

# Transport properties of two-dimensional hole gas in a $\text{Ge}_{1-x}\text{Si}_x/\text{Ge}/\text{Ge}_{1-x}\text{Si}_x$ quantum well in a vicinity of metal–insulator transition

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Observation of a low-temperature transition from metallic ( $\partial\rho/\partial T > 0$ ) to insulator ( $\partial\rho/\partial T < 0$ ) behavior of resistivity  $\rho(T)$  induced by a perpendicular magnetic field  $B$  is reported for a two-dimensional (2D) hole system confined within Ge layers of a  $p\text{-Ge}_{1-x}\text{Si}_x/\text{Ge}/\text{Ge}_{1-x}\text{Si}_x$  heterostructure. Essential feature of this system is that it is described by the Luttinger Hamiltonian with the  $g$ -factor highly anisotropic for orientations of magnetic field perpendicular and parallel to the 2D plane ( $g_{\perp} \gg g_{\parallel}$ ). The positive magnetoresistance revealed scales as a function of  $B/T$ . We attribute this finding to suppression of the triplet channel of electron-electron (hole-hole) interaction due to Zeeman splitting in the hole spectrum.

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## 1. Introduction

For the diffusive motion of electron in disordered conductors the quantum corrections to the Drude conductivity  $\sigma_0$  appear due to both the single-particle weak localization (WL) effects and the disorder modified electron-electron interaction (EEI) [1,2]. Recently an observation of apparent metal-insulator in high mobility semiconductor heterostructures (see the pioneer work [3] and references [4,5] for an extensive review) has provoked a breakthrough in the theory of EEI effects for two-dimensional (2D) disordered systems [6,7]. A general theory of the interaction induced quantum corrections to the conductivity tensor of 2D electrons is developed at  $kT \ll \varepsilon_F$  for arbitrary relation between  $kT$  and  $\hbar/\tau$  (where  $\tau$  is the elastic mean free time  $\varepsilon_F$  — the Fermi energy), in the whole range of temperatures from diffusive ( $kT\tau/\hbar \ll 1$ ) to the ballistic ( $kT\tau/\hbar \gg 1$ ) regimes both for short-range (point-like) [6] and long-range (smooth) [7] random impurity potentials.

According to these latest theories, a linear increase of resistivity  $\rho$  with temperature in high-mobility Si-MOSFETs at large values of  $\sigma \gg e^2/h$ , which for a decade has been considered as a signature of the „anomalous metallic“ state, can now be described quantitatively in terms of interaction effects in the ballistic regime [8]. But the non-monotonous temperature dependence of  $\rho(T)$  near the conjectural conductor–insulator transition (at  $\sigma \geq e^2/h$ ) [8–10] does not have yet a generally accepted understanding. It is the subject of our investigation, realized on  $p\text{-Ge}_{1-x}\text{Si}_x/\text{Ge}/\text{Ge}_{1-x}\text{Si}_x$  heterostructures.

Also we investigate magnetoresistance in a perpendicular to the 2D plane magnetic field  $B$  where both the Zeeman splitting and weak localisation dephasing effects should be taken into account. We extensively use some ideas exploited for interpretation of the experimental data on  $\rho(B, T)$  dependencies of samples with parameters in a vicinity of conjectural conductor–insulator transition in high-mobility 2D semiconductor systems [9,11–13].

