTS} + \frac{\tau}{2}\dot{\sigma}, & \dot{\sigma} \leq \frac{2\sigma_{UTS}}{\tau}, \\ \sqrt{2\sigma_{UTS}\tau\dot{\sigma}}, & \dot{\sigma} > \frac{2\sigma_{UTS}}{\tau}. \end{cases} \tag{2}$$

The least squares method (LSM) was used to calculate the optimal values of the parameter τ, which minimize the root-mean-square deviation of the calculated dependence (2) on the experimental points. Incubation time values were determined for each group of sampling rate values and curves of the maximum breaking stress vs. stress increase rate were plotted. Fig. 4 shows the calculated and experimental dependences of the maximum tensile strength on stress increase rate for the sampling rate of the breaking stress time profile equal to 500 and 100 kHz. The calculated curves were plotted according to criterion (1) taking into account the parameters $\sigma_{UTS} = 250$ MPa and $\tau_{500\,\mathrm{kHz}} = 9.9\,\mu$s, $\tau_{100\,\mathrm{kHz}} = 7.6\,\mu$s. The parameters for the other variations of the averaging time segment are $\tau_{250\,\mathrm{kHz}} = 9.5\,\mu$s, $\tau_{200\,\mathrm{kHz}} = 8.9\,\mu$s and $\tau_{125\,\mathrm{kHz}} = 8.3\,\mu$s respectively. The average parameter value for the whole point array is $\tau = 8.8\,\mu$s. The LSM provides point estimation, but cannot be used to determine the degree of reliability for the interval of the calculated parameter, particularly in the conditions of input data variability. It should be noted that the average value of the incubation time parameter was close to the value determined for the data array generated for the sampling rate of breaking stress time profile equal to 200 kHz, which corresponding to

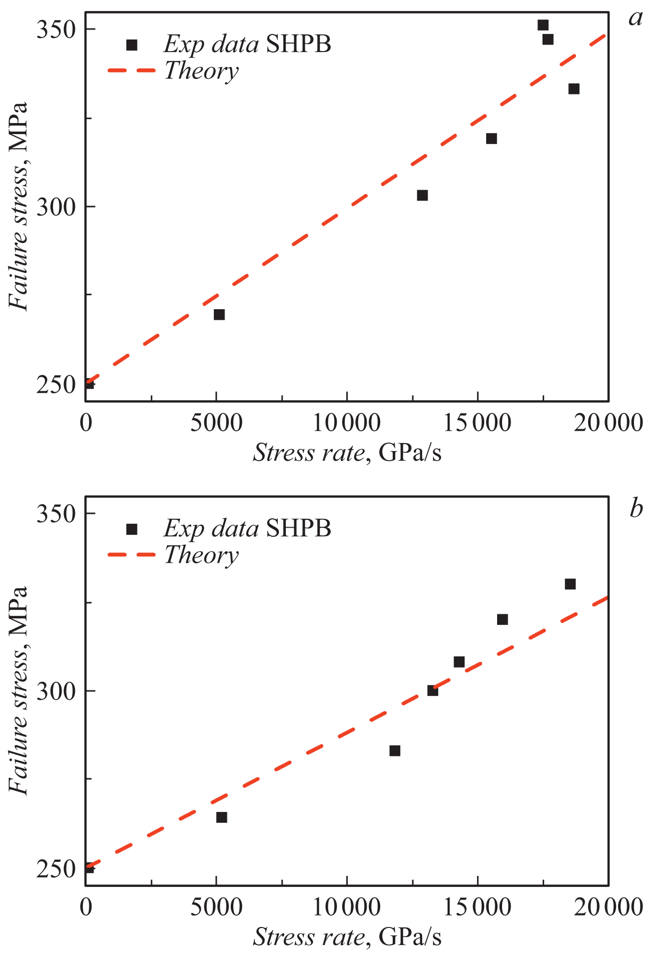

Figure 4. Calculated and experimental dependences of the maximum tensile strength on stress increase rate for the sampling rate of the breaking stress time profile *a* — 500 and *b* — 100 kHz.

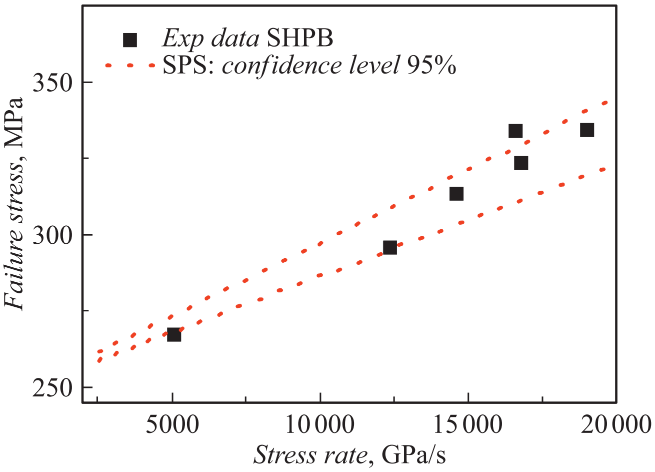

Figure 5. Calculated (by the sign-perturbed sums method) dependence of the maximum tensile strength on stress increase rate for the sampling rate of the breaking stress time profile equal to 200 kHz as compared to the experimental data.

the frequency threshold of the signal amplifier at 100× amplification. When frequency increases, the original signal is distorted while passing through the amplifier, and when frequency decreases, signal distortion is numeric due to excessive averaging.

The incubation values, calculated for different sampling rates, pertain to the interval $[7.6; 9.9]\,\mu$s. To proceed with the study, the obtained range of values was compared to the confidence interval determined by the sign-perturbed sums method. This method has previously shown good results when solving problems of system parameter estimation in conditions of noise uncertainty and insufficiency of data. It has been shown that the SPS-method can be reasonably applied within the framework of the structural-time approach to estimate incubation time values [14,15]. Calculation was performed using the experimental data obtained for the time profile sampling rate of 200 kHz. For the 95% confidence level, an estimate for the incubation time was calculated in the form of interval $\tau \in [7.33; 9.53]\,\mu$s (Fig. 5), which almost coincides with the previously obtained range of values for different sampling rates. The following conclusion can be drawn: estimation of the incubation time value by the sign-perturbed sums method makes it possible to avoid significant deviations of the calculated value of the incubation time parameter from its weighted average even in the conditions of few experimental points. The given example has shown that the confidence interval, determined by the sign-perturbed sums method, corresponds to the interval obtained by adding a deviation to the signal characteristics; this deviation corresponds both to excessive averaging and to actual interference generated by the amplifier at high frequencies. The model experiment has allowed for artificial generation of an extended set of „experimental“ points and for verification of the calculation results within the framework of the structural-time approach through determination of a confidence interval.

Conclusion

Strength characteristics of the material in the quasi-static and dynamic loading conditions have been studied in the paper by the example of M1 copper alloy. Breaking stress chronograms have been experimentally obtained on a Kolsky-method unit using split Hopkinson pressure bars, with various striker speeds. It has been demonstrated that the material strength characteristics cannot be unambiguously determined according to the initial charts.

A procedure for test data analysis has been developed; it includes an automated algorithm for experimental data array digitization and estimation of the rate dependence of strength. The structural-time approach using the SPS-algorithm was been applied to prediction of the critical failure state at a random dynamic load.

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Conflict of interest

The authors declare that they have no conflict of interest.

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