Exciton polaritons in quantum-dot photonic crystals

© E.L. Ivchenko, Y. Fu*, M. Willander*

A.F. Ioffe Physico-Technical Institute,
194021 St. Petersburg, Russia
* Laboratory of Physical Electronics and Photonics, MC2, Department of Physics,
University of Gothenburg and Chalmers University of Technology, Fysikgränd 3,
S-412 96, Gothenburg, Sweden

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We present a theory of the photonic band structure of three-dimensional arrays of quantum dots (QDs). A system of Maxwell and material equations is solved and the dispersion equation for exciton polaritons is derived making allowance for a nonlocal dielectric response of quasi-zero-dimensional excitons confined in QDs. The reflection and transmission coefficients are calculated for a single plane, a pair of planes and a stack of equidistant planes of QDs. Two different approaches are proposed to perform the calculation. One of them is based on recurrent equations relating the reflection coefficients for $N + 1$ and N planes, and in other approach the Bloch solutions for an infinite QD lattice are used.

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I. Introduction

In bulk crystals, a photon and an exciton mix in the dispersion-crossover region, losing their identity in a combined quasi-particle called the exciton polariton. Exciton polaritons were intensively studied in the 60's and 70's, their manifestation in various optical phenomena, including light reflection and transmission, photoluminescence and resonant light scattering, are well-established and documented (see e.g. the contributed volume [1] and references therein). Renewed interest and recent important developments in this field [2–7] were stimulated by technological achievements in fabrication of high-quality multi-layered heterostructures, multiple quantum wells (MQWs) and superlattices (SLs). Moreover, the concept of exciton polariton has undergone a substantial modification, in particular with respect to long-period MQW structures containing a finite number of wells [8–18]. The present paper outlines the framework for similar studies of structures containing regular arrays of quantum dots (QDs). The shift from long-period MQWs to 3D lattices of quantum dots allows to bridge the gap between multilayered structures and photonic crystals. The latter are defined as periodic dielectric structures with the period being comparable to the wavelength of the visible-range electromagnetic waves. In the simplest realization, a photonic crystal is thought of as a periodic lattice of dielectric spheres of dielectric constant ε_a embedded in a uniform dielectric background ε_b (see reviews [19,20]). Other potential realizations are a three-dimensional (3D) lattice of resonant two-level atoms [21] or semiconductor microcrystals embedded into the pores of periodic porous materials [22] (see also [23]).

Here we study the photonic (or more precisely, exciton-polaritonic) band structure of 3D periodic arrays of QDs or simply QD lattices and the light reflection from a finite number of QD planes. The excitonic states in a single QD are quasi-zero-dimensional due to the quantum-confinement effect and we consider a narrow frequency region near a particular exciton size-quantization level. In the resonant frequency region the dielectric response to an electromagnetic wave is nonlocal and the main goal of the work is to develop a theory which makes allowance for such kind of nonlocality.

In Sect. II we derive the dispersion equation for exciton polaritons in a 3D QD lattice. The reflection from and transmission through a single plane containing quadratic QD lattice are considered in Sect. III. The relation between the exciton-polariton dispersion equation and single-plane reflection and transmission coefficients is established in Sect. IV. The reflection from a pair of QD planes and from a stack of QD planes is considered respectively in Sects. V and VI. The derived theory can be used as well for the description of nuclear resonant scattering of γ quanta by artificial nuclear multilayers (see [24–26]).

II. Bloch solutions in three-dimensional quantum-dot lattices

We start from the Maxwell equations

$$\Delta \mathbf{E} - \text{grad div } \mathbf{E} = -\left(\frac{\omega}{c}\right)^2 \mathbf{D},$$

$$\text{div } \mathbf{D} = 0 \qquad (1)$$

for the electric field $\mathbf{E}$ and the displacement vector $\mathbf{D}$. The nonlocal material equation relating $\mathbf{D}$ and $\mathbf{E}$ is taken in the form (see [27])

$$\mathbf{D}(\mathbf{r}) = \varepsilon_b \mathbf{E}(\mathbf{r}) + 4\pi \mathbf{P}_{exc}(\mathbf{r}), \qquad (2)$$

$$4\pi \mathbf{P}_{exc}(\mathbf{r}) = T(\omega) \sum_{\mathbf{a}} \Phi_{\mathbf{a}}(\mathbf{r}) \int \Phi_{\mathbf{a}}(\mathbf{r}') \mathbf{E}(\mathbf{r}') d\mathbf{r}'. \qquad (3)$$

Here $\mathbf{a}$ are the lattice translation vectors enumerating quantum dots, $\Phi_{\mathbf{a}}(\mathbf{r}) = \Phi_0(\mathbf{r} - \mathbf{a})$ is the envelope function

$\Psi_{exc}(\mathbf{r}_e-\mathbf{a},\mathbf{r}_h-\mathbf{a})$ of an exciton excited in the $\mathbf{a}$th QD at coinciding electron and hole coordinates: $\Phi_{\mathbf{a}}(\mathbf{r})=\Psi_{exc,\mathbf{a}}(\mathbf{r},\mathbf{r})$. The other notations are

$$T(\omega)=2\pi\frac{\varepsilon_b\omega_{LT}\omega_0 a_B^3}{\omega_0^2-\omega^2}\approx\frac{\varepsilon_b\omega_{LT}\pi a_B^3}{\omega_0-\omega}, \qquad (4)$$

ω_{LT} and a_B are the exciton longitudinal-transverse splitting and Bohr radius in the corresponding bulk semiconductor, ω_0 is the QD-exciton resonance frequency, ε_b is the background dielectric constant which is assumed to coincide with the dielectric constant of the barrier material. In the following we neglect the overlap of exciton envelope functions $\Psi_{\mathbf{a}}$ and $\Psi_{\mathbf{a}'}$ with $\mathbf{a}\neq\mathbf{a}'$ so that excitons excited in different dots are assumed to be coupled only via electromagnetic field.

It follows from Eq. (2) that $\operatorname{div}\mathbf{E}=-(4\pi/\varepsilon_b)\operatorname{div}\mathbf{P}_{exc}$ which allows to rewrite the first Eq. (1) as

$$\Delta\mathbf{E}(\mathbf{r})+k^2\mathbf{E}(\mathbf{r})=-4\pi k_0^2\left(1+k^{-2}\operatorname{grad}\operatorname{div}\right)\mathbf{P}_{exc}(\mathbf{r}), \quad (5)$$

where $k_0=\omega/c$, $k=k_0n_b=\omega n_b/c$ and $n_b=\sqrt{\varepsilon_b}$. We seek for Bloch-like solutions of Eq. (8) satisfying the translational symmetry

$$\mathbf{E}_{\mathbf{q}}(\mathbf{r}+\mathbf{a})=\exp(i\mathbf{qa})\mathbf{E}_{\mathbf{q}}(\mathbf{r}),$$

$$\mathbf{P}_{exc,\mathbf{q}}(\mathbf{r}+\mathbf{a})=\exp(i\mathbf{qa})\mathbf{P}_{exc,\mathbf{q}}(\mathbf{r}), \qquad (6)$$

where the wave vector $\mathbf{q}$ is defined within the first Brillouin zone. The exciton-polariton dispersion $\omega(\mathbf{q})$ can be shown to satisfy the equation

$$\operatorname{Det}\|\delta_{\alpha\beta}-R_{\alpha\beta}(\omega,\mathbf{q})\|=0, \qquad (7)$$

where $\alpha,\beta=x,y,z$, $\delta_{\alpha\beta}$ is the Kronecker symbol and, for QD lattices,

$$R_{\alpha\beta}(\omega,\mathbf{q})=\frac{k_0^2T(\omega)}{v_0}\sum_{\mathbf{g}}\frac{I_{\mathbf{q}+\mathbf{g}}^2S_{\alpha\beta}(\mathbf{q}+\mathbf{g})}{(\mathbf{q}+\mathbf{g})^2-k^2}, \qquad (8)$$

$$I_{\mathbf{Q}}=\int\Phi_0(\mathbf{r})e^{i\mathbf{Qr}}d\mathbf{r},\quad S_{\alpha\beta}=\delta_{\alpha\beta}-\frac{Q_\alpha Q_\beta}{k^2}, \qquad (9)$$

$\mathbf{g}$ are the reciprocal lattice vectors and v_0 is the volume of the lattice primitive cell.