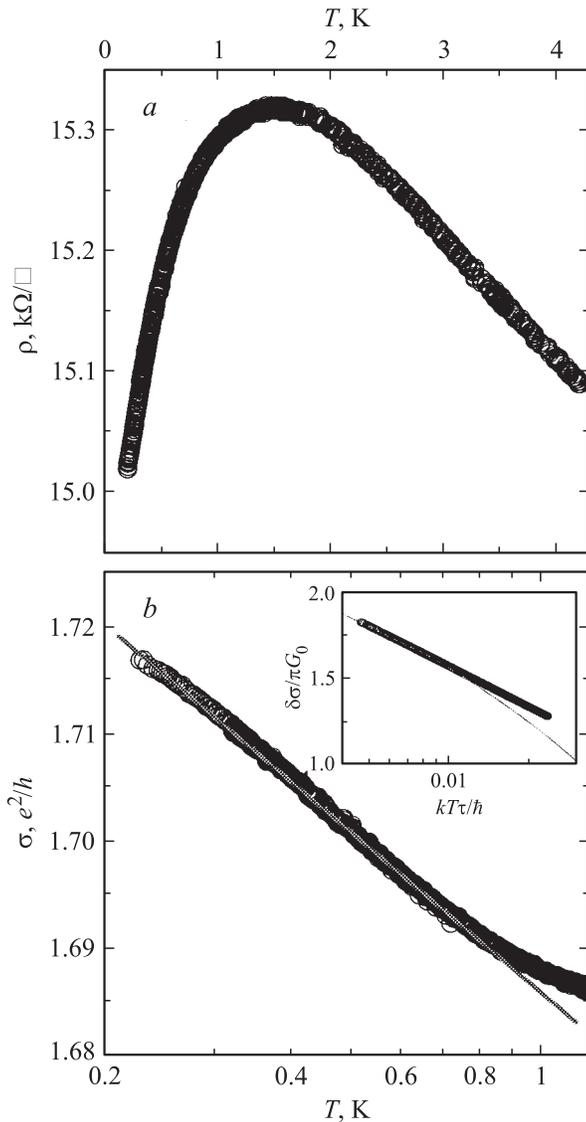
## 2. Experimental results and discussion

Transport properties of the multilayer  $p\text{-Ge}/\text{Ge}_{1-x}\text{Si}_x$  heterostructures with the quantum well (Ge layer, QW) width  $d_w \gtrsim 150 \text{ \AA}$  and hole density per layer  $p_s \gtrsim 3 \times 10^{11} \text{ cm}^{-2}$  have been investigated in our previous works [14 a,b]. Here the experimental data are presented and analyzed for two similar samples of the same system with narrower —  $d_w = 80 \text{ \AA}$  — Ge layers and lower  $p_s = 1.1(1.4) \times 10^{11} \text{ cm}^{-2}$ . Other structural parameters are: the number of  $(\text{Ge} + \text{Ge}_{1-x}\text{Si}_x)$  periods  $N = 51$ ;  $x \approx 0.20$ ; the barrier  $(\text{Ge}_{1-x}\text{Si}_x)$  width  $d_b = 150 \text{ \AA}$ . This width is large enough to treat the barriers as impenetrable for holes and Ge QWs as independent, in accordance with our quantum Hall effect data. The hole mobility  $\mu = 4.0(3.1) \times 10^3 \text{ cm}^2/(\text{V} \cdot \text{s})$  [the latter and  $p_s$  obtained from zero-field resistivity  $\rho_0 = 15(16)k\Omega/\square$  (per layer) and low field Hall  $\rho_{xy}(B)$  at 4.2 K]. This give  $\varepsilon_F\tau/\hbar = 0.85(0.8)$ . The central part of each barrier is doped with boron at concentration  $\sim 10^{17} \text{ cm}^{-3}$  so that the undoped spacers of about  $44 \text{ \AA}$  are on its both sides. The longitudinal and Hall resistivities have been investigated in magnetic fields configured perpendicular to the layers  $B \leq 5 \text{ T}$  at

