

Transmission distribution, $\mathcal{P}(\ln T)$, of 1D disordered chain: low- T tail

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We demonstrate that the tail of transmission distribution through 1D disordered Anderson chain is a strong function of the correlation radius of the random potential, a , even when this radius is much shorter than the de Broglie wavelength, k_F^{-1} . The reason is that the correlation radius defines the phase volume of the trapping configurations of the random potential, which are responsible for the low- T tail. To see this, we perform the averaging over the low- T disorder configurations by first introducing a finite lattice spacing $\sim a$, and then demonstrating that the prefactor in the corresponding functional integral is exponentially small and depends on a even as $a \rightarrow 0$. Moreover, we demonstrate that this restriction of the phase volume leads to the dramatic change in the shape of the tail of $\mathcal{P}(\ln T)$ from universal Gaussian in $\ln T$ to a simple exponential (in $\ln T$) with exponent depending on a . Severity of the phase-volume restriction affects the shape of the low- T disorder configurations transforming them from almost periodic (Bragg mirrors) to periodically-sign-alternating (loose mirrors).

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1. Introduction

All the states in one dimension are localized at the scale of a mean free path, l_ϵ . This means that the typical value of transmission through a 1D region of a length \mathcal{L} is $T \sim \exp(-2\mathcal{L}/l_\epsilon)$. Since T is exponentially small, the subject of recent theoretical studies [1,2] is the distribution of the log-transmission, $\mathcal{P}(\ln T)$ (and also violation of the „orthodox“ 1D localization [3] for certain correlated disorders [4]). These studies are mainly focused on the body of the distribution $\mathcal{P}(\ln T)$. A separate issue is the question about the far tail of the distribution, i.e. the behavior of $\mathcal{P}(\ln T)$ at $|\ln T| \gg 2\mathcal{L}/l_\epsilon$. This question is directly related to a more general concept of the anomalously localized states in disordered conductors [5–8]. In Ref. [9] and in subsequent paper [10] it was asserted that the small- T tail is dominated by specific configurations of the disorder, $V(x)$, namely, the Bragg mirrors. These configurations are illustrated in Fig. 1. The potential $V(x) = 2\mathcal{V}\cos(2k_F x)$ opens a gap $2\mathcal{V}$ centered at energy $\epsilon = k_F^2/2$. The corresponding wave function oscillates with a period π/k_F and decays as $\exp(-\gamma x)$, where $\gamma = \mathcal{V}/(2k_F) \ll k_F$ is the decrement. Then we have $|\ln T| = 2\gamma\mathcal{L} = \mathcal{V}\mathcal{L}/k_F$. The important assumption adopted in Refs. [9,10] is that, with exponential accuracy, $\mathcal{P}(\ln T)$ can be found by substituting $2\mathcal{V}\cos(2k_F x)$ into the „white-noise“ probability, $\exp[-\{l_\epsilon \int_0^{\mathcal{L}} dx V(x)^2/4k_F^2\}]$, of the fluctuation $V(x)$. This yields [9,10] $|\ln \mathcal{P}(\ln T)| = (l_\epsilon \ln^2 T)/2\mathcal{L}$. Remarkably, the result coincides with the asymptote of the „exact“ solution obtained by Altshuler and Prigodin [11] using the Berezinskii technique [3].

The Bragg mirror configurations, $V(x) = 2\mathcal{V}\cos(2k_F x)$, emerged in Refs. [9,10] upon applying the optimal fluctuation approach [12,13]. This approach was specifically

designed to deal with situations when the result is determined by a particular disorder configuration. The above log-normal expression for $\mathcal{P}(T)$ corresponds to the saddle point of the functional integral over disorder configurations. Obviously, the statistical weight of an ideal Bragg mirror is zero. Rigorous application of the optimal fluctuation approach implies taking into account the configurations close to optimal. This procedure corresponds to the calculation of the prefactor in the functional integral. In most cases [14–17] the prefactor behaves as a power law and, thus, cannot compete with the main exponent.

In the present paper we demonstrate that the situation depicted in Fig. 1, a differs drastically from Refs. [9,10] due to a large size of the optimal fluctuation. Resulting from this large size, the large number of „degrees of freedom“ makes the prefactor exponentially small, so that, the final result for $\mathcal{P}(\ln T)$ is determined by the competition of the prefactor and the main exponent. More specifically, as illustrated in Fig. 1, b, c , weakly perturbed Bragg mirrors include fluctuations with phase varying along x . These fluctuations are „dangerous“, in the sense, that they result in spatial modulation of the gap center (Fig. 1, c) and, thus, suppress the decrement γ . Large size of a mirror translates into a large statistical weight of these dangerous fluctuations, i.e. it severely restricts the weight of the efficient Bragg mirrors.

As we demonstrate in the present paper, due to the reasons listed above, the proper application of the optimal fluctuation approach, i.e. taking prefactor into account, has dramatic consequences for the shape of the tail of $\mathcal{P}(\ln T)$. Namely,

(i) The log-normal result [9,10] has a „universal“ form, in the sense, that it contains only the mean free path, l_ϵ . Thus, it is insensitive to the actual value of the correlation radius a of the disorder, (as long as $a \ll l_\epsilon$). In contrast, we demonstrate that, with prefactor taken into account, $\mathcal{P}(\ln T)$ depends on a exponentially strongly even for $a \ll l_\epsilon$.

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(ii) It was assumed in Refs. [9,10] that the optimal Bragg-mirror fluctuation extends over the entire region \mathcal{L} . This is indeed the case for Gaussian form of $\mathcal{P}(\ln T)$. However, with $\mathcal{P}(\ln T)$ having non-Gaussian form, it turns out that the optimal fluctuation corresponds to the Bragg mirror occupying only a part, $L_{\text{opt}} < \mathcal{L}$, of the interval \mathcal{L} , as illustrated in Fig. 2. The underlying reason for this is that the prefactor makes the Bragg mirrors very „costly“.

The paper is organized as follows. In Sec. 2 we introduce the discretization, which is always mandatory for the functional integration. We choose the lattice spacing to be finite, which is the most convenient discretization for averaging over disorder configurations of the Braggmirror type. In Sec. 3 the functional integral, which defines $\mathcal{P}(\ln T)$, is calculated with prefactor in the domain, where the Bragg mirrors dominate the low- T configurations. In Sec. 4 we consider the low-energy domain, where the Bragg mirrors, being too costly, become inefficient. We demonstrate that relevant low- T disorder configurations in this domain are the loose mirrors, which are periodically-

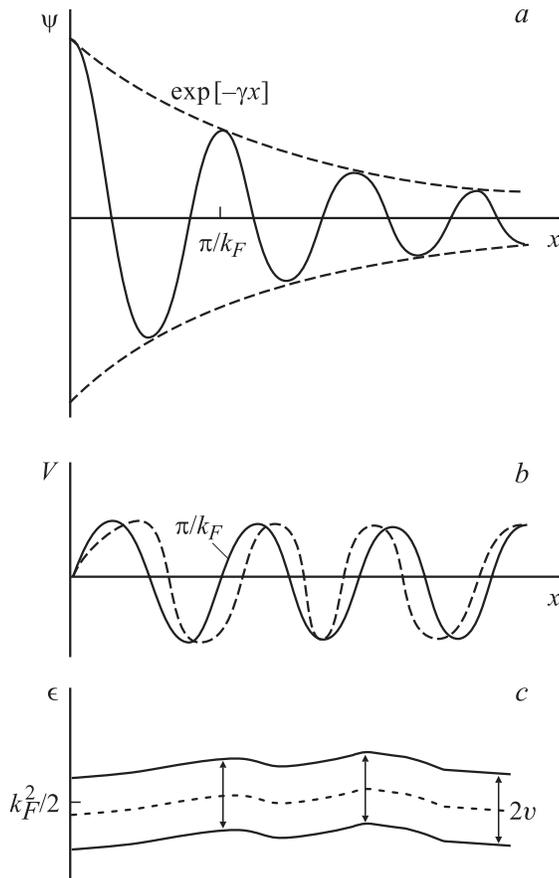


Figure 1. *a* — schematic illustration of the decay of the wave function within the Bragg mirror. *b*: solid line — potential fluctuation corresponding to an „ideal“ Bragg mirror; dashed line — „real“ Bragg mirror with fluctuating phase. *c* — fluctuations of phase result in the fluctuations of position of the gap center (dashed line) leaving the width of the gap (solid lines) unchanged.

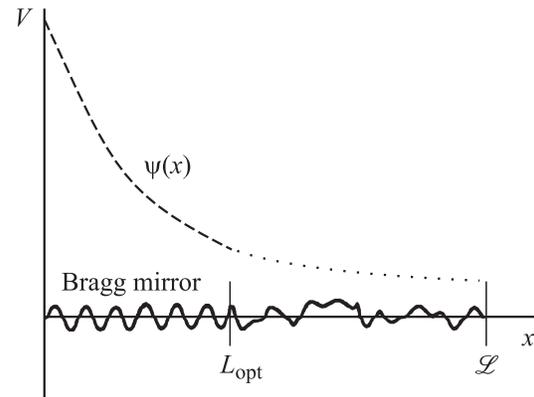


Figure 2. Disorder configuration in which the Bragg mirror occupies only a part L_{opt} of the total length \mathcal{L} . The decay of the envelope of the wave function within the mirror and in the rest of the sample is illustrated with dashed and dotted lines, respectively.

sign-alternating on-site energies. As we show in Sec. 4, such loose mirrors form a well-defined subspace in the space of all possible realizations of the on-site energies. In particular, they dominate the functional integral for $\mathcal{P}(\ln T)$, which we calculate with prefactor. In Sec. 5 we turn to the continuous limit $a \rightarrow 0$. In contrast to Refs. [9,10], we find that, due to the exponentially small prefactor in the functional integral, it is loose mirrors, extending over a part of the chain, rather than the Bragg mirrors, occupying the entire chain [9,10], that dominate $\mathcal{P}(\ln T)$ in this limit. In the Sec. 6 we trace the reason why the loose mirrors are not captured in the standard analytical techniques in 1D.

2. General considerations

2.1. Discretization procedure

To calculate the prefactor of $\mathcal{P}(\ln T)$, it is necessary, as in any functional integration, to adopt some sort of discretization [15]. In this paper we simply introduce a finite lattice constant (equal to 1), and a finite hopping constant (equal to 2), so that the problem reduces to 1D Anderson model. The discrete on-site energies, V_m , are random numbers; their distribution function, $P(V_m)$, has a characteristic scale $\Delta \ll 1$, which we identify with r.m.s.

$$\Delta = \left[\int_{-\infty}^{\infty} dV_m V_m^2 P(V_m) \right]^{1/2}. \quad (1)$$

The discrete version of the ideal „continuous“ Bragg mirror $V(x) = 2\mathcal{V} \cos(2k_F x)$ has the period n and corresponds to the on-site energies $V_m = 2\mathcal{V} \cos(2\pi m/n)$. Then the discrete analog of the energy $k_F^2/2$ has the form

$$\epsilon_n = 4 \sin^2(\pi/2n), \quad (2)$$

where the energy is measured from the band edge (equal to -2). To approximate the continuum, unlike Ref. [18],

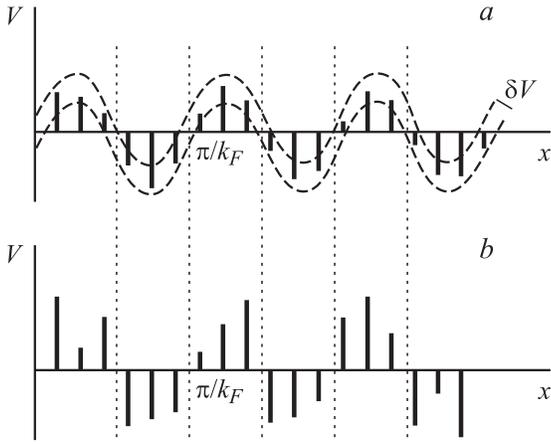


Figure 3. *a* — weakly disturbed Bragg mirror on a lattice; δV is the tolerance in the on-site energies. *b* — loose mirror with „rigidity“ in signs of the on-site energies.

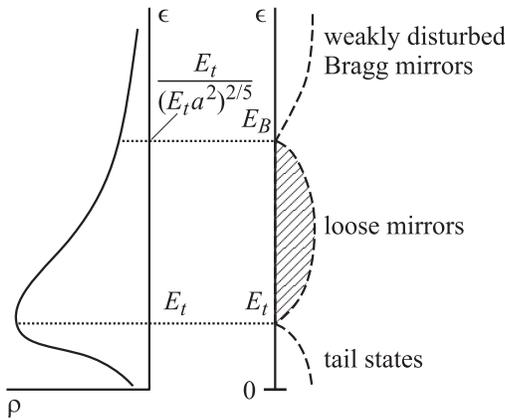


Figure 4. Left: schematic plot of the 1D density of states ρ smeared by disorder; for energies $\epsilon < E_t$ the states are strongly localized. Right: low- T disorder configurations have the form as in Fig. 3, *b* within the energy domain $E_t < \epsilon < E_B$ and the form as in Fig. 3, *a* for energies $\epsilon > E_B$.

we will focus on the energy interval close to the band edge, i.e. $n \gg 1$. On the other hand, the energy should be well above the fluctuation-tail domain, $\epsilon < E_t$, where E_t is determined from the following consideration. As follows from the golden rule, for $n \gg 1$, the mean free path is equal to $l_\epsilon = 8\epsilon_n/\Delta^2$. Then the conductance, G_ϵ , can be written as $G_\epsilon = \epsilon^{1/2}l_\epsilon$. The upper boundary of the fluctuation-tail domain is determined by the condition $G_{E_t} \approx 1$, which yields $E_t \approx \Delta^{4/3}$. The fact that we consider energies above E_t sets the lower bound for the values of n , namely, $n \ll \Delta^{-2/3}$.

Once the discretization procedure is specified, the averaging over disorder realizations is well-defined. In particular, to calculate the statistical weight of the Bragg mirrors, providing a given value of $\ln T$, one has to integrate $P(V_m)$ over the deviations of the on-site energies from $V_m = 2\mathcal{V}\cos(2\pi m/n)$ with a restriction that the

log-transmission for the set $\{V_m\}$ is fixed and equal to $\ln T$. Translating the claim made in Refs. [9,10] into the „discrete“ language, this weight is simply equal to $\prod_m P(2\mathcal{V}\cos(2\pi m/n))$, i.e. the deviations of V_m from $2\mathcal{V}\cos(2\pi m/n)$, that are responsible for the prefactor, can be neglected within exponential accuracy. Below we test this assertion by explicit calculation of the prefactor. The result of this test can be summarized as follows.

- 1) Weakly disturbed Bragg mirrors (see Fig. 3, *a*) are indeed the dominating disorder configurations, providing a given value of $\ln T$, only above certain energy, $E_B \approx \Delta^{4/5}$, i.e. $n \approx \Delta^{-2/5}$, as illustrated in Fig. 4.
- 2) Even for energies bigger than E_B , the prefactor is exponentially small. Whether or not it competes with the main exponent [9,10] depends on the length \mathcal{L} of disordered region.
- 3) Our most important finding is that within a parametrically wide energy domain, $E_B > \epsilon_n > E_t$, the low- T disorder configurations are dominated by the novel entity, which we call „loose mirrors“. They are illustrated in Fig. 3, *b* and represent the alternating regions of equal length $n/2$; within each region the values of V_m are either random, but strictly positive or random, but strictly negative. The „phase volume“ of these configurations is much bigger than that of the Bragg mirrors. On the other hand, for these configurations at large n the sign „rigidity“ of V_m within each half-period is sufficient to provide the Bragg reflection.

2.2. Optimal length of the Bragg mirror for a given length of the chain

Taking prefactor into account has a dramatic effect on the structure of the optimal fluctuation. To clarify this point, suppose that the Bragg mirror extends not over entire distance \mathcal{L} , but only over the interval $L < \mathcal{L}$, see Fig. 2. Denote with T_L the transmission of the mirror. Then for the transmission of the entire interval \mathcal{L} we have

$$|\ln T| = |\ln T_L| + 2 \left(\frac{\mathcal{L} - L}{l_\epsilon} \right), \quad (3)$$

where the second term describes the transmission through the region outside the Bragg mirror (Fig. 2). It is apparent that the first term in Eq. (3) increases with L , whereas the second term decreases with L . This suggests the following procedure to determine the optimal length of the mirror. Denote with $\mathcal{P}_L(\ln T)$ the distribution function of T_L . Then the distribution function of the total transmission for a given L can be written as $\mathcal{P}_L \{ \ln T + 2(\mathcal{L} - L)/l_\epsilon \}$. The fact that the Bragg mirror has an optimal length can be expressed in the form

$$|\ln \mathcal{P}(\mathcal{L}, \ln T)| = \min_L \left| \ln \mathcal{P}_L \left\{ \ln T + 2 \left(\frac{\mathcal{L} - L}{l_\epsilon} \right) \right\} \right|. \quad (4)$$

It is seen from Eq. (4) that the calculation of the small- T tail of the net transmission of the entire interval \mathcal{L} reduces to the calculation of the function $\mathcal{P}_L(\ln T)$, which is the

characteristic of the Bragg mirror only. If now we use for $\mathcal{P}_L(\ln T)$ the result [9,10] $\mathcal{P}_L(\ln T) = \exp[-(L_\epsilon \ln^2 T)/2L]$, which was obtained neglecting the prefactor, then the minimum in Eq. (4) would correspond to $L = \mathcal{L}$, i.e. to the Bragg mirror extending over the entire interval \mathcal{L} . Below we demonstrate that, once the prefactor is taken into account, the true optimal fluctuation corresponds to a „short“ Bragg mirror, $L < \mathcal{L}$, within a parametrically wide interval of $|\ln T|$.

3. Weakly distorted Bragg mirrors

3.1. Calculation of the functional integral

Here we consider the case when the deviations, δV_m , of the on-site energies, V_m , from the optimal values, $V_m = 2\mathcal{V} \cos(2\pi m/n)$, are relatively small. For concreteness we choose the Gaussian distribution of the on-site energies, $P(V) = \pi^{-1/2} \Delta^{-1} \exp(-V^2/\Delta^2)$. To calculate the prefactor due to small deviations, δV_m , we adopt the assumption that δV_m are homogeneously distributed within a small interval (tolerance) $\delta V \ll \Delta$ (Fig. 3, *a*). On the one hand, this assumption leads to a drastic simplification of the calculation. On the other hand, as we will see below, it yields an asymptotically correct result.

With homogeneously distributed δV_m , the statistical weight of distorted Bragg mirror, \mathcal{P}_L , can be easily expressed through the tolerance δV ,

$$\begin{aligned} \mathcal{P}_L &= \left(\frac{\delta V}{\Delta}\right)^L \exp\left[-\left(\frac{1}{\Delta^2}\right) \sum_m V_m^2\right] \\ &= \exp\left[-L \ln\left(\frac{\Delta}{\delta V}\right) - \frac{\gamma^2 L}{2\Delta^2}\right]. \end{aligned} \quad (5)$$

In Eq. (5) we have assumed that δV not only smaller than Δ , but even stronger condition $\delta V \ll \Delta^2/\mathcal{V}$ is met. We will check this condition below.

We now incorporate the fluctuations δV_m into the log-transmission of the Bragg mirror, $\ln T$. As it was pointed out above, random shifts, $\delta \epsilon_i$, of the gap center reduce the decrement $\gamma = \mathcal{V}/2k_F = \mathcal{V}n/2\pi$ within each period. This is due to the local detuning from the Bragg resonance. Quantitatively, the reduction of the decrement γ can be expressed as

$$\gamma(\delta \epsilon_i) = \gamma \sqrt{1 - \left(\frac{\delta \epsilon_i}{\mathcal{V}}\right)^2}. \quad (6)$$

As a result, instead of $2\gamma L$ in the absence of fluctuations, the expression for $|\ln T|$ is modified to

$$|\ln T| = 2n \sum_i \gamma(\delta \epsilon_i) \approx 2\gamma L - n\gamma \sum_i \left(\frac{\delta \epsilon_i}{\mathcal{V}}\right)^2. \quad (7)$$

Consider now a given period, i , containing n sites. Denote with $V_m^{(i)}$ the on-site energies within this period. Then the

shift, $\delta \epsilon_i$, of the gap center for this period can be expressed through $V_m^{(i)}$ via a discrete Fourier transform as follows

$$\delta \epsilon_i = \left(\frac{\pi}{n}\right)^2 \frac{\sum_{m=1}^n V_m^{(i)} \sin(\pi m/n)}{\sum_{m=1}^n V_m^{(i)} \cos(\pi m/n)}, \quad (8)$$

where the summation is performed over the sites within the i -th period. Obviously, for an ideal Bragg mirror, $V_m = 2\mathcal{V} \cos(2\pi m/n)$, we obtain from Eq. (8) that $\delta \epsilon_i = 0$. In the presence of fluctuations δV_m the typical value of $\delta \epsilon_i$ is proportional to δV and can be estimated from Eq. (8) as follows. The numerator is the sum of n random numbers, each being $\sim \delta V$. Thus, the typical value of the numerator is $n^{1/2} \delta V$. On the other hand, the denominator is equal to $n\mathcal{V}/2$. Then we obtain

$$\delta \epsilon_i = \frac{C \epsilon_{2n}}{(2n)^{1/2}} \left(\frac{\delta V}{\mathcal{V}}\right) = \frac{\pi^2 C}{2^{1/2} n^{5/2}} \left(\frac{\delta V}{\mathcal{V}}\right), \quad (9)$$

where the constant C is of the order of 1.

Looking at Eq. (6), it might seem that the condition $\delta V \ll \mathcal{V}$ of the weak distortion of the Bragg mirror by fluctuations, and the condition $\delta \epsilon_i \ll \mathcal{V}$ of the weak reduction of the decrement are quite different. It turns out, as we will see later, that $\delta V \ll \mathcal{V}$ insures that $|\gamma_i - \gamma| \ll \gamma$, and thus justifies the expansion of $\gamma(\delta \epsilon_i)$ used in Eq. (7). Substituting Eq. (9) into Eq. (7) we get

$$\begin{aligned} |\ln T| - \frac{\gamma L n}{\pi} &\approx -C^2 \frac{\gamma L}{2\pi} \left(\frac{\epsilon_{2n} \delta V}{\mathcal{V}^2}\right)^2 \\ &\approx -8\pi^3 C^2 \left(\frac{L \delta V^2}{n^4 \mathcal{V}^3}\right). \end{aligned} \quad (10)$$

Using the fact that the r.h.s. in Eq. (10) is much smaller than $|\ln T|$, we can express \mathcal{V} through $|\ln T|$ as follows

$$\mathcal{V} = \frac{\pi |\ln T|}{nL} + 8\pi C^2 \frac{L^3 \delta V^2}{n^2 |\ln^3 T|}. \quad (11)$$

Further steps are straightforward. Using Eq. (11), we can rewrite the exponent in Eq. (5) as

$$\begin{aligned} |\ln \mathcal{P}_L| &= \left\{ L \ln\left(\frac{\Delta}{\delta V}\right) + \frac{\gamma^2 L}{2\Delta^2} \right\} \\ &= \frac{\pi^2 \ln^2 T}{2n^2 \Delta^2 L} + \left\{ L \ln\left(\frac{\Delta}{\delta V}\right) + \frac{8\pi^2 C^2 L^3 \delta V^2}{n^3 \Delta^2 \ln^2 T} \right\}. \end{aligned} \quad (12)$$

Now it is easy to see that there exists the optimal tolerance

$$\delta V = \delta V_{\text{opt}} = \frac{\gamma \Delta n^{5/2}}{4\pi^2} = \frac{n^{3/2} \Delta |\ln T|}{4\pi C L}, \quad (13)$$

for which $|\ln \mathcal{P}_L|$ is minimal and is equal to

$$|\ln \mathcal{P}_L| = \frac{\ln^2 T}{2\Delta^2 L} \left(\frac{\pi}{n}\right)^2 + L\Lambda(T), \quad (14)$$

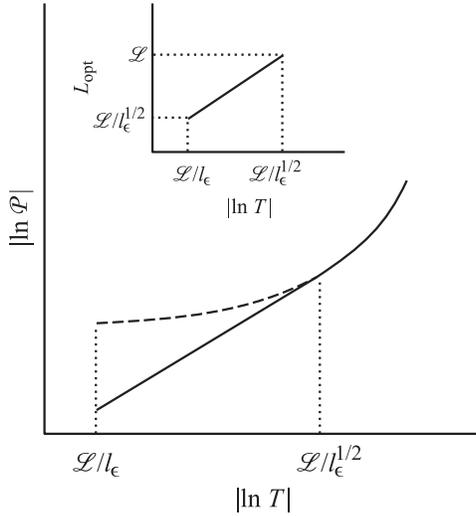


Figure 5. Solid line: our main result of Eqs. (18), (19) for the low- T tail of the transmission distribution. Dashed line: log-normal $\mathcal{P}(T)$ of Refs. [9,10]. Inset: the portion of the chain occupied with Bragg (loose) mirror.

where

$$\Lambda(T) = \ln \left(\frac{\Delta}{\delta V_{\text{opt}}} \right) = \ln \left(\frac{4\pi CL}{n^{3/2} |\ln T|} \right) \quad (15)$$

depends on T very weakly. It is also seen from Eq. (15) that C enters into the final result only as a factor under the logarithm, so that our assumption about homogeneous distribution of δV_m is justified.

Now, in order to calculate the tail of the transmission distribution, $\mathcal{P}(\mathcal{L}, \ln T)$, we substitute Eq. (14) into Eq. (4):

$$|\ln \mathcal{P}_L| = \min_L \left\{ \frac{1}{2L} \left(\frac{\pi}{n\Delta} \right)^2 \times \left[|\ln T| + 2 \left(\frac{\mathcal{L} - L}{l_\epsilon} \right) \right]^2 + L\Lambda(T) \right\}. \quad (16)$$

Next we perform minimization with respect to L . This yields the following equation for the optimal $L = L_{\text{opt}}$

$$L_{\text{opt}} = \frac{\pi |\ln T|}{\sqrt{2\Lambda n\Delta}} \left[1 + 2 \left(\frac{\mathcal{L} - L_{\text{opt}}}{l_\epsilon |\ln T|} \right) \right] \times \left[1 - \frac{2}{\Lambda} \left(\frac{\pi}{n\Delta} \right)^2 \frac{|\ln T|}{l_\epsilon L_{\text{opt}}} \left\{ 1 + 2 \left(\frac{\mathcal{L} - L_{\text{opt}}}{l_\epsilon |\ln T|} \right) \right\} \right]^{-1/2}. \quad (17)$$

Since we are interested in anomalously low transmissions, $|\ln T| \gg \mathcal{L}/l_\epsilon$, the second term in the first square brackets in Eq. (17) is small. The second term in the second square brackets contains an additional parameter $\sim |\ln T|/L_{\text{opt}}(n^2\Delta^2 l_\epsilon)$. Since $l_\epsilon = 8\epsilon_n/\Delta^2$, the combination $n^2\Delta^2 l_\epsilon$ is ~ 1 . Thus, the above parameter reduces to $|\ln T|/L_{\text{opt}}$, which is also small. More precisely, it is of the order of $l_\epsilon^{-1/2}$. Neglecting second terms in both square

brackets, and substituting L_{opt} from Eq. (17) into Eq. (16), we arrive at the final result

$$|\ln \mathcal{P}(\ln T, \mathcal{L})| = \left(\frac{\pi\sqrt{2\Lambda}}{n\Delta} \right) |\ln T|, \quad (18)$$

$$\left(\frac{\sqrt{2\Lambda}}{\pi} \right) n\Delta\mathcal{L} > |\ln T| > n^2\Delta^2\mathcal{L},$$

$$|\ln \mathcal{P}(\ln T, \mathcal{L})| = \frac{1}{2\mathcal{L}} \left(\frac{\pi}{n\Delta} \right)^2 \ln^2 T + \mathcal{L}\Lambda(T),$$

$$|\ln T| > \left(\frac{\sqrt{2\Lambda}}{\pi} \right) n\Delta\mathcal{L}. \quad (19)$$

It is instructive to rewrite the above result in terms of energy, $\epsilon_n \approx (\pi/n)^2$, and conductance, $G_n \approx 1/(n^3\Delta^2)$,

$$|\ln \mathcal{P}(\ln T, \mathcal{L})| = \left(\frac{2\pi^3\Lambda G_n}{\sqrt{\epsilon_n}} \right)^{1/2} |\ln T|,$$

$$\left(\frac{2\Lambda\sqrt{\epsilon_n}}{\pi^3 G_n} \right)^{1/2} \mathcal{L} > |\ln T| > \left(\frac{\sqrt{\epsilon_n}}{\pi G_n} \right) \mathcal{L}, \quad (20)$$

$$|\ln \mathcal{P}(\ln T, \mathcal{L})| = \frac{\pi^3}{2\mathcal{L}} \left(\frac{G_n}{\sqrt{\epsilon_n}} \right) \ln^2 T + \mathcal{L}\Lambda(T),$$

$$|\ln T| > \left(\frac{2\Lambda\sqrt{\epsilon_n}}{\pi^3 G_n} \right)^{1/2} \mathcal{L}. \quad (21)$$

The behavior of Eqs. (18), (19) is illustrated schematically in Fig. 5. We see, that the log-normal dependence of Refs. [9,10] takes place only for very small transmission coefficients $|\ln T| > \mathcal{L}/\sqrt{l_\epsilon}$. Only in this domain the Bragg mirror extends over the entire interval, and the prefactor (second term in Eq. (19)) is smaller than the main exponent. Within a wide domain $\mathcal{L}/\sqrt{l_\epsilon} > |\ln T| > \mathcal{L}/l_\epsilon$ the probability, $\mathcal{P}(\mathcal{L}, \ln T)$, behaves as a simple exponent. The underlying reason for such a behavior is that the dependences of the main exponent and of the prefactor on L are opposite. As a result, there exists an optimal mirror length

$$L = L_{\text{opt}} = \frac{\pi |\ln T|}{\sqrt{2\Lambda n\Delta}}, \quad (22)$$

which leads to the result Eq. (18). Simple exponent Eq. (18) corresponds to the situation when L_{opt} is shorter than the length of the interval \mathcal{L} . More precisely, the portion of the interval \mathcal{L} , occupied by the Bragg mirror, is given by $L_{\text{opt}}/\mathcal{L} = \pi |\ln T|/\sqrt{2\Lambda n\Delta}$. Within the domain $\mathcal{L}/\sqrt{l_\epsilon} > |\ln T| > \mathcal{L}/l_\epsilon$ this portion changes from 1 to a small value $(\Delta/G)^{1/3} \ll 1$, as illustrated in the inset in Fig. 5.

3.2. Justifications of the assumptions

The above calculation was based on three assumptions.

- 1) $\delta V \ll \Delta^2/\mathcal{V}$; we used this condition in the expression Eq. (5) for the probability \mathcal{P}_L .
- 2) $\delta V \ll \mathcal{V}$; this is the condition that the Bragg mirror is well defined. It was also used in deriving Eq. (5).
- 3) $\delta\epsilon_i \ll \mathcal{V}$; this condition was used in expansion Eq. (7).

From the result Eq. (13) of the above calculation we find $\delta V_{\text{opt}}/\mathcal{V} = \Delta n^{5/2}/(4\pi^2)$. Thus, the *assumption 2*) is valid under the condition

$$\Delta n^{5/2} \ll 1. \quad (23)$$

Below we demonstrate that the same condition Eq. (23) guarantees the validity of the other two assumptions.

Assumption 1). Within the domain $\mathcal{L}/\sqrt{l_\epsilon} > |\ln T| > \mathcal{L}/l_\epsilon$, where $\mathcal{P}(\mathcal{L}, \ln T)$ behaves as a simple exponent, the amplitude of the Bragg mirror, given by $\mathcal{V} = \pi |\ln T|/nL_{\text{opt}}$, is equal to $\mathcal{V} = \sqrt{2\Lambda}\Delta$ and does not depend on \mathcal{L} , where, as follows from Eq. (15), $\Lambda = \Lambda(L_{\text{opt}}) = \ln(4\pi^2 C/\sqrt{2\Lambda}\Delta n^{5/2}) \approx \ln(1/\Delta n^{5/2})$. To check the assumption 1) we rewrite the ratio $\mathcal{V}\delta V/\Delta^2$ as $(\delta V/\mathcal{V})(\mathcal{V}^2/\Delta^2) = (\Delta n^{5/2}/4\pi^2)(\mathcal{V}^2/\Delta^2) = \frac{1}{2\pi^2} \Delta n^{5/2} \times \ln(1/\Delta n^{5/2})$. We see that $\mathcal{V} \ll \Delta^2/\delta V$ holds under the condition $\Delta n^{5/2} \ll 1$, which is precisely the condition (23). *Assumption 3*). From Eq. (9) we have the following estimate for the ratio $\delta\epsilon_i/\mathcal{V} \ll 1$:

$$\frac{\delta\epsilon_i}{\mathcal{V}} = \frac{\pi^2 C}{2^{1/2} n^{5/2}} \left(\frac{\delta V}{\mathcal{V}^2} \right). \quad (24)$$

Substituting into this equation the optimal value $\delta V_{\text{opt}} = \mathcal{V}\Delta n^{5/2}/(4\pi^2)$, we obtain $\delta\epsilon_i/\mathcal{V} \sim \Delta/\mathcal{V} = [2\Lambda(L_{\text{opt}})]^{-1/2}$. On the other hand, $\Lambda(L_{\text{opt}}) = \ln(1/\Delta n^{5/2})$ is large under the same condition Eq. (23). This large logarithm justifies the *assumption 3*).

In conclusion of this Section we would like to make the following two remarks.

- 1) The expression for the decrement $\gamma = \mathcal{V}n/2\pi$ is the result of the two-wave approximation, within which propagating and Bragg-reflected waves are coupled only in the first order, i.e. by a single harmonics of periodic potential. For two-wave approximation to be valid, the second-order coupling matrix elements must be much smaller than \mathcal{V} . The estimate for these second-order elements is $\sim \mathcal{V}^2/\epsilon_n \sim \mathcal{V}^2 n^2$. Thus, the two-wave approximation is valid if $\mathcal{V}^2 n^2 \ll \mathcal{V}$, i.e. $\mathcal{V} n^2 \ll 1$. As it is seen from Eq. (23), $\mathcal{V} \ll \Delta^{1/2} n^{-5/4}$. Thus $\mathcal{V} n^2 \ll [\Delta^{1/2} n^{-5/4}] n^2 = [\Delta n^{5/2}]^{1/2} / n^{1/2}$. We see that the applicability condition of the two-wave approximation is weaker than the main condition $\Delta n^{5/2} \ll 1$.
- 2) The condition Eq. (23) implies that the energy ϵ_n exceeds $\Delta^{4/5}$. This, in turn, suggests that the conductance $G_n = k_F l_\epsilon$ for $\epsilon = \epsilon_n$ is equal to $G_n = (\Delta^2 n^3)^{-1}$, and is large by virtue of this condition.

4. „Loose“ mirrors

4.1. Density of the loose mirrors

We now turn to the case of low energies. More precisely, we consider the domain $E_B > \epsilon_n > E_t$ (Fig. 4). The upper boundary of this domain corresponds to $\Delta n^{5/2} \approx 1$, whereas the lower boundary corresponds to $\Delta n^{3/2} \approx 1$. For energies $\epsilon_n > E_B$ the transmission is dominated by weakly disturbed

Bragg mirrors, as discussed in the previous Section. For energies $\epsilon < E_t$ we have $G_\epsilon < 1$, i.e. these energies correspond to the tail states.

As we enter the low energy (large- n) domain, the key component of the above scenario of weakly disturbed mirrors gets violated. Namely, at $n \sim \Delta^{-2/5}$ we have $\delta V \sim \mathcal{V}$. This implies that almost sinusoidal Bragg mirror cannot retain its role as an optimal fluctuation which is responsible for low- T values.

In general, optimal fluctuation constitutes a saddle point in the functional space. In the previous Section, by demonstrating that the disorder configurations contributing to the functional integral differ weakly (by $\delta V \ll \mathcal{V}$) from the optimal configuration, we have justified that the saddle point is well defined, or, in other words, the expansion around the saddle point yields a narrow width of the Gaussian in the functional space. In this Section we demonstrate that in the energy domain $E_B > \epsilon_n > E_t$ there exists a well-defined subspace of all realizations of the on-site energies, $\{V_m\}$, which assumes the role of a saddle point. We dub the elements of this subspace as „loose“ mirrors. A loose mirror is a configuration of alternating regions of n random, but positive V_m and n random, but negative V_m . It is illustrated in Fig. 3. Obviously, the statistical weight of the loose mirrors is small. It is easy to see that this weight is equal to 2^{-L} . Most importantly, despite the randomness of V_m , the fact of the sign rigidity within each interval of length, n , is sufficient for the formation of the Bragg gap with the well-defined width, and thus, for generating the low transmission coefficients.

The key element of calculation of $\mathcal{P}(\ln T)$ in the regime of weakly distorted Bragg mirrors was the expansion Eq. (7), which expressed the fact that the decrement γ weakly fluctuates from period to period. It turns out that in the regime of loose mirrors, $\Delta n^{5/2} \gg 1$, these fluctuations are also weak. This can be seen from Eq. (24). Since in the regime of loose mirrors the only scale for \mathcal{V} and δV is Δ , Eq. (24) yields $\delta\epsilon_i/\mathcal{V} \sim (\Delta n^{5/2})^{-1}$, which is small in the regime of loose mirrors. Thus, the expansion Eq. (7) is applicable in this regime as well.

To calculate the distribution $\mathcal{P}(\ln T)$ in the regime of loose mirrors, the calculation in the previous Section should be modified in the following way. For loose mirrors the „period“ consists of interval of n positive V_m followed by an interval of n negative V_m . The magnitude of the gap $2\mathcal{V}$ and corresponding decrement, $\gamma = n\mathcal{V}/2\pi$, are determined by discrete Fourier component of this realization of the on-site energies. Then the expansion analogous to Eq. (10) takes the form

$$|\ln T| \approx \frac{\mathcal{V}Ln}{\pi}. \quad (25)$$

Then the corresponding expression for \mathcal{V} , analogous to Eq. (11), reads

$$\mathcal{V} = \frac{\pi |\ln T|}{nL}. \quad (26)$$

It is obvious that the typical value of \mathcal{V} is $\sim \Delta$ with variance is $\sim \Delta/n^{1/2}$. It can be demonstrated that the full distribution

of \mathcal{V} is given by

$$p(\mathcal{V}) = \left(\frac{n}{\pi\beta_2\Delta^2} \right)^{1/2} \exp \left[-\frac{n(\mathcal{V} - \beta_1\Delta)^2}{\beta_2\Delta^2} \right], \quad (27)$$

where the constants β_1 and β_2 depend on the actual distribution $P(V)$. For Gaussian $P(V)$ they assume the values $\beta_1 = 4\pi^{-3/2}$ and $\beta_2 = 1/2 - 1/\pi$.

Analogously to Eq. (5), the actual calculation reduces to optimization with respect to L of the product

$$\mathcal{P}_L = \left(\frac{1}{2} \right)^L [p(\mathcal{V})]^{L/n} = \left(\frac{1}{2} \right)^L \left[p \left(\frac{\pi |\ln T|}{nL} \right) \right]^{L/n}, \quad (28)$$

where the power L/n emerges from the product over periods. With Gaussian $p(\mathcal{V})$, given by Eq. (27), this optimization can be performed analytically in a similar fashion as in the previous Section, yielding

$$L_{\text{opt}} = \frac{\pi |\ln T|}{n\Delta\beta_0^{1/2}}, \quad (29)$$

where $\beta_0 = \beta_2 \ln 2 + \beta_1^2$ is the constant of the order of 1. The corresponding value of the gap width is

$$\mathcal{V}_{\text{opt}} = \frac{\pi |\ln T|}{nL_{\text{opt}}} = \Delta\beta_0^{1/2}. \quad (30)$$

The resulting Eq. (29) is quite similar to Eq. (22), and differs only by replacement of the logarithmic factor 2Δ by a constant β_0 , which is of the order of 1. Correspondingly, the final results for $\mathcal{P}(\ln T, \mathcal{L})$ are quite similar to Eqs. (20), (21):

$$|\ln \mathcal{P}(\ln T, \mathcal{L})| = \beta_{\text{eff}} \left(\frac{2\pi^3 G_n}{\sqrt{\epsilon_n}} \right)^{1/2} |\ln T|, \quad (31)$$

$$\frac{\mathcal{L}}{\pi} \left(\frac{\beta_0 \sqrt{\epsilon_n}}{\pi G_n} \right)^{1/2} > |\ln T| > \left(\frac{\sqrt{\epsilon_n}}{\pi G_n} \right) \mathcal{L},$$

$$|\ln \mathcal{P}(\ln T, \mathcal{L})| = \left(\frac{\pi^3 G_n}{\beta_2 \epsilon_n^{1/2} \mathcal{L}} \right) \ln^2 T - \left(\frac{2\pi^3 \beta_1^2 G_n}{\beta_2^2 \sqrt{\epsilon_n}} \right)^{1/2} |\ln T| - \left(\frac{\beta_0}{\beta_2} \right) \mathcal{L}, \quad |\ln T| > \frac{\mathcal{L}}{\pi} \left(\frac{\beta_0 \sqrt{\epsilon_n}}{\pi G_n} \right)^{1/2}, \quad (32)$$

where $\beta_{\text{eff}} = [\beta_0^{1/2} - \beta_1]/\beta_2$. For Gaussian distribution of the on-site energies we have $\beta_{\text{eff}} \approx 0.46$. The results Eqs. (31), (32) were obtained assuming that loose mirrors are well-defined entities, in the sense, that the subspace that they constitute within the functional space has a sharp boundary. In the next subsection we examine the width of this boundary and demonstrate that this width is indeed relatively small.

4.2. Tolerance of the loose mirrors

In order to examine to what extent the loose mirrors are well defined, we consider below two generic sources of violation of the sign rigidity, which are illustrated in Fig. 6.

1) We allow the on-site energies within „positive“ periods to assume slightly negative values, restricted by $-W$ (see Fig. 6, *a*), and the on-site energies within „negative“ periods to assume small positive values, restricted by $W \ll \Delta$. This allowance increases exponentially the number of configurations constituting the loose mirrors. On the other hand, such an allowance suppresses the gap. As a result of these competing trends, there exists an optimal value of W , that maximizes $\mathcal{P}(\ln T, \mathcal{L})$.

2) We allow a small portion, κ , of on-site energies to assume the „wrong“ sign preserving their magnitude $\sim \Delta$. This allowance also increases the number of loose mirrors and suppresses the gap. Thus there exists an optimal $\kappa \ll 1$, which we calculate below.

The quantitative characteristics of the „quality“ of the loose mirror is the fluctuation, $\delta\epsilon$, of the gap center due to the above violations, which is analogous to the tolerance δV of Bragg mirror in the previous Section.

The enhancement of the portion of the loose mirrors due to allowance W can be estimated as $(1 + W/\Delta)^{L_{\text{opt}}} \approx \exp(WL_{\text{opt}}/\Delta)$. This is an exponential „gain“ in $\mathcal{P}(\ln T, \mathcal{L})$. The „loss“ due to suppression of the gap, similarly to Eq. (12), can be expressed as $\sim \exp[-L_{\text{opt}}(\delta\epsilon/\Delta)^2]$. The relation between $\delta\epsilon$ and W can be established from Eq. (8). Indeed, all the terms in numerator are of the same sign and of the order of W , while the corresponding terms in denominator are also sign-preserving and $\sim \Delta$. Thus we

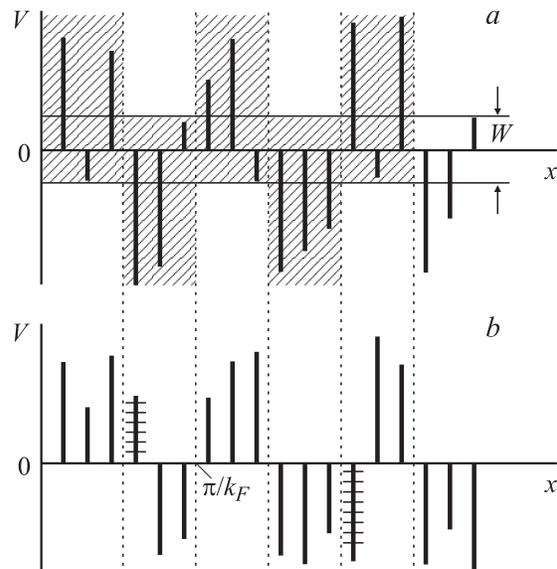


Figure 6. Possible violations of the sign rigidity of a loose mirror: *a* — small ($\sim W \ll \Delta$) on-site energies with „wrong“ sign are allowed; *b* — sparse „large“ ($\sim \Delta$) on-site energies having the „wrong“ sign (hash-marked lines) are allowed.

have $\delta\epsilon \sim (\pi/n)(W/\Delta)$. Finally, the product

$$\begin{aligned} & \exp \left[L_{\text{opt}} \left(\frac{W}{\Delta} \right) \right] \exp \left[-L_{\text{opt}} \left(\frac{\delta\epsilon}{\Delta} \right)^2 \right] \\ &= \exp \left\{ L_{\text{opt}} \left[\left(\frac{W}{\Delta} \right) - \left(\frac{\pi W}{n\Delta^2} \right)^2 \right] \right\} \quad (33) \end{aligned}$$

of the gain and loss has a maximum at $W_{\text{opt}} \approx n^2\Delta^3$. We see, that the allowance is relatively small, since $W_{\text{opt}}/\Delta = n^2\Delta^2 = (1/n)(E_t/\epsilon_n)^{3/2} \ll 1$. This suggests that the allowance does not change the result Eq. (31), since the correction to $\ln \mathcal{P}(\ln T)$ due to allowance, W_{opt} , amounts to a small portion $\sim W_{\text{opt}}/\Delta \ll 1$ of the main exponent Eq. (31).

Strictly speaking, the optimal value of the allowance W is well defined if the exponent in Eq. (33) is much bigger than one. Upon substituting W_{opt} into Eq. (33), we obtain $\delta|\ln \mathcal{P}| \sim n^2\Delta^2 L_{\text{opt}} \approx n\Delta|\ln T|$. From here we conclude, that for large enough $|\ln T| > 1/(n\Delta)$, the allowance W indeed leads to $\delta|\ln \mathcal{P}| \gg 1$. For smaller $|\ln T| \lesssim 1/(n\Delta)$ the gain does not play a role, so that the allowance W is determined exclusively by the second term in the exponent in Eq. (33). This term falls off at characteristic $W = n\Delta^2/L_{\text{opt}}^{1/2}$. This allowance is bigger than W_{opt} , since $W/W_{\text{opt}} \approx (n\Delta|\ln T|)^{-1/2} > 1$, but still smaller than Δ . The latter can be seen if we rewrite $W/\Delta = n\Delta/L_{\text{opt}}^{1/2}$ in the form $W/\Delta \approx (W_{\text{opt}}/\Delta)^{3/4} |\ln T|^{-1/2}$, which is the product of two small numbers. The fact that for small $|\ln T| \lesssim 1/(n\Delta)$ the allowance increase with $|\ln T|$ can be interpreted qualitatively as follows. The smaller is $|\ln T|$, the less effort is required to create a disorder configuration with low transmission T .

The enhancement of the portion of the loose mirrors due to allowance, κ , is equal to $(2^\kappa)^{L_{\text{opt}}/n} = \exp[\kappa(L/n) \ln 2]$. The loss can be estimated analogously to the case (i). Namely, due to the sites with wrong sign of on-site energies the ratio $\sum_{m=1}^n V_m^{(i)} \sin(\pi m/n) / \sum_{m=1}^n V_m^{(i)} \cos(\pi m/n)$ in Eq. (8) is $\sim \kappa$. This yields the estimate $\delta\epsilon \sim (\pi/n)\kappa$, so that, analogously to (i), the product of gain and loss can be written as

$$\exp \left\{ L_{\text{opt}} \left[\kappa - \left(\frac{\pi\kappa}{n\Delta} \right)^2 \right] \right\}. \quad (34)$$

This product is maximal for $\kappa = \kappa_{\text{opt}} = n^2\Delta^2$. We see that $\kappa_{\text{opt}} \approx W_{\text{opt}}/\Delta$, and thus is small, as discussed above.

The fact that the optimal allowances W_{opt}/Δ and κ_{opt} are small justifies that loose mirrors are well-defined entities.

5. Continuous limit

In this Section we establish the relation between the above consideration on the lattice and the results of Refs. [9,10] obtained within the continuous approach. To establish this relation, we restore the lattice constant, a , in the dispersion law, i. e. $\epsilon(k)a^2 = 4 \sin^2(ka/2)$, where k

is the momentum. For lattice constant $a = 1$ the dimensionless parameter that separates the regimes of weakly disturbed (Eqs. (18), (19)) and loose (Eqs. (31), (32)) mirrors was equal to $n\Delta^{2/5}$. To incorporate the arbitrary lattice constant, it is convenient to first express this parameter through the conductance G for $a = 1$. From the relation $G_n = n^{-3}\Delta^{-2}$ we find $\Delta^{2/5} = n^{-3/5}G_n^{-1/5}$. Thus, $n\Delta^{2/5} = n^{2/5}/G_n^{1/5}$. For arbitrary a , the number n should be replaced by $(ka)^{-1}$, while G_n should be replaced by $G(k) = [\epsilon(k)/E_t]^{3/2}$, where E_t is the upper boundary of the tail states. For $a = 1$ this boundary is expressed through Δ as $E_t = \Delta^{4/3}$. Thus, the parameter $n\Delta^{2/5}$ for arbitrary a transforms into $G(k)^{-1/5}(ka)^{-2/5}$. It is seen that this parameter contains the lattice constant and in the white noise limit $a \rightarrow 0$, considered in Refs. [9,10], it is much bigger than 1, which corresponds to the regime of loose mirrors. We thus conclude, that for small a the distribution function $\mathcal{P}(\ln T)$ is given by Eqs. (31), (32). With a restored, these expressions take the form

$$|\ln \mathcal{P}(\ln T, \mathcal{L})| = \beta_{\text{eff}} \left[\frac{2\pi^3 G(k)}{ka} \right]^{1/2} |\ln T|, \quad (35)$$

$$\frac{\beta_0^{1/2}}{\pi^{3/2}} \left[\frac{k\mathcal{L}}{G^{1/2}(ka)^{1/2}} \right] > |\ln T| > \frac{k\mathcal{L}}{\pi G},$$

$$\begin{aligned} |\ln \mathcal{P}(\ln T, \mathcal{L})| &= \left[\frac{\pi^3 G}{\beta_2 \mathcal{L} \sqrt{\epsilon}} \right] \ln^2 T - \left[\frac{2\pi^3 \beta_1^2 G}{\beta_2^2 a \sqrt{\epsilon}} \right]^{1/2} |\ln T| \\ &- \left(\frac{\beta_0}{\beta_2} \right) \frac{\mathcal{L}}{a}, \quad |\ln T| > \frac{\beta_0^{1/2}}{\pi^{3/2}} \left[\frac{k\mathcal{L}}{G^{1/2}(ka)^{1/2}} \right]. \quad (36) \end{aligned}$$

The result of Refs. [9,10] correspond to the first term of Eq. (36). We see, however, that this result obtained neglecting the prefactor does not survive the white-noise limit $a \rightarrow 0$. Formally, taking the prefactor into account shifts the domain of applicability of the log-normal distribution to very small transmission coefficients, $|\ln T| > (\beta_0 k \mathcal{L}^2 / \pi^3 G a)^{1/2}$, so that this domain vanishes when $a \rightarrow 0$. Physically, the result of Refs. [9,10] does not apply in the white-noise limit due to the huge phase fluctuations, that are allowed for small a . These fluctuations forbid the formation of a Bragg mirror with a weakly distorted sinusoidal shape of Fig. 3, *a*. Instead, the relevant fluctuations have the form of loose mirrors shown in Fig. 3, *b*, where only the positions of sign changes are adjusted to the de Broglie wave length, $2\pi/k_F$, of the electron. In this regime the tail of $\mathcal{P}(\ln T)$ is described by a simple exponent Eq. (35) with a non-universal coefficient $\propto a^{-2}$. Clearly, in the „continuous language“, the lattice constant a should be identified with the smallest scale in the problem, namely, the correlation length of the random potential. Thus, we arrive at the conclusion that the correlation length determines the coefficient in front of $|\ln T|$ in the leading term of $\mathcal{P}(\ln T)$. In terms of the correlation length and dimensionless conductance, the portion of the sample occupied by the loose mirror is given by $|\ln T|(Ga/\beta_0 k \mathcal{L}^2)^{1/2}$.

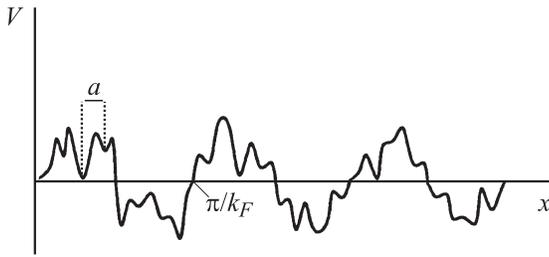


Figure 7. „Continuous“ realization of a loose mirror for short-range disorder with correlation radius $a \ll k_F^{-1}$.

Finally, we establish the energy interval, where the loose mirrors, and thus Eqs. (35), (36), determine the far tail of $\mathcal{P}(\ln T)$. For this purpose, we equate the parameter $G(k)^{-1/5}(ka)^{-2/5}$ to 1 and express the energy E_B in Fig. 4 through the correlation length a as $E_B \approx E_t/(E_t a^2)^{2/5}$. This yields the sought interval

$$E_t < \epsilon < \frac{E_t}{(E_t a^2)^{2/5}}. \quad (37)$$

Recall that E_t , the position of the boundary of the tail states, does not depend on a . This is valid for small a , such that $(E_t a^2) \ll 1$, i.e. the interval (37) is broad. For Eq. (37) to apply, we should require that at the upper boundary of the interval (37) the corresponding momentum, k_B , is much smaller than the inverse correlation length. It is easy to see that this is indeed the case, since $k_B a = a E_B^{1/2} = (E_t a^2)^{3/10} \ll 1$.

Fig. 7 illustrates the main qualitative outcome of our consideration. Namely, for a short-range potential with a correlation radius $a \ll k_F^{-1}$, the low- T disorder configurations for energies within the interval Eq. (37) have the shape of loose mirrors depicted schematically in this figure.

6. Conclusion

In conclusion, let us address the relation between our results and the analytical results in 1D, predicting the shape of $\mathcal{P}(\ln T)$. Neither of the „exact“ techniques [3,19] allows to pinpoint the actual disorder configuration, responsible for low- T values. Although they are believed to be exact, each of these techniques contain a step at which mirror-like configurations are lost. Let us illustrate this point using the Berezinskii technique [3] as an example. In Fig. 8, a three-impurity scattering configuration, employed in Ref. [3] (see also [20]) to make the case for complete localization in 1D, is depicted. As was explained by Berezinskii [3], the key ingredient of the technique in Ref. [3] is the observation that the scattering paths I and II correspond to the same accumulated phase

$$\phi = 2k_F(x_2 + x_3 - 2x_1), \quad (38)$$

and, thus, interfere constructively for any ϕ . However, within the „exact“ technique, the value of ϕ is assumed to

be random, and the averaging over ϕ is performed. Similar procedure is a key element of the technique in Ref. [19]. Calculating the higher-order diagrams in Ref. [11] takes into account increasingly large number of possibilities of constructive interference of different paths, all of which are of the type as in Fig. 8 (correspond to the same accumulated random phase). However, each step involves averaging over this phase. In contrast, the Bragg-like configurations are those sparse realizations, for which the phase accumulated upon traversing the period, π/k_F , first forwards, and then backwards, is not random, but close to 2π . Thus, in our opinion, the complete coincidence of the estimate for $\mathcal{P}(\ln T)$ based on the Bragg mirrors and of the result [11] is accidental.

Finally, let us briefly formulate the main message of the present paper. Creating low- T disorder configuration in 1D demands this configuration to possess a long-range order, adjusted to the wave vector, k_F . Ideal configurations with such a long-range order are of the Bragg-mirror type. However, they are very „costly“ to maintain over a large distance. This is due to the phase fluctuations, those tend to violate the Bragg condition. These fluctuations are not captured at the stage of calculating the saddle point. They show up at the next stage, i.e. calculating the prefactor. We have demonstrated that loose mirrors, illustrated in Fig. 7, in which the long-range order is present, but relaxed are much „cheaper“ to create than the Bragg mirrors. On the other hand, as follows from the analysis that we have performed, a loose mirror, shown in Fig. 7, still constitutes an efficient low- T disorder configuration. The smaller is the correlation radius a of the disorder, the wider is the energy interval within which loose mirrors dominate the low- T tail of the transmission distribution.

Lastly, we are not aware of any numerical work in which the tail of $\mathcal{P}(\ln T)$ was studied close to (but well above) the band edge. Recent simulations are mostly focused at the body of the distribution both in 1D [1,21] and in 2D [22,23], and are aimed at testing the scaling hypothesis. With regard to the tail of the transmission distribution, the related characteristics, namely, the density of anomalously localized states, was studied numerically only in two [24] and three [25–28] dimensions.

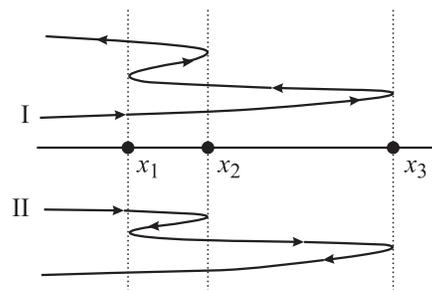


Figure 8. Illustration of the simplest building block of the Berezinskii technique Ref. [3]. In course of moving along the trajectories, I and II, an electron accumulates the same random phase $\phi = 2k_F(x_2 + x_3 - 2x_1)$.

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