Eqs. (7), (8) can be derived by using the two equivalent approaches: (a) to express the exciton dielectric polarization $\Phi_0(\mathbf{r})$ in terms of the electric field, $\mathbf{E}(\mathbf{r})$, and find solutions of the wave equation for $\mathbf{E}(\mathbf{r})$; (b) by using Green's function of the wave equation, to express the electric field in terms of the exciton polarization and write a system of self-consistent equations describing electric-field-mediated coupling between the excitons excited in different quantum dots. In the first approach, we substitute Eq. (3) into Eq. (5) and expand the vector function $\mathbf{E}_{\mathbf{q}}(\mathbf{r})$ in the Fourier series as follows:

$$\mathbf{E}_{\mathbf{q}}(\mathbf{r})=\sum_{\mathbf{g}}e^{i(\mathbf{q}+\mathbf{g})\mathbf{r}}\,\mathbf{E}_{\mathbf{q}+\mathbf{g}}. \qquad (10)$$

The integral in Eq. (2) can be transformed into

$$\int\Phi_{\mathbf{a}}(\mathbf{r})\,\mathbf{E}(\mathbf{r})d\mathbf{r}=e^{i\mathbf{qa}}\sum_{\mathbf{g}}I_{\mathbf{q}+\mathbf{g}}\,\mathbf{E}_{\mathbf{q}+\mathbf{g}}\equiv e^{i\mathbf{qa}}\mathbf{\Lambda}. \qquad (11)$$

The sum $\sum_{\mathbf{a}}\Phi_{\mathbf{a}}(\mathbf{r})e^{i\mathbf{qa}}$ satisfies the translational symmetry similar to Eq. (6) and can be presented as

$$\sum_{\mathbf{a}}\Phi_{\mathbf{a}}(\mathbf{r})\,e^{i\mathbf{qa}}=\sum_{\mathbf{g}}e^{i(\mathbf{q}+\mathbf{g})\mathbf{r}}\,\frac{I_{\mathbf{q}+\mathbf{g}}^*}{v_0}. \qquad (12)$$

The system of linear equations for the space harmonics $\mathbf{E}_{\mathbf{q}+\mathbf{g}}$ can be written in the form

$$\left[(\mathbf{q}+\mathbf{g})^2-k^2\right]\mathbf{E}_{\mathbf{q}+\mathbf{g}}=T(\omega)\,k_0^2\frac{I_{\mathbf{q}+\mathbf{g}}^*}{v_0}\,\hat{S}(\mathbf{q}+\mathbf{g})\mathbf{\Lambda}, \qquad (13)$$

where the vector $\mathbf{\Lambda}$ is introduced in Eq. (11) and $\hat{S}(\mathbf{Q})\mathbf{\Lambda}$ is a vector with the components $S_{\alpha\beta}(\mathbf{Q})\Lambda_\beta$. Dividing both parts of Eq. (13) by $(\mathbf{q}+\mathbf{g})^2-k^2$, multiplying them by $I_{\mathbf{q}+\mathbf{g}}$ and summing over $\mathbf{g}$ we arrive at the vector equation $\mathbf{\Lambda}=\hat{R}(\omega,\mathbf{q})\mathbf{\Lambda}$, where the matrix $\hat{R}$ is defined by Eq. (8), and hence at the dispersion equation (7).

In the second approach, we use Green's function

$$G(\mathbf{r}-\mathbf{r}')=\frac{\exp\left(ik|\mathbf{r}-\mathbf{r}'|\right)}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

$$=\frac{1}{V}\sum_{\mathbf{Q}}\frac{\exp\left[i\mathbf{Q}(\mathbf{r}-\mathbf{r}')\right]}{\mathbf{Q}^2-k^2}, \qquad (14)$$

satisfying the differential equation

$$(\Delta+k^2)G(\mathbf{r}-\mathbf{r}')=-\delta(\mathbf{r}-\mathbf{r}'). \qquad (15)$$

Here V is the lattice volume. Green's function allows to express $\mathbf{E}(\mathbf{r})$ via the polarization as

$$\mathbf{E}(\mathbf{r})=4\pi k_0^2T(\omega)\int d\mathbf{r}'G(\mathbf{r}-\mathbf{r}')\left(1+k^{-2}\operatorname{grad}\operatorname{div}\right)\mathbf{P}_{exc}(\mathbf{r}'). \qquad (16)$$

Now Eq. (3) is presented in the form

$$4\pi\mathbf{P}_{exc}(\mathbf{r})=\sum_{\mathbf{a}}\mathbf{p}_{\mathbf{a}}\Phi_{\mathbf{a}}(\mathbf{r}), \qquad (17)$$

where

$$\mathbf{p}_{\mathbf{a}}=T(\omega)\int\Phi_{\mathbf{a}}(\mathbf{r}')\,\mathbf{E}(\mathbf{r}')\,d\mathbf{r}'. \qquad (18)$$

For the Bloch solutions (6) one has $\mathbf{p}_{\mathbf{a}}=e^{i\mathbf{qa}}\mathbf{p}_0$. Taking $\mathbf{a}=0$ in Eq. (18) and using Eqs. (16), (17) we obtain

$$\mathbf{p}_0=T(\omega)\int d\mathbf{r}'\Phi_0(\mathbf{r}')\int d\mathbf{r}\,G(\mathbf{r}'-\mathbf{r})\left(1+k^{-2}\operatorname{grad}\operatorname{div}\right)$$

$$\times\sum_{\mathbf{a}'}\mathbf{p}_0\,\Phi_{\mathbf{a}'}(\mathbf{r})\,e^{i\mathbf{qa}'}. \qquad (19)$$

If we now use Eqs. (9), (12) and the integral presentation of Green's function we finally come to the equation $\mathbf{p}_0=\hat{R}(\omega,\mathbf{q})\mathbf{p}_0$ and re-derive Eq. (7).

Dispersion equations written in terms of $R_{\alpha\beta}$ for different $\mathbf{K}$ points in the Brillouin zone of a face-centered-cubic QD lattice

$\mathbf{K}$ $(2\pi/a)$	Nonzero components of $R_{\alpha\beta}$	Dispersion equations
Γ (0, 0, 0)	$R_{xx}=R_{yy}=R_{zz}$	$R_{xx}=1$
X (0, 0, 1)	$R_{xx}=R_{yy},\quad R_{zz}$	$R_{xx}=1,\quad R_{zz}=1$
L (1/2, 1/2, 1/2)	$R_{\alpha\alpha}=R_{xx},\quad R_{\alpha\beta}=R_{xy}(\alpha\neq\beta)$	$R_{xx}-R_{xy}=1,\quad R_{xx}+2R_{xy}=1$
W (1/2, 0, 1)	$R_{xx},\quad R_{yy}=R_{zz}$	$R_{xx}=1,\quad R_{yy}=1$
K (3/4, 0, 3/4)	$R_{xx}=R_{zz},\quad R_{yy},\quad R_{xz}=R_{zx}$	$R_{xx}\pm R_{xz}=1,\quad R_{yy}=1$
U (1/4, 1/4, 1)	$R_{xx}=R_{yy},\quad R_{zz},\quad R_{xy}=R_{yx}$	$R_{xx}\pm R_{xy}=1,\quad R_{zz}=1$

The numerical calculation is performed for spherical QDs with the radius R exceeding the Bohr radius a_B in which case we have

$$I_{\mathbf{Q}}=\pi\left(\frac{2R}{a_B}\right)^{3/2}\frac{\sin QR}{QR\left[\pi^2-(QR)^2\right]}. \tag{20}$$