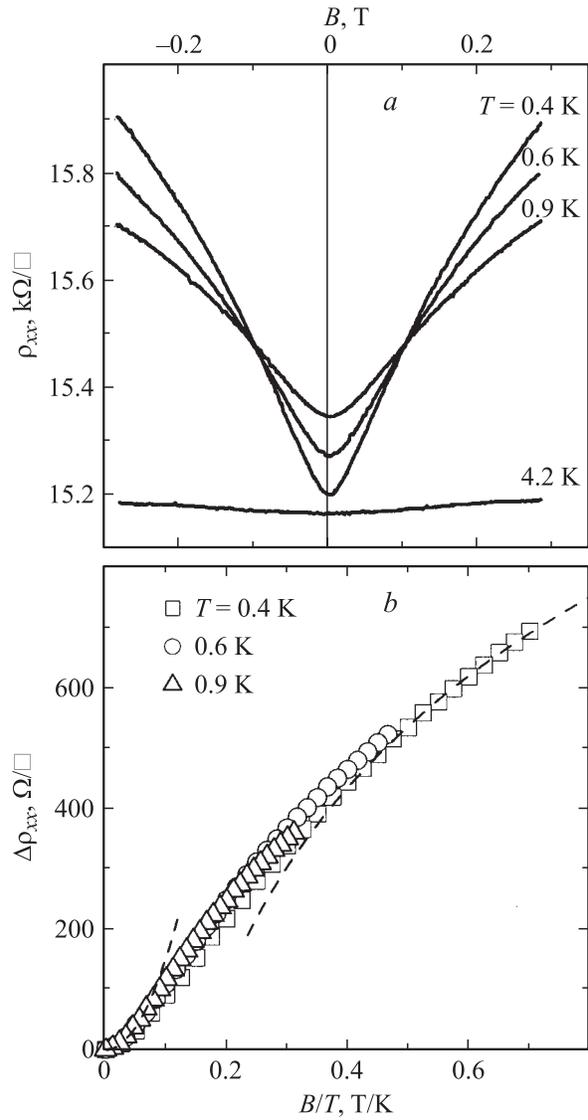
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temperatures  $T = 0.2\text{--}4.2\text{ K}$ . The data for the two samples are alike, so we shall concentrate on results for one on them. A brief description of the experimental results with preliminary interpretation has been done in the Materials of the 15th Ural International Winter School on the Physics of Semiconductors [14 c].

In our previous works [14 b] on samples of the same heterosystem with higher hole densities and mobilities ( $k_F\lambda = 2\varepsilon_F\tau/\hbar \gtrsim 10$ ,  $k_F$  — the Fermi quasimoment,  $\lambda$  — the mean free path) the logarithmic drop of  $\rho$  with temperature increase up to 20 K has only been observed. For the samples investigated here, having  $k_F\lambda \gtrsim 1$ , a non-monotonous temperature behavior of zero-field resistivity is revealed (Fig. 1, a). The „metallic“ behavior ( $\partial\rho/\partial T > 0$ )



**Figure 1.** *a* — temperature dependence of zero-field resistivity, *b* — zero-field conductivity as a function of  $\ln T$ . Solid line is the fit by Eq. (7) with  $p = 1$  and  $\lambda = 0.69$ . Inset to Fig. 1, *b*: points — experimental data for  $T = 0.2\text{--}1\text{ K}$ ; line — the result of calculations according to Eqs. (2), (3) with  $\gamma_2 = 2.2$  ( $F_0^\sigma = -0.69$ ).



**Figure 2.** *a* — dependences of resistivity on perpendicular to plane magnetic field at different temperatures. *b* — magnetoresistivity as a function of  $B/T$ . Dashed lines are the fit by Eqs. (3) and (6) in the low field and high field limits.

takes place from 0.2 to 1.5 K and changes to the „insulating“ behavior ( $\partial\rho/\partial T < 0$ ) at higher temperatures. In the „metallic“ region at  $T \lesssim 1\text{ K}$ ,  $\rho(T)$  dependence may be described by the logarithmic law (Fig. 1, *b*).

Fig. 2 shows magnetoresistance (MR) data at  $T = 0.4\text{--}0.9\text{ K}$  and  $T = 4.2\text{ K}$ . Note that *positive* magnetoresistance (PMR) is observed at all temperatures and the upturn of MR is the sharper the lower is temperature (Fig. 2, *a*). For the lowest temperatures  $T \leq 0.9\text{ K}$ , the magnetoresistance curves scales as a function of  $B/T$  (Fig. 2, *b*).

The condition  $\varepsilon_F\tau/\hbar \approx 1$  for the samples investigated in this study, formally indicates that we are near the critical region of the low-temperature transition from insulating to conducting behavior, which has been seen experimentally in

a variety of high-mobility semiconductor systems [3–5]. We will take it into account in our analysis.

