Missing understanding of the phase factor between valence-electron and hole operators

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This paper provides the long-missing foundation to connect semiconductor and atomic notations and to support results incorrectly obtained by doing as if semiconductor electrons possessed an orbital angular momentum. We here show that the phase factor between valence-electron destruction operator and hole creation operator is the same as the one between particle and antiparticle in quantum relativity, namely $\hat{a}_m = (-1)^{j-m}\hat{b}^\dagger_{-m}$ provided that $m = (j, j-1 \cdots, -j)$ labels the degenerate states of the $(2j+1)$-fold electron level at hand. This result is remarkable because (i) the hole is definitely not a naive antiparticle due to the remaining valence electrons; (ii) unlike atomic electrons in a central potential, semiconductor electrons in a periodic crystal do not have *orbital angular* momentum $\mathbf{L} = \mathbf{r} \wedge \mathbf{p}$ nor *angular* momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}$. Consequently, (j, m) for semiconductor electrons merely are convenient notations to label the states of a degenerate level. To illustrate the physical implications, we discuss the interband couplings between photons and semiconductor, in terms of valence electrons and of holes: the phase factor is crucial to establish that bright excitons are in a spin-singlet state.

Keywords: Valence-band degeneracy, spin-orbit interaction, phase factor.

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