Then Eq. (8) can be transformed into

$$R_{\alpha\beta}(\Omega,\mathbf{K})=\xi\,\frac{\Omega^2}{\Omega^2-1}\,\sigma_{\alpha\beta}(\Omega,\mathbf{K}), \tag{21}$$

$$\sigma_{\alpha\beta}(\Omega,\mathbf{K})=\sum_{\mathbf{b}}\frac{f\big(|\mathbf{K}+\mathbf{b}|R\big)S_{\alpha\beta}(\mathbf{K}+\mathbf{b})}{\Omega^2-\Omega^2(\mathbf{K}+\mathbf{b})}, \tag{22}$$

$$\Omega=\frac{\omega}{\omega_0},\qquad \xi=\frac{64}{\pi}\frac{\omega_{LT}}{\omega_0}\left(\frac{R}{a}\right)^3,$$

$$f(x)=\left[\frac{\pi^2\sin x}{x(\pi^2-x^2)}\right]^2, \tag{23}$$

$\Omega(\mathbf{Q})=cQ/\omega_0 n_b$. Eq. (7) is equivalent to the three separate equations $R_j(\Omega,\mathbf{K})=1$ where R_j $(j=1,2,3)$ are eigenvalues of the matrix $R_{\alpha\beta}$. The further simplification follows taking into account a small value of the parameter ξ in Eq. (21) since, in semiconductors, the ratio ω_{LT}/ω_0 typically lies between 10^{-4} and 10^{-3}. Then one can change the factor $\Omega/(\Omega+1)$ in Eq. (21) by 1/2.

Figure 1 shows the photonic band structure for the face-centered-cubic QD lattices with the radius $R=a/4$, $\omega_{LT}/\omega_0=5\times10^{-4}$ and $P=(\pi\sqrt{3}c/a\omega_0 n_b)^3=1.1$.

Note that in this case the lattice constant a and the unit-cell volume v_0 are related by $v_0=a^3/4$. For high-symmetry points of the Brillouin zone, the symmetry imposes certain relations between the $R_{\alpha\beta}$ components and the eigenvalues R_j can be readily expressed via these components as illustrated in the Table 1 for the points Γ, X, L, W, K and U. According to Fig. 1 the dispersion on the Λ line is characterized by a giant anticrossing between the branches of bare transverse photon and exciton modes. At the X point, the gap is determined by the separation between the longitudinal and lower transverse branches, it is still remarkable and exceeds $0.5\omega_{LT}$. However near the points U and

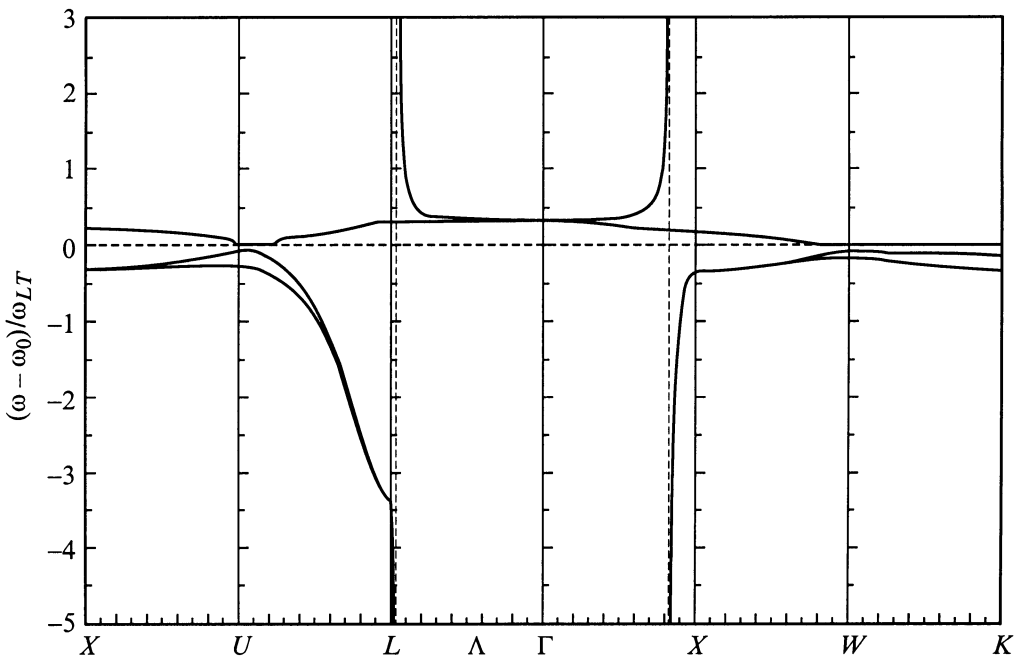

Figure 1. Exciton-polariton dispersion near the exciton resonance frequency ω_0 in a face–centered–cubic lattice of spherical QDs characterized by the following set of parameters: $P=1.1$, $R/a=1/4$ and $\omega_{LT}/\omega_0=5\times10^{-4}$. The dashed lines show the photon dispersion in the empty lattice, i.e. for $\omega_{LT}=0$, the dotted horizontal line indicates the value $\omega=\omega_0$.

W the exciton-polariton branches converge and the gap almost disappears. Note that the anticrossing can be described with a high accuracy by retaining in the sum over $\mathbf{b}$ in Eq. (22) the two terms due to $\mathbf{b} = 0,\ -(4\pi/a)(0,0,1)$ for the Δ points and $\mathbf{b} = 0,\ -(2\pi/a)(1,1,1)$ for the Λ points.

III. Reflection from and transmission through a planar array of quantum dots

We start the analysis of the resonant light reflection from an array of dots regularly packed in one plane. For simplicity we consider here a quadratic lattice of spherical or cubic quantum dots and normal incidence of the light. In this case the integral (9) can be reduced to

$$I_{\mathbf{Q}} = \int \Phi_0(\mathbf{r}) \cos \mathbf{Qr} d\mathbf{r}$$
$$= \int \Phi_0(\mathbf{r}) \cos Q_x x \cos Q_y y \cos Q_z z\, d\mathbf{r} \qquad (24)$$

and is real if $\mathbf{Q}$ is a purely real or imaginary vector. First we consider the normal incidence of the light on the planar array of quantum dots. The electric field can be written in the form

$$\mathbf{E}(\mathbf{r}) = \sum_{\mathbf{b}} \mathbf{E}_{\mathbf{b}}(z)\, \exp(i\,\mathbf{b}\boldsymbol{\rho}), \qquad (25)$$

where $\mathbf{b} = l\mathbf{b}_1 + m\mathbf{b}_2$ are the two-dimensional reciprocal-lattice vectors.

The integral in Eq. (3) can be transformed into

$$\int \Phi_{\mathbf{a}}(\mathbf{r})\mathbf{E}(\mathbf{r})\, d\mathbf{r} = \sum_{\mathbf{b}} e^{i\mathbf{ba}} \int \mathbf{E}_{\mathbf{b}}(z)\Phi(\boldsymbol{\rho}, z)\exp(i\,\mathbf{b}\boldsymbol{\rho})d\boldsymbol{\rho}dz$$
$$= \sum_{\mathbf{b}} \int \varphi_{\mathbf{b}}(z)\mathbf{E}_{\mathbf{b}}(z)dz \equiv \boldsymbol{\Lambda}_1, \qquad (26)$$

where

$$\varphi_{\mathbf{b}}(z) = \int \Phi(\boldsymbol{\rho}, z)\exp(i\,\mathbf{b}\boldsymbol{\rho})d\boldsymbol{\rho} \qquad (27)$$

and we used the identity $\exp(i\mathbf{ba}) = 1$. We will also use the expansion

$$\sum_{\mathbf{a}} \Phi_{\mathbf{a}}(\mathbf{r}) = \frac{1}{a^2}\sum_{\mathbf{b}} \varphi_{\mathbf{b}}(z)\exp(i\,\mathbf{b}\boldsymbol{\rho}), \qquad (28)$$

where a^2 is the unit cell area.