## 2.1. $\rho(B, T)$ dependencies at $T < 1\text{ K}$

It is essential that at  $\varepsilon_F\tau/\hbar \approx 1$  the temperature range  $kT \ll \varepsilon_F$  inevitably corresponds to the diffusive regime for EEI effect,  $kT\tau/\hbar \ll 1$ . Using the Shubnikov–de-Haas data for effective mass  $m/m_0 = 0.08$  ( $m_0$  is the free electron mass) [14a] we have Fermi energy  $\varepsilon_F \approx 3.0\text{ meV}$  and  $\hbar/\tau \approx 3.5\text{ meV}$  (the elastic mean free time  $\tau \approx 1.9 \times 10^{-13}\text{ s}$ ) for investigated sample. Then estimation gives  $kT\tau/\hbar \approx 2.5 \times 10^{-2}T[\text{K}]$  and, hence, at  $T \lesssim 1\text{ K}$  we have:  $kT\tau/\hbar \lesssim 0.025$  (the more detailed estimations see below).

The observed resistivity dependences  $\rho(B, T)$  may be attributed to the quantum conductivity corrections due to both WL  $\delta\sigma_{\text{WL}}$  and EEI  $\delta\sigma_{\text{ee}}$ . For the interaction effect at  $B = 0$ , we have [6]:

$$\delta\sigma_{\text{ee}} = \delta\sigma_{\text{c}} + \delta\sigma_{\text{T}}, \quad (1)$$

where

$$\delta\sigma_{\text{c}} = \frac{e^2}{2\pi^2\hbar} \left\{ \frac{2\pi kT\tau}{\hbar} \left[ 1 - \frac{3}{8} f(T\tau) \right] + \ln \frac{kT\tau}{\hbar} \right\} \quad (2)$$

is the charge channel correction (which combines the Fock and singlet part of Hartree contributions) and

$$\delta\sigma_{\text{T}} = \frac{e^2}{2\pi^2\hbar} \left\{ -3\gamma_2 \frac{2\pi kT\tau}{\hbar} \left[ 1 - \frac{3}{8} t(T\tau, \gamma_2) \right] - 3\lambda \ln \frac{kT\tau}{\hbar} \right\} \quad (3)$$

is the triplet channel correction (triplet part of the Hartree contribution). Contributions from the charge and triplet channels have different sign favoring lotalization or antilocalization, respectively.

Here the parameter  $\gamma_2$  is the Fermi-liquid amplitude normalized by the density of states [9,15],

$$\lambda = \frac{1 + \gamma_2}{\gamma_2} \ln(1 + \gamma_2) - 1 \quad (4)$$

and the dimensionless functions  $f(x)$  and  $t(x, \gamma_2)$  describe the crossover between ballistic ( $kT\tau/\hbar \gg 1$ ) and diffusive ( $kT\tau/\hbar \ll 1$ ) regimes. In designations of Ref. 6:

$$\gamma_2 = -\frac{F_0^\sigma}{1 + F_0^\sigma}, \quad -1 < F_0^\sigma < 0.$$

In the diffusive approximation, electron-electron (e–e) quantum correction (the second terms in Eqs. (2), (3)) have been well known earlier [1,2,15].

For WL effect at  $B = 0$  and  $\tau_\varphi \gg \tau$ , we have [1,2]:

$$\delta\sigma_{\text{WL}}(T) = -\frac{e^2}{2\pi^2\hbar} \beta \ln \frac{\tau_\varphi}{\tau}, \quad (5)$$

where  $\tau_\varphi(T) \propto T^{-p}$  ( $p > 0$ ) is a phase breaking time and prefactor  $\beta$  is 1 in the first approximation in  $1/k_F\lambda$ . In equation (5) we disregarded a mechanism of antilocalization

due to the heavy- and light hole sates mixing developed in [16,17] for  $p$ -type quantum wells with strong spin-orbit interaction because of a smallness of parameter  $p_s d_w^2 = 0.07$  in our system. At intermediate and small values of  $k_F\lambda$ , the higher-order corrections to (5) should be taken into account. In a recent work [18] the terms of the second and third orders in  $1/k_F\lambda$  are shown to cancel out in the interference correction at zero magnetic field. This means that the temperature dependence of  $\delta\sigma_{\text{WL}}$  at  $B = 0$  is just the same as for the case  $k_F\lambda \gg 1$ , down to low enough values of

$$\sigma = (2 - 3) \frac{e^2}{2\pi^2\hbar}$$

— see Eq. (5) with  $\beta = 1$ .

