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# Double negative media based on antiferromagnetic semiconductors for the terahertz frequency range

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The paper presents the results of a theoretical study of the dispersion characteristics of electromagnetic waves (EMW) existing in a transversely magnetized antiferromagnetic (AFM) semiconductor with loss. An AFM semiconductor is an infinite bi-gyrotropic medium, the effective material parameters of which are twice negative in several frequency ranges. It was found that these frequency bands are in the terahertz range, and there are four backward EMEs in them, two of which are TE waves, and the other two are TM waves.

Keywords: left-handed media, antiferromagnets, semiconductors, spin waves.

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Double negative media (or "left-handed" media) are a variety of metamaterials whose dielectric permittivity  $(\varepsilon)$  and magnetic permeability  $(\mu)$  are simultaneously negative [1]. Due to this, such a medium obtains a negative refraction index, and a backward electromagnetic wave (EMW) propagates in it. As of now, the double negative media have been created for both the microwave and terahertz (THz) frequency ranges by using sub-wave elements (thin metal wires and open ring resonators) located typically in a nonmagnetic matrix [2].

Paper [3] has shown that a magnetic-field-controllable double negative medium for the THz range can be created using a natural magnetic material which is an antiferromagnet (ANF). AFM is known to be a  $\mu$ -negative medium whose magnetic permeability is negative in two frequency bands belonging to the THz-range because of the presence of two sublattices and strong internal magnetic field [4]. The combination of  $\mu$ -negative AFM properties with the properties of a  $\varepsilon$ -negative medium, for which a periodic grid of thin conductive wires was used, allowed realization of a double negative medium without involving additional sub-wave elements in the form of open ring resonators.

Another no less interesting candidate for creating a magnetic-field-controllable double negative medium is bigyrotropic medium [5]. It is known that in bi-gyrotropic medium the interrelation between electric and magnetic field vectors and electric and magnetic inductance vectors is actualized via material parameters of the medium (dielectric permittivity and magnetic permeability) that are describable by Hermitian second-order tensors [6]. As an example of a bi-gyrotropic material, a magnetic semiconductor may be regarded, which possesses anisotropic properties of a magnetized magnet ( $\mu$ -negative medium) and magnetically active electron plasma ( $\varepsilon$ -negative medium). Authors of [1] were the first who paid attention to the possibility of creating a double negative medium from a magnetic semiconductor. In [7], an electrodynamic model of transversely magnetized ferromagnetic semiconductor was used to demonstrate the transformation of a slow EMW dispersion characteristic (DC) from the positive normal state to the negative abnormal one when the plasma frequency changes its position with respect to the ferromagnetic resonance and antiresonance frequencies. However, no physical explanation for the backward EMW emergence was given.

The goal of this work is to demonstrate the possibility of creating a double negative medium based on an AFM semiconductor in the THz range and investigate the influence of losses upon the backward EMW dispersion characteristics.

Consider the bi-gyrotropic material schematically represented in Fig. 1, *a*. This is a magnetized AFM in which there exist free charge carriers in the form of magnetized "cold" single–component (electron) plasma. The constant external magnetic field  $\mathbf{H}_0$  is applied along axis 0Z. In this material that is infinite in the 0X direction, EMW propagates along the 0Y axis. Dielectric properties of the bi-gyrotropic material are described by the tensor of high–frequency dielectric permittivity that is defined as follows (taking into account losses caused by electron collisions) [5–8]:

$$\overset{\leftrightarrow}{\varepsilon} = \varepsilon_f \begin{pmatrix} \varepsilon & -jg & 0\\ jg & \varepsilon & 0\\ 0 & 0 & \eta \end{pmatrix},$$
 (1)

 $\varepsilon_f = \varepsilon_0 \varepsilon_r$  is the absolute dielectric permittivity of the medium,  $\varepsilon_0 = 1/(\mu_0 c^2)$  is the electric constant,  $\mu_0$  is the magnetic constant,  $\varepsilon_r$  is the relative dielectric permittivity of the medium,

$$\begin{split} \varepsilon &= 1 - \omega_{pe}^2 (\omega - j v_e) / \{ \omega [(\omega - j v_e)^2 - \omega_{ce}^2] \}, \\ g &= \omega_{pe}^2 \omega_{ce} / \{ \omega [(\omega - j v_e)^2 - \omega_{ce}^2] \}, \\ \eta &= 1 - \omega_{pe}^2 / \omega (\omega - j v_e), \end{split}$$



**Figure 1.** *a* — schematic representation of a transversely magnetized infinite bi-gyrotropic medium. *b* — frequency dependences of effective high–frequency electric permittivity and magnetic permeability of the transversely magnetized medium where TE waves (upper panel) and TM waves (lower panel) exist. The effective medium parameters were calculated for  $H_0 = 79.58$  kA/m,  $H_E = 2.86$  MA/m,  $H_A = 636.64$  kA/m,  $N = 10^{19}$  cm<sup>-3</sup>,  $M_S = 1.16$  T,  $\varepsilon_r = 6.9$ .

 $\omega_{pe} = \sqrt{4\pi Ne^2/m_e}$  is the electron plasma frequency,  $\omega_{ce} = |eB_0|/(m_ec)$  is the electron cyclotron frequency,  $\omega = 2\pi f$  is the circular frequency, f is the linear frequency,  $\nu_e$  is the electron collision frequency, N is the electron concentration in plasma,  $e/m_e$  is the specific electron charge,  $B_0$  is the constant magnetic inductance. Magnetic properties are described by the tensor of high-frequency magnetic permeability in the following form:

$$\vec{\mu} = \begin{pmatrix} \mu & j\mu_a & 0\\ -j\mu_a & \mu & 0\\ 0 & 0 & 1 \end{pmatrix},$$
 (2)

where the diagonal  $\mu$  and off-diagonal  $\mu_a$  tensor components for an AFM with an "easy" anisotropy axis coinciding with the 0Z axis and with losses equal for both sublattices may be presented as follows [3,4]:

$$egin{aligned} &\mu = 1 + 8\pi\gamma_S^2 M_S H_A[(\omega_+ + jlpha\omega)(\omega_- + jlpha\omega) \ &-\omega^2]/\{[(\omega_+ + jlpha\omega)^2 - \omega^2]](\omega_- + jlpha\omega)^2 - \omega^2]\}, \ &\mu_a = 8\pi\gamma_S^2 M_S H_A \omega(\omega_- - \omega_+)/\{[(\omega_+ + jlpha\omega)^2 - \omega^2]] \ & imes [(\omega_- + jlpha\omega)^2 - \omega^2]\}, \end{aligned}$$

where  $\gamma_S$  is the average g-factor,  $M_S$  is the average static magnetization of the sublattices,  $H_A$  is the anisotropy field,  $\omega_+ = \gamma_S(H_C + H_0)$ ,  $\omega_- = \gamma_S(H_C - H_0)$  are the antiferromagnetic resonance frequencies,  $H_C = [H_A(2H_E + H_A)]^{1/2}$  is the field of sublattice "flip"  $H_E$  is the field of the uniform exchange interaction between the sublattices,  $\alpha$  is the parameter of losses caused by the processes of wave relaxation in AFM.

Solving the electrodynamic problem in the uniform plane wave approximation for a transversely magnetized infinite bi-gyrotropic medium, obtain two dispersion equations (DE) one of which describes the TE wave characteristics, the other is for the TM waves. DE for the TE waves looks as follows:

$$k = k_0 \left( \mu_{eff\perp}^{\text{TE}} \varepsilon_{eff\perp}^{\text{TE}} \right)^{1/2}, \tag{3}$$

where k is the EMW wave number in the medium,  $k_0 = \omega/c$  is the EMW wave number in vacuum,  $\mu_{eff\perp}^{\text{TE}}$  and  $\varepsilon_{eff\perp}^{\text{TE}}$  are the effective magnetic permeability and dielectric permittivity defined for the medium with TE waves as follows:

$$\mu_{eff\perp}^{\rm TE} = (\mu^2 - \mu_a^2)/\mu, \tag{4}$$

$$\varepsilon_{eff\perp}^{\rm TE} = \varepsilon_f \eta. \tag{5}$$

DE for the TM waves looks as follows:

$$k = k_0 \left( \mu_{eff\perp}^{\rm TM} \varepsilon_{eff\perp}^{\rm TM} \right)^{1/2}, \tag{6}$$

where  $\mu_{eff\perp}^{\text{TM}}$  and  $\varepsilon_{eff\perp}^{\text{TM}}$  are the effective magnetic permeability and dielectric permittivity defined for the medium with TM-waves as follows:

$$\mu_{eff\perp}^{\rm TM} = \mu, \tag{7}$$

$$\varepsilon_{eff\perp}^{\rm TM} = \varepsilon_f (\varepsilon^2 - g^2) / \varepsilon.$$
 (8)

The conditions at which  $\mu_{eff\perp}^{\rm TE} < 0$  ( $\alpha = 0$ ) can be written as

$$\omega_{\perp 1} < \omega < \omega_{ar1}, \tag{9a}$$

$$\omega_{\perp 2} < \omega < \omega_{ar2}, \tag{9b}$$



**Figure 2.** Dispersion characteristics of fast (solid lines 1, 1') and slow (solid lines 2, 2', 3, 3' and dashed lines) of TE waves (a, c) and TM waves (b, d) existing in the transversely magnetized AFM at N = 0 and  $\alpha = 0$  (a, b) and in AFM semiconductor with  $N = 10^{19}$  cm<sup>-3</sup> at  $v_e = 0$ ,  $\alpha = 0$  (solid lines) and  $v_e = 10^{14}$  rad · Hz,  $\alpha = 4 \cdot 10^{-2}$  (dashed lines) (c, d). Calculations presented in panel (c) were performed for  $\alpha = 4 \cdot 10^{-2}$  and  $v_e = 0$ , those in (d) are for  $v_e = 10^{14}$  rad · Hz and  $\alpha = 0$ . The calculations were performed at  $H_0 = 79.58$  kA/m,  $H_E = 2.86$  MA/m,  $H_A = 636.64$  kA/m,  $M_S = 1.16$  T,  $\varepsilon_r = 6.9$ .

where

$$\omega_{ar1,2} = [\pm(\omega_+ - \omega_-) + D_1^{1/2}]/2$$

are the AFM antiresonance frequencies,

$$\omega_{\perp 1,2} = [(\omega_+^2 + \omega_-^2 + 8\pi\gamma_S^2 M_S H_A \pm D_2^{1/2})/2]^{1/2}$$

are the AFM resonance frequencies in the case of transverse magnetization,

$$D_{1} = (\omega_{+} + \omega_{-})^{2} + 32\pi\gamma_{S}^{2}M_{S}H_{A},$$
  
$$D_{2} = (\omega_{+}^{2} + \omega_{-}^{2} + 8\pi\gamma_{S}^{2}M_{S}H_{A})^{2}$$
  
$$- 4\omega_{+}\omega_{-}(\omega_{+}\omega_{-} + 8\pi\gamma_{S}^{2}M_{S}H_{A}).$$

Two backward TE waves can exist at the frequencies to be determined from conditions (9a) and (9b) only

if the following condition is met at these frequencies:  $\varepsilon_{eff\perp}^{\text{TE}} < 0 \ (\nu_e = 0)$ . This situation is possible if

$$\omega_{\perp 1,2} < \omega < \omega_{ar1,2} \leqslant \omega_{pe}. \tag{10}$$

The conditions at which  $\mu_{eff\perp}^{TM} < 0$  ( $\alpha = 0$ ) can be written as

$$\omega_{\parallel 1} < \omega < \omega_{\perp 1}, \tag{11a}$$

$$\omega_{\parallel 2} < \omega < \omega_{\perp 2}, \tag{11b}$$

where  $\omega_{||1,2} = \omega_{\pm}$  -are the AFM resonance frequencies in the case of longitudinal magnetization.

Two backward TM waves can exist at the frequencies to be determined from (11a) and (11b) if the following condition is met at these frequencies:  $\varepsilon_{eff\perp}^{\text{TM}} < 0$  ( $\nu_e = 0$ ).

This situation is possible if, for instance, when

$$\omega_{||1,2} < \omega < \omega_{\perp 1,2} \leqslant \omega_{\varepsilon}, \tag{12}$$

where  $\omega_{\varepsilon} = \left[ -\omega_{ce} + (\omega_{ce}^2 + 4\omega_{pe}^2)^{1/2} \right] / 2.$ 

Fig. 1, b presents frequency dependences of effective medium parameters; frequency bands where backward TE and TM waves can exist are highlighted by shading. Fig. 2 presents DCs of TE and TM waves calculated based on relations (3) and (6) taking into account (4), (5), (7), (8) for the AFM semiconductor to which one of the europium monochalcogenides relates, namely, europium telluride (EuTe) [9,10]. For comparison, this figure also presents DCs of TE and TM waves calculated for an AFM in which there are no free charge carriers (N = 0) and losses. Calculations presented in Figs. 2, a, b show that the AFM contains slow EMWs with the normal dispersion only. The presence of electron plasma in AFM results in that slow EMWs get converted into backward waves belonging to the THz frequency band where effective parameters of the medium are double negative (Figs. 2, c, d and 1, b). The calculations showed that losses caused by the processes of spin-wave relaxation affect mainly DCs of TE waves, while losses in plasma affect only DCs of TM waves. In the first case, deceleration of the lower-frequency backward TE-wave decreases, while, vice versa, that of the higher-frequency backward TE wave increases at a fixed frequency. As for the backward TM-waves, deceleration of both the high-frequency and low-frequency waves decreases with enhancing losses in plasma.

In conclusion, notice that the obtained results are of interest for developing THz-spintronics functional devices.

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## **Conflict of interests**

The authors declare that they have no conflict of interests.

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