The function $\mathbf{E}_{\mathbf{b}}(z)$ satisfies the equation

$$\left(\frac{d^2}{dz^2}+k_{\mathbf{b}}^2\right)\mathbf{E}_{\mathbf{b}}(z) = -\frac{k_0^2}{a^2}T(\omega)\left(1+k^{-2}\,\mathrm{grad\,div}\right)_{\mathbf{b}}\varphi_{\mathbf{b}}(z)\boldsymbol{\Lambda}_1, \qquad (29)$$

where

$$k_{\mathbf{b}} = \sqrt{k^2 - \mathbf{b}^2}, \quad \left(\frac{\partial^2}{\partial r_\alpha \partial r_\beta}\right)_{\mathbf{b}} = -K_\alpha K_\beta,$$
$$K_x = b_x, \qquad K_y = b_y, \qquad K_z = -i\frac{\partial}{\partial z}. \qquad (30)$$

The solution can be presented as

$$\mathbf{E}_{\mathbf{b}}(z) = \mathbf{E}^{(0)}e^{ikz}\delta_{\mathbf{b},0} + \frac{ik_0^2}{2k_{\mathbf{b}}a^2}T(\omega)$$
$$\times \int dz' e^{ik_{\mathbf{b}}|z-z'|}\left(1 + k^{-2}\,\mathrm{grad\,div}\right)_{\mathbf{b}}\varphi_{\mathbf{b}}(z')\boldsymbol{\Lambda}_1, \qquad (31)$$

where $\mathbf{E}^{(0)}$ is the amplitude of the initial wave. Multiplying the both parts of Eq. (31) by $\varphi_{\mathbf{b}}(z)$ and integrating over z we obtain

$$\boldsymbol{\Lambda}_1 = \boldsymbol{\Lambda}_1^0 + \sum_{\mathbf{b}} \frac{ik_0^2}{2k_{\mathbf{b}}a^2}T(\omega)\int dz\,dz' e^{ik_{\mathbf{b}}|z-z'|}\varphi_{\mathbf{b}}(z)$$
$$\times \left(1 + k^{-2}\,\mathrm{grad\,div}\right)_{\mathbf{b}}\varphi_{\mathbf{b}}(z')\boldsymbol{\Lambda}_1, \qquad (32)$$

where

$$\boldsymbol{\Lambda}_1^0 = \mathbf{E}^{(0)}\int \varphi_0(z)e^{ikz}dz = \mathbf{E}^{(0)}\int \varphi_0(z)\cos kz\,dz. \qquad (33)$$

Let us denote by β the star of the vector $\mathbf{b}$. If $\mathbf{b} = l\mathbf{b}_1 + m\mathbf{b}_2$, the star β contains the vectors $\pm l\mathbf{b}_1 \pm m\mathbf{b}_2$, $\pm m\mathbf{b}_1 \pm l\mathbf{b}_2$ of equal moduli. For $l \neq m \neq 0$ the star consists of eight vectors, otherwise it has four vectors ($l = m \neq 0$ or $l = 0$, $m \neq 0$ or $l \neq 0$, $m = 0$) and one vector in the particular case $l = m = 0$. Then the second term in the right-hand side of Eq. (32) can be rewritten

$$T(\omega)\sum_\beta \frac{ik_0^2 n_\beta}{2k_\beta a^2}\iint dz\,dz' e^{ik_\beta|z-z'|}\varphi_\beta(z)$$
$$\times \left[\left(1 - \frac{\beta^2}{2k^2}\right)\boldsymbol{\Lambda}_{1,\parallel} + \left(1 - \frac{1}{k^2}\frac{\partial^2}{\partial z'^2}\right)\boldsymbol{\Lambda}_{1,\perp}\right]\varphi_\beta(z'),$$

where $\boldsymbol{\Lambda}_{1,\parallel}$, $\boldsymbol{\Lambda}_{1,\perp}$ are vectors with the components $(\Lambda_{1,x},\ \Lambda_{1,y},\ 0)$ and $(0,\ 0,\ \Lambda_{1,z})$ respectively, n_β is the number of vectors in the star β and $\beta^2 = |\mathbf{b}|^2$. Taking into account that $\boldsymbol{\Lambda}_1^0 = (\Lambda_{1,x}^0,\ \Lambda_{1,y}^0,\ 0)$ we obtain

$$\boldsymbol{\Lambda}_1 = \boldsymbol{\Lambda}_1^0\Bigg[1 - T(\omega)\sum_\beta \frac{ik_0^2 n_\beta}{2k_\beta a^2}\left(1 - \frac{\beta^2}{2k^2}\right)$$
$$\times \int dz\,dz' e^{ik_\beta|z-z'|}\varphi_\beta(z)\varphi_\beta(z')\Bigg]^{-1}$$
$$= \boldsymbol{\Lambda}_1^0\frac{\omega_0 - \omega - i\Gamma}{\tilde{\omega}_0 - \omega - i(\Gamma + \Gamma_0)}. \qquad (34)$$

Here $\tilde{\omega}_0$ is the normalized exciton resonant frequency, the difference between $\tilde{\omega}_0$ and ω_0 consists of two terms

$$\delta\omega_1 = \omega_{LT}\frac{k^2\pi a_B^3}{2a^2}\sum_{\beta\in B_1}\frac{n_\beta}{k_\beta}\left(1 - \frac{\beta^2}{2k^2}\right)$$
$$\times \int dz\,dz' \sin k_\beta|z - z'|\,\varphi_\beta(z)\,\varphi_\beta(z'),$$
$$\delta\omega_2 = -\omega_{LT}\frac{k^2\pi a_B^3}{2a^2}\sum_{\beta\in B_2}\frac{n_\beta}{\varkappa_\beta}\left(1 - \frac{\beta^2}{2k^2}\right)$$
$$\times \int dz\,dz' e^{-\varkappa_\beta|z-z'|}\,\varphi_\beta(z)\,\varphi_\beta(z'), \qquad (35)$$

B_1 and B_2 represent stars β with real and imaginary k_β respectively, $\varkappa_\beta = \mathrm{Im}\,k_\beta$. The exciton radiative damping rate is given by

$$\Gamma_0 = \omega_{LT} \frac{k^2 \pi a_B^3}{2a^2} \sum_{\beta \in B_1} \frac{n_\beta}{k_\beta} \left(1 - \frac{\beta^2}{2k^2}\right) \lambda_\beta^2, \tag{36}$$

$$\lambda_\beta = \int \varphi_\beta(z) \cos k_\beta z \, dz = \int \Phi_0(\mathbf{r}) \cos(\mathbf{b}\boldsymbol{\rho} + k_\beta z) \, d\mathbf{r}. \tag{37}$$

The reflected and transmitted light waves are written as

$$\sum_{\mathbf{b}} \mathbf{E}_{\mathbf{b}}^{(r)} \exp\left[i(\mathbf{b}\boldsymbol{\rho} - k_{\mathbf{b}} z)\right] \text{ and } \sum_{\mathbf{b}} \mathbf{E}_{\mathbf{b}}^{(t)} \exp\left[i(\mathbf{b}\boldsymbol{\rho} + k_{\mathbf{b}} z)\right]. \tag{38}$$

The amplitudes $\mathbf{E}_{\mathbf{b}}^{(r)}$, $\mathbf{E}_{\mathbf{b}}^{(t)}$ are given by

$$\mathbf{E}_{\mathbf{b}}^{(r)} = i \frac{k^2 \pi a_B^3}{2k_{\mathbf{b}} a^2} \frac{\omega_{LT} \lambda_{\mathbf{b}} \lambda_0}{\tilde{\omega}_0 - \omega - i(\Gamma + \Gamma_0)} \left(1 - \frac{\mathbf{K}_r \hat{} \mathbf{K}_r}{k^2}\right) \mathbf{E}^{(0)},$$

$$\mathbf{E}_{\mathbf{b}}^{(t)} = \left[\delta_{\mathbf{b},0} + i \frac{k^2 \pi a_B^3}{2k_{\mathbf{b}} a^2} \frac{\omega_{LT} \lambda_{\mathbf{b}} \lambda_0}{\tilde{\omega}_0 - \omega - i(\Gamma + \Gamma_0)} \left(1 - \frac{\mathbf{K}_t \hat{} \mathbf{K}_t}{k^2}\right)\right] \mathbf{E}^{(0)}, \tag{39}$$

where $(\mathbf{K}\hat{}\mathbf{K}\mathbf{E})_i = K_i \Sigma_j K_j E_j$, $\mathbf{K}_r = (b_x, b_y, -k_{\mathbf{b}})$, $\mathbf{K}_t = (b_x, b_y, k_{\mathbf{b}})$. While deriving Eq. (39) we took into account that $\Lambda_z^{(0)} = 0$ and

$$\int e^{ik_{\mathbf{b}} z} (-id/dz) \varphi_{\mathbf{b}}(z) \, dz = -k_{\mathbf{b}} \int e^{ik_{\mathbf{b}} z} \varphi_{\mathbf{b}}(z) \, dz = -k_{\mathbf{b}} \lambda_{\mathbf{b}}.$$

One can check that (39) satisfies the energy-flux conservation law. Really, for zero dissipation, i. e. for $\Gamma = 0$, we have

$$\sum_{\mathbf{b} \in B_1} k_{\mathbf{b}} \left[|\mathbf{E}_{\mathbf{b}}^{(r)}|^2 + |\mathbf{E}_{\mathbf{b}}^{(t)}|^2\right] = k |\mathbf{E}^{(0)}|^2. \tag{40}$$

Equations (35)–(39) are original, previously analytical results for the reflectivity of a planar QD array were obtained only for particular limiting cases [27].

Note that

$$\mathbf{E}_\beta^{(r,t)} = \sum_{\mathbf{b} \in \beta} \mathbf{E}_{\mathbf{b}}^{(r,t)} \parallel \mathbf{E}^{(0)} \perp z.$$

It follows then that, in a more general case $\mathbf{E}_{\mathbf{b}}^{(0)} \neq 0$ for $\mathbf{b} \neq 0$ but

$$\mathbf{E}_\beta^{(0)} = \sum_{\mathbf{b} \in \beta} \mathbf{E}_{\mathbf{b}}^{(0)} \parallel \mathbf{E}^{(0)} \equiv \mathbf{E}_0^{(0)},$$

a value of

$$\boldsymbol{\Lambda}_1^0 = \sum_\beta \lambda_\beta \mathbf{E}_\beta^{(0)} \tag{41}$$

is oriented along $\mathbf{E}^{(0)}$ and Eq. (34) is valid as well. As a result, we obtain

$$\mathbf{E}_\beta^{(r)} = \sum_{\beta'} r_{\beta\beta'} \mathbf{E}_{\beta'}^{(0)}, \quad \mathbf{E}_\beta^{(t)} = \sum_{\beta'} t_{\beta\beta'} \mathbf{E}_{\beta'}^{(0)} \tag{42}$$

with

$$r_{\beta\beta'} = i \frac{k^2 \pi a_B^3 n_\beta}{2k_\beta a^2} \frac{\omega_{LT} \lambda_\beta \lambda_{\beta'}}{\tilde{\omega}_0 - \omega - i(\Gamma + \Gamma_0)} \left(1 - \frac{\beta^2}{2k^2}\right),$$

$$t_{\beta\beta'} = \delta_{\beta\beta'} + r_{\beta\beta'}. \tag{43}$$

Since $\mathbf{E}_\beta^{(0)}$, $\mathbf{E}_\beta^{(r)}$, $\mathbf{E}_\beta^{(0)}$ are parallel to $\mathbf{E}^{(0)}$ one can omit the vector symbols for these quantities.

In the following it is convenient to have the reflection and transmission referred to the planes shifted by some distance $d/2$ to the left and to the right with respect to the quantum-dot plane, i. e. the field on the left-hand side is written as $E_{\beta,+} \exp\left[ik_\beta(z + d/2)\right] + E_{\beta,-} \exp\left[-ik_\beta(z + d/2)\right]$ and the field on the right-hand side is written as $E'_{\beta,+} \exp\left[ik_\beta(z - d/2)\right] + E'_{\beta,-} \exp\left[-ik_\beta(z - d/2)\right]$. The corresponding reflection and transmission coefficients are related with (43) by

$$\tilde{r}_{\beta\beta'} = s_\beta s_{\beta'} r_{\beta\beta'}, \qquad \tilde{t}_{\beta\beta'} = s_\beta s_{\beta'} t_{\beta\beta'},$$

$$s_\beta = \exp\left(ik_\beta d/2\right). \tag{44}$$

IV. Exciton-polariton dispersion in terms of $\tilde{r}_{\beta\beta'}$ and $\tilde{t}_{\beta\beta'}$

We show here that the dispersion equation (7) for exciton-polaritons with $\mathbf{q} = (0, 0, q)$ can be independently derived by using the reflection and transmission coefficients for a single plane of quantum dots. Taking into account that, for the polaritons in an infinite primitive cubic (PC) lattice, the amplitudes $E_{\beta,\pm}$ and $E'_{\beta,\pm}$ at the planes $z = -a/2$ and $z = a/2$ are related by the Bloch condition $E'_{\beta,\pm} = \exp(iqa) E_{\beta,\pm}$ and using the definition of $\tilde{r}_{\beta\beta'}, \tilde{t}_{\beta\beta'}$ for $d = a$ we can write

$$E_{\beta,-} = \tilde{r}_{\beta\beta'} E_{\beta',+} + \left(s_\beta^2 \delta_{\beta\beta'} + \tilde{r}_{\beta\beta'}\right) e^{iqa} E_{\beta',-},$$

$$e^{iqa} E_{\beta,+} = \left(s_\beta^2 \delta_{\beta\beta'} + \tilde{r}_{\beta\beta'}\right) E_{\beta',+} + \tilde{r}_{\beta\beta'} e^{iqa} E_{\beta',-}. \tag{45}$$

The latter equations can be rewritten as

$$\left(1 - e^{iqa} s_\beta^2\right) E_{\beta,-} = \left(e^{iqa} - s_\beta^2\right) E_{\beta,+} = \tilde{r}_{\beta\beta'} \left(E_{\beta',+} + e^{iqa} E_{\beta',-}\right)$$

or

$$E_{\beta,+} + e^{iqa} E_{\beta,-} = \eta_\beta \tilde{r}_{\beta\beta'} \left(E_{\beta',+} + e^{iqa} E_{\beta',-}\right), \tag{46}$$

where

$$\eta_\beta = \frac{1}{e^{iqa} - s_\beta^2} + \frac{1}{e^{-iqa} - s_\beta^2}.$$

Note that the β'-dependence of $\tilde{r}_{\beta\beta'}$ is governed by the product $\lambda_{\beta'} s_{\beta'}$ and one can present this amplitude coefficient in the form

$$\tilde{r}_{\beta\beta'} = U_\beta \lambda_{\beta'} s_{\beta'}, \tag{47}$$

where U_β is β'-independent. Now we multiply Eq. (46) by $\lambda_\beta s_\beta$, sum over β and eventually come to the equation

$$\left(1 - \sum_\beta U_\beta \eta_\beta \lambda_\beta s_\beta\right) \sum_{\beta'} \lambda_{\beta'} s_{\beta'} \left(E_{\beta',+} + e^{iqa} E_{\beta',-}\right) = 0$$

which can be reduced to

$$\tilde{\omega}_0-\omega-i(\Gamma+\Gamma_0)=i\,\frac{k^2\omega_{LT}\pi a_B^3}{2a^2}\sum_{\beta}\frac{n_\beta\lambda_\beta^2}{k_\beta}\left(1-\frac{\beta^2}{2k^2}\right)s_\beta^2\eta_\beta. \tag{48}$$

The equivalence between Eqs. (7) and (48) follows immediately if we observe that

$$s_\beta^2\eta_\beta=\frac{\cos qa-e^{ik_\beta a}}{\cos k_\beta a-\cos qa}=-1-\frac{i\sin k_\beta a}{\cos k_\beta a-\cos qa} \tag{49}$$

and, for a PC lattice and for $\mathbf{q}\parallel[001]$,

$$\sum_{\mathbf{g}}\frac{I_{\mathbf{q}+\mathbf{g}}^2 S_{xx}(\mathbf{q}+\mathbf{g})}{(\mathbf{q}+\mathbf{g})^2-k^2}=\iint d\mathbf{r}d\mathbf{r}'\Phi_0(\mathbf{r})\Phi_0(\mathbf{r}')$$

$$\times\sum_{\mathbf{g}}\frac{e^{i(\mathbf{q}+\mathbf{g})(\mathbf{r}-\mathbf{r}')}}{(\mathbf{q}+\mathbf{g})^2-k^2}S_{xx}(\mathbf{q}+\mathbf{g})=\sum_\beta n_\beta\left(1-\frac{\beta^2}{2k^2}\right)$$

$$\times\iint dz\,dz'\varphi_\beta(z)\varphi_\beta(z')F(z-z',q,k_\beta), \tag{50}$$

where

$$F(\zeta,q,K)=\frac{a}{2K}\left[-\sin K|\zeta|+\frac{\sin Ka}{\cos Ka-\cos qa}\right.$$

$$\left.\times\left(\cos K\zeta+i\frac{\sin qa}{\sin Ka}\sin K\zeta\right)\right]. \tag{51}$$

V. Optical reflection from a pair of (001) quantum-dot planes

For two d-spaced quantum-dot planes (001), the normal-incidence reflection coefficient can be expanded as follows:

$$\tilde{r}^{(2)}_{\beta'\beta}=\tilde{r}_{\beta'\beta}+\tilde{t}_{\beta'\beta_2}\tilde{r}_{\beta_2\beta_1}\tilde{t}_{\beta_1\beta}+\tilde{t}_{\beta'\beta_4}\tilde{r}_{\beta_4\beta_3}\tilde{r}_{\beta_3\beta_2}\tilde{r}_{\beta_2\beta_1}\tilde{t}_{\beta_1\beta}+\ldots \tag{52}$$

Taking into account the representation (47) we obtain

$$\tilde{r}_{\beta_4\beta_3}\tilde{r}_{\beta_3\beta_2}\tilde{r}_{\beta_2\beta_1}=U_{\beta_4}\lambda_{\beta_3}s_{\beta_3}U_{\beta_3}\lambda_{\beta_2}s_{\beta_2}U_{\beta_2}\lambda_{\beta_1}s_{\beta_1}$$

$$=\left(\sum_{\beta_3}\tilde{r}_{\beta_3\beta_3}\right)^2\tilde{r}_{\beta_4\beta_1}.$$

It follows then that

$$\tilde{r}^{(2)}_{\beta'\beta}=\tilde{r}_{\beta'\beta}+\frac{\tilde{t}_{\beta'\beta_2}\tilde{r}_{\beta_2\beta_1}\tilde{t}_{\beta_1\beta}}{1-(\Sigma_{\beta_3}\tilde{r}_{\beta_3\beta_3})^2}. \tag{53}$$

In a similar way one can show that

$$\tilde{t}_{\beta'\beta_2}\tilde{r}_{\beta_2\beta_1}\tilde{t}_{\beta_1\beta}=\left(e^{ik_{\beta'}d}\delta_{\beta'\beta_2}+\tilde{r}_{\beta'\beta_2}\right)\tilde{r}_{\beta_2\beta_1}\left(e^{ik_\beta d}\delta_{\beta_1\beta}+\tilde{r}_{\beta_1\beta}\right)$$

$$=e^{ik_\beta d}e^{ik_{\beta'}d}\tilde{r}_{\beta'\beta}+\left(e^{ik_{\beta'}d}+e^{ik_\beta d}\right)\tilde{r}_{\beta'\beta_1}\tilde{r}_{\beta_1\beta}+\tilde{r}_{\beta'\beta_2}\tilde{r}_{\beta_2\beta_1}\tilde{r}_{\beta_1\beta}$$

$$=e^{i(k_\beta+k_{\beta'})d}\tilde{r}_{\beta'\beta}+\left(e^{ik_{\beta'}d}+e^{ik_\beta d}\right)\tilde{r}_{\beta'\beta}V+\tilde{r}_{\beta'\beta}V^2, \tag{54}$$

where $V=\Sigma_\beta\tilde{r}_{\beta\beta}$. Thus, we finally obtain

$$\tilde{r}^{(2)}_{\beta'\beta}=\tilde{r}_{\beta'\beta}\left[1+\frac{(e^{ik_\beta d}+V)(e^{ik_{\beta'}d}+V)}{1-V^2}\right]. \tag{55}$$

In particular it follows then that

$$\tilde{r}^{(2)}_{00}=\tilde{r}_{00}\left[1+\frac{(e^{ikd}+V)^2}{1-V^2}\right],$$

$$\tilde{r}_{00}=e^{ikd}\frac{i\Gamma_0}{\tilde{\omega}_0-\omega-i(\Gamma+\Gamma_0)}. \tag{56}$$

Hereafter we use the parameter a_{Br} defined by the resonant Bragg condition

$$\frac{\pi}{a_{Br}}=\frac{\omega_0}{c}n_b. \tag{57}$$

According to (43), (44), for $a<2a_{Br}$, we have

$$V=\frac{i\Gamma_0e^{ikd}+A}{\tilde{\omega}_0-\omega-i(\Gamma+\Gamma_0)}, \tag{58}$$

where

$$A=\frac{k^2\omega_{LT}\pi a_B^3}{2a^2}\sum_{\beta\neq0}\frac{n_\beta\lambda_\beta^2}{\varkappa_\beta}\left(1-\frac{\beta^2}{2k^2}\right)e^{-\varkappa_\beta d}$$

and $\varkappa_\beta=\sqrt{\beta^2-k^2}$. From Eqs. (56), (58) we obtain

$$\tilde{r}^{(2)}_{00}=e^{ikd}2i\Gamma_0\,\frac{(\omega-\tilde{\omega}_0+i\Gamma)\cos kd+\Gamma_0\sin kd-A}{\left[\omega-\tilde{\omega}_0+i(\Gamma+\Gamma_0)\right]^2-(i\Gamma_0e^{ikd}+A)^2}. \tag{59}$$

For the resonant Bragg double-plane structure with $d=a=a_{Br}$, this equation reduces to

$$\tilde{r}^{(2)}_{00}(d=a_{Br})=\frac{-2i\Gamma_0}{\tilde{\omega}_0-A-\omega-i(\Gamma+2\Gamma_0)}. \tag{60}$$

We see that the reflection coefficient $\tilde{r}^{(2)}_{00}\,(d=a_{Br})$ differs from the single-plane case by replacement of Γ_0 into $2\Gamma_0$, similarly to the resonant Bragg double QWs [8], and of $\tilde{\omega}_0$ into $\tilde{\omega}_0-A$.

VI. Optical reflection from a stack of quantum-dot planes

Here we consider the reflection from a system of N parallel (001) planes which is nothing more than a layer of the PC lattice of quantum dots. The first approach can be based on recurrent equations relating the reflection coefficients for $N+1$ and N planes:

$$\tilde{r}^{(N+1)}=\tilde{r}+\tilde{t}P^{(N)}\tilde{t}, \tag{61}$$

where the matrix $P^{(N)}_{\beta'\beta}$ satisfies the equation

$$P^{(N)}=\tilde{r}^{(N)}\left(I+\tilde{r}P^{(N)}\right),$$

I is the unit matrix: $I_{\beta'\beta}=\delta_{\beta'\beta}$, and for the sake of shortness we omit the indices β,β'. The similar equations for a semiinfinite lattice can be presented in the form

$$\tilde{r}^{(\infty)}=\tilde{r}+\tilde{t}P^{(\infty)}\tilde{t},\quad P^{(\infty)}=(\tilde{r}+\tilde{t}P^{(\infty)}\tilde{t})\left(I+\tilde{r}P^{(\infty)}\right). \tag{62}$$

In fact this approach was used in the previous Section to calculate the reflectivity from two QD planes.

In an alternative approach we divide the space into the following three parts: (I) $z < z_L$, (II) $z_L < z < z_R$ and (III) $z_R < z$, where the planes $z = z_L = -a/2$ and $z = z_R = Na - a/2$ are shifted by the halfperiod from the leftmost and rightmost quantum dots respectively. The secondary electric field appearing as a result of the diffraction from the quantum dot lattice allows the expansion

$$\mathbf{E}(\mathbf{r}; z \leqslant z_L) = e^{ik(z-z_L)}\mathbf{E}_0 + \sum_{\mathbf{g}_-} e^{i\mathbf{g}_-(\mathbf{r}-\mathbf{r}_L)}\mathbf{E}_{\mathbf{g}_-},$$

$$\mathbf{E}(\mathbf{r}; z_L \leqslant z \leqslant z_R) = \sum_{\mathbf{b}} \exp(i\mathbf{b}\boldsymbol{\rho})\mathbf{E}_{\mathbf{b}}(z),$$

$$\mathbf{E}(\mathbf{r}; z \geqslant z_R) = \sum_{\mathbf{g}_+} e^{i\mathbf{g}_+(\mathbf{r}-\mathbf{r}_R)}\mathbf{E}_{\mathbf{g}_+}. \tag{63}$$

Here $\mathbf{E}_0$ is the amplitude of the primary wave, $\mathbf{r}_L = (0, 0, z_L)$, $\mathbf{r}_R = (0, 0, z_R)$,

$$g_{\pm,x} = b_x = \frac{2\pi l}{a}, \qquad g_{\pm,y} = b_y = \frac{2\pi m}{a},$$

$$g_{\pm,z} = \pm k_{\mathbf{b}} = \pm\sqrt{k^2 - b_x^2 - b_y^2}.$$

In the region II the field is a superposition of two Bloch solutions

$$\mathbf{E}(\mathbf{r}; z_L \leqslant z \leqslant z_R) = \mathbf{E}_{\mathbf{q}}(\mathbf{r}) + \mathbf{E}_{-\mathbf{q}}(\mathbf{r}), \tag{64}$$

where $\mathbf{q} = (0, 0, q)$ satisfies the dispersion equation (7). In the region I in addition to the primary wave, $\mathbf{E}_0$, and specularly reflected wave, $\mathbf{E}_r = \mathbf{E}_{\mathbf{g}_-}$ with $\mathbf{g}_- = (0, 0, -k)$, there are the space harmonics oscillating in the plane (x, y). Among the latter those which satisfy the condition $k > (2\pi/a)\sqrt{l^2 + m^2}$ are diffracted waves propagating in the region I without decay. The harmonics with $k < (2\pi/a)\sqrt{l^2 + m^2}$ decay with increasing distance from the left-hand side interface. In the region III, in addition to the transmitted wave $\mathbf{E}_t$ with $\mathbf{g}_+ = (0, 0, k)$, there exist free and decaying diffracted waves with $l^2 + m^2 \neq 0$. From the field continuity at $z = z_L$ and $z = z_R$ we obtain the boundary conditions

$$\Pi_{0,+}\mathbf{E}_{\mathbf{b}=0}(z_L) = \mathbf{E}_0, \qquad \Pi_{\mathbf{b},-}\mathbf{E}_{\mathbf{b}}(z_L) = \mathbf{E}_{\mathbf{g}_-},$$

$$\Pi_{\mathbf{b},+}\mathbf{E}_{\mathbf{b}}(z_R) = \mathbf{E}_{\mathbf{g}_+}, \tag{65}$$

$$\Pi_{\mathbf{b},+}\mathbf{E}_{\mathbf{b}\neq 0}(z_L) = 0, \qquad \Pi_{\mathbf{b},-}\mathbf{E}_{\mathbf{b}}(z_R) = 0, \tag{66}$$

where we introduced the projection operators

$$\Pi_{\mathbf{b},\pm} = \frac{1}{2}\left(1 \pm \frac{1}{ik_{\mathbf{b}}}\frac{d}{dz}\right).$$

Note that Eqs. (66) mean that, under the normal incidence, there are no incoming waves with $\mathbf{b} \neq 0$.

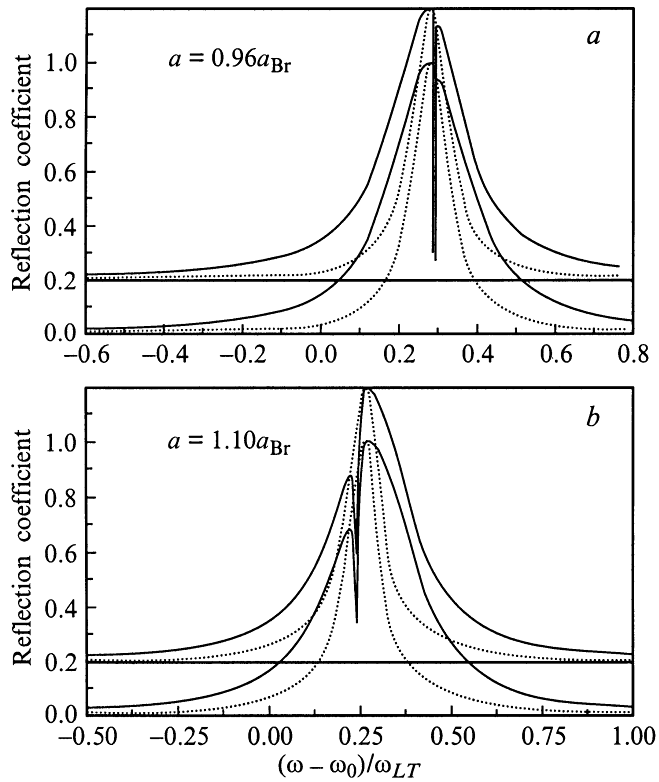

Figure 2. The reflectance from single (dotted) and double ($d = a$, solid) planes containing the quadratic QD lattice with two different periods, $a = 0.96\,a_{Br}$ (a) and $a = 1.10\,a_{Br}$ (b). The spectra are calculated by using Eq. (59) and Eq. (68). For the sake of convenience the latter are vertically shifted by 0.2.

We expand $\mathbf{E}_{\mathbf{q}}(\mathbf{r})$, $\mathbf{E}_{-\mathbf{q}}(\mathbf{r})$ in the Fourier series (10) and take into account the boundary conditions (65), (66) for $\mathbf{b} = 0$. The latter can be rearranged and written as

$$E_0 + E_r = \sum_n \left[E_{q+b_n}\, e^{i(q+b_n)z_L} + E_{-q+b_n}\, e^{i(-q+b_n)z_L}\right],$$

$$E_0 - E_r = \sum_{n,\pm} \frac{\pm q + b_n}{k}\, E_{\pm q+b_n} e^{i(\pm q+b_n)z_L},$$

$$E_t = \sum_{n,\pm} E_{\pm q+b_n} e^{i(\pm q+b_n)z_R}$$

$$= \sum_{n,\pm} \frac{\pm q + b_n}{k} E_{\pm q+b_n} e^{i(\pm q+b_n)z_R}, \tag{67}$$

where $b_n = 2\pi n/a$ and, for $\mathbf{b} = 0$, one can use the scalar form for representing the field amplitudes. Solving Eqs. (67) we come to

$$\tilde{r}^{(N)} = \frac{A_+A_-(1 - e^{i2Nqa})}{A_+^2 - A_-^2 e^{i2Nqa}}, \quad \tilde{t}^{(N)} = \frac{(A_+^2 - A_-^2)e^{iNqa}}{A_+^2 - A_-^2 e^{i2Nqa}}, \tag{68}$$

where

$$A_\pm = F\left(-\frac{a}{2}, q, k\right) \pm \frac{1}{ik} F'\left(-\frac{a}{2}, q, k\right)$$

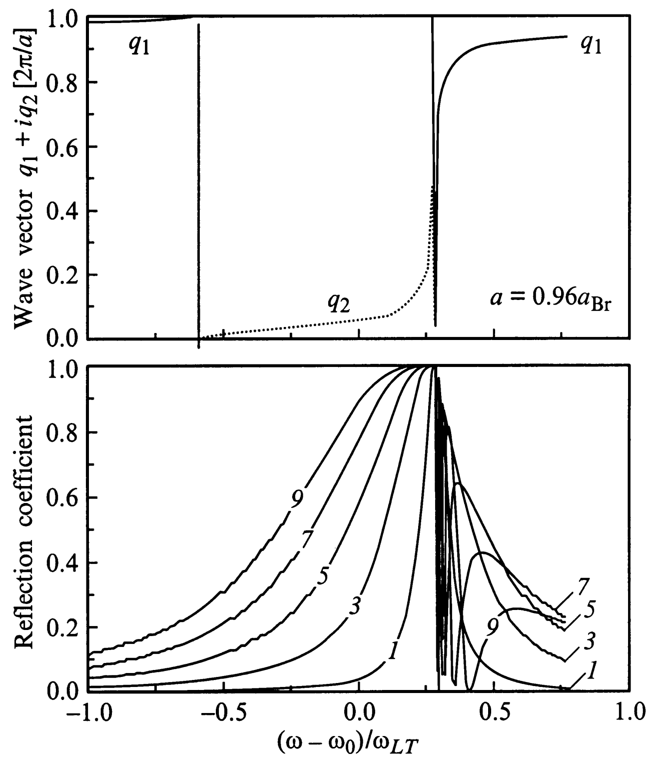

Figure 3. The dispersion of exciton polaritons propagating in the infinite QD lattice along the [001] direction (upper graph) and the reflectance from a stack of N QD planes with the spacing $d = a$ and the period $a = 0.96\, a_{Br}$ for $N = 1, 3, 5, 7, 9$ (lower graph).

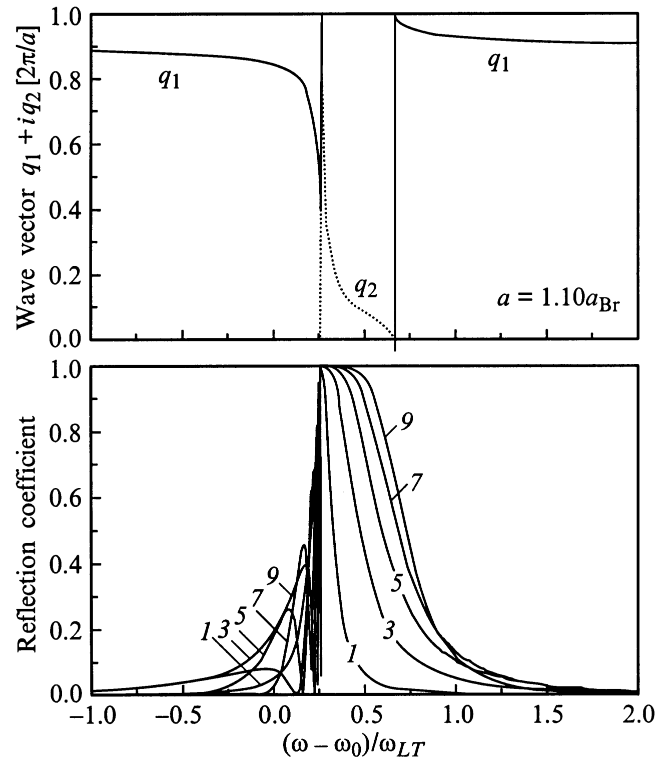

Figure 5. The same as Fig. 3 but for the period $a = 1.10\, a_{Br}$.

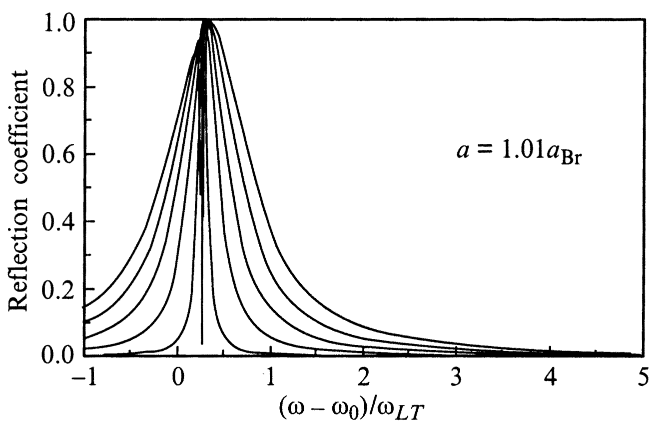

Figure 4. The reflectance from a stack of N QD planes with the spacing $d = a$ and the period $a = 1.01\, a_{Br}$ for $N = 1, 3, 5, 7, 9$.

and the function $F(\zeta, q, k)$ is defined by Eq. (51). One can straightforwardly show that

$$\frac{ikF(-a/2, q, k)}{F'(-a/2, q, k)} = \tan\frac{qa}{2}\cot\frac{ka}{2}.$$

Efficiency of the both approaches is demonstrated in Fig. 2 which shows reflection spectra from a single plane and double planes containing the quadratic lattice of spherical QDs. In case of the double-plane structures, the interplane spacing, d, is taken to coincide with the in-plane period $a = 0.96\, a_{Br}$ [Fig. 2, a] and $a = 1.10\, a_{Br}$ [Fig. 2, b]. The chosen values of the QD radius and the bulk longitudinal-transverse splitting are $R = a/4.001$ and $\omega_{LT}/\omega_0 = 5 \times 10^{-4}$, the nonradiative exciton damping is neglected: $\Gamma = 0$. For convenience the spectra calculated by using Eq. (59) and Eq. (68) are shifted by 0.2 along the vertical axis. One can see that the two approaches give identical results. The spectral dips reflect the fact that at the frequency $\omega = \tilde{\omega}_0 + A - \Gamma_0 \sin kd$ the numerator in Eq. (59) vanishes.

Figs. 3, 4 and 5 show the dependence of the reflection spectra on the number of QD planes, N. The parameters R and ω_{LT}/ω_0 are the same as in Fig. 2, the interface spacing d equals to the in-plane period $a = 0.96\, a_{Br}$, $1.01\, a_{Br}$ and $1.10\, a_{Br}$ respectively. The upper panels of Figs. 3 and 5 present the dispersion curves of exciton polaritons propagating along the [001] principal axis of the corresponding 3D primitive-cubic QD lattice. Note that within forbidden gap the polariton wavevector is imaginary, $q_1 = 0$, $q_2 \neq 0$. The period $d = a = 1.01\, a_{Br}$ is almost satisfying the Bragg condition at the exciton resonance frequency ω_0. One can see from Fig. 4 that in this case the halfwidth of the reflection spectrum is almost linearly increasing as a function of N, similarly to the enhancement by a factor of N of the radiative damping of the superradiant mode in resonant Bragg MQW structures [8].

VII. Conclusion

In conclusion, we have developed a theory of exciton polaritons in QD regular structures and calculated the resonant reflection spectra from a stack of N planes containing quadratic-lattice arrays of spherical QDs. The theory fills the gap existing between long-period multiple quantum well structures and photonic crystals. It can be also used to generalize the theory of resonant diffraction of γ-radiation by nuclei from bulk crystals [28] to synthesized multilayers like the nuclear multilayer $[^{57}\mathrm{Fe}(22\,\text{Å})/\mathrm{Sc}(11\,\text{Å})/\mathrm{Fe}(22\,\text{Å})/\mathrm{Sc}(11\,\text{Å})] \times 25$ studied by Chumakov et al. [26].

The developed theory takes into account a contribution of only one confined-exciton resonance which is valid if the separation between the exciton size-quantization levels is much larger than the bulk value of the exciton longitudinal-transverse splitting, ω_{LT}. In the opposite limit of extremely large bulk-exciton translational effective mass one can use the local material relation $\mathbf{D}(\mathbf{r}) = \varepsilon(\mathbf{r}, \omega)\mathbf{E}(\mathbf{r})$ as it was done by Sigalas et al. [29] for phonon-polaritons in a two-dimensional lattice consisting of semiconductor cylinders.

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