For temperature dependent part of conductivity at  $B = 0$ , we then have from (2), (3) and (5):

$$\delta\sigma(T) = \delta\sigma_{\text{c}}(T) + \delta\sigma_{\text{T}}(T) + \delta\sigma_{\text{WL}}(T). \quad (6)$$

Comparing the observed temperature dependence of  $\sigma$  in a region of „metallic“ conductivity at  $T < 1\text{ K}$  (see the inset to Fig. 1, *b*) with Eq. (6) we conclude that such a behavior is an evidence of the predominant role of the antilocalizing contribution by triplet channel  $\delta\sigma_{\text{T}}(T)$ . From fitting of (6) with  $p = 1$  [2,19],  $\beta = 1$  to the experimental curve, we have  $\gamma_2 = 2.2$  or  $F_0^\sigma = -\gamma_2/(1 + \gamma_2) = -0.69$ .

Now let us estimate the relative contributions of „ballistic“ (linear in  $T$ ) and „diffusive“ (logarithmic) terms in our case. It follows from Eqs. (2) and (3) that at  $T < 0.8\text{ K}$  ( $kT\tau/\hbar < 0.02$ ) the linear in  $T$  contributions amount to less than 4% for the charge channel and less than 8% for the triplet channel (with  $F_0^\sigma = -0.69$ ). Hence, we are in effect at a diffusive limit in the low temperature region,  $T \lesssim 1\text{ K}$ .

By the way, in a diffusive approximation it is convenient to estimate the Fermi-liquid parameter  $\gamma_2$  using the following simple expression [9]:

$$\frac{d\tilde{\sigma}}{d\xi} = \beta p + 1 - 3\lambda, \quad (7)$$

where  $\tilde{\sigma} = (2\pi^2\hbar/e^2)\sigma$ ,  $\xi = \ln(kT\tau/\hbar)$ . Fitting Eq. (7) to the experimental points at  $T < 1\text{ K}$  (see Fig. 1, *b*) gives  $\lambda = 0.69$ , and, according to Eq. (4), we have the same value of  $\gamma_2 = 2.2$  with the corresponding accuracy ( $\lesssim 10\%$ ).

The dependence of  $\delta\sigma_{\text{WL}}$  on a perpendicular field at  $B \ll B_{\text{tr}}$ ,  $B_\varphi \ll B_{\text{tr}}$  (with the so called transport field  $B_{\text{tr}} = \hbar c/4eD\tau$ ,  $B_\varphi = \hbar c/4dD\tau_\varphi$ ,  $D$  — diffusion constant) is described by the expression [20]:

$$\delta\sigma_{\text{WL}}(B) = \alpha \frac{e^2}{2\pi^2\hbar} \left[ \Psi \left( \frac{1}{2} + \frac{B_\varphi}{B} \right) - \ln \frac{B_\varphi}{B} \right] \quad (8)$$

with  $\Psi(x)$  — the Digamma function (see also the note to Eq. (5) about a spin-orbit contribution).

The calculation to the first order in  $1/k_F\lambda$  gives the prefactor  $\alpha = 1$  (without the spin-orbit interaction and with no magnetic impurities) [20]. The second-order terms in

$1/k_F\lambda$ , both in the WL contribution and in the correction induced by the mutual effect of WL and  $e-e$  interaction, have been analyzed in Ref. 18. It was shown that for  $B_T \ll B \ll B_{tr}$  (where  $B_T = B_\phi x$ ,  $x \equiv kT\tau_\phi/\hbar$ ) the shape of the magnetic field dependence of  $\delta\sigma_{WL}$  remains the same as in Eq. (8) but the prefactor  $\alpha$  is modified in the following way:

$$\alpha = 1 - 2 \frac{G_0}{\sigma} \quad (9)$$

with  $G_0 \equiv e^2/2\pi^2\hbar$ . As it will be shown below, in our case we have  $kT\tau_\phi/\hbar \approx 1$  and  $B_T \approx B_\phi$ . At  $B \ll B_\phi$  the prefactor  $\alpha \approx 1$ , as for  $B = 0$  [18].

For our sample  $B_{tr} \approx 1.5$  T, so that at  $B \leq 0.3$  T the inequality  $B \ll B_{tr}$  is rather well satisfied. Eq. (8) leads to a *negative* magnetoresistance (NMR) due to suppression of WL by a magnetic field (dephasing effect). Note that  $\delta\sigma_{WL}$  depends on the ratio  $B/B_\phi$  and thus for  $p = 1$  it is a function of  $B/T$  only.

In our systems with a large  $g$ -factor, spin-splitting effects should be taken into account. For the  $B$ -dependent (Zeeman) part of the total EEI contribution we have [1,21–23]:

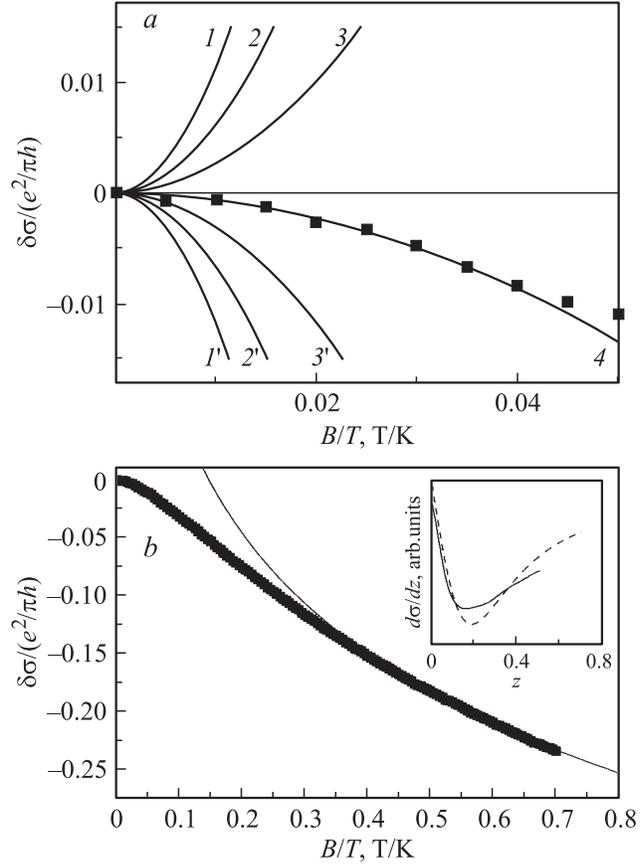
$$\delta\sigma_{ee}(B, T) \equiv \delta\sigma_z(B, T) = \frac{e^2}{2\pi^2\hbar} G(B/B_z) \quad (10)$$

with  $B_z \equiv kT/g\mu_B$ ,  $g$  — is the electron Lande factor and  $\mu_B$  is the Bohr magneton.

The function  $G(B/B_z)$  in Eq. (10) describes the effect of Zeeman splitting of EEI that leads to a PMR due to suppression of a great part of antilocalizing triplet contribution into  $\delta\sigma_{ee}$ . The expression for it in diffusive approximation was first deduced by Lee and Ramakrishnan for weak EEI ( $\gamma_2 \ll 1$ ) [21] and then by Castellani, Di Castro and Lee for any value of  $\gamma_2$  [22]. At present, the  $G(B/B_z)$  expression for arbitrary strength of interaction is anew derived, both in diffusive and ballistic regimes [23].

Important is that, according to Ref. 23 (see formulas (9), (10) of that reference), the argument of  $G$ -function,  $B/B_z = g\mu_B B/kT$ , depends only on the bare electron  $g$ -factor, which is not renormalized by the Fermi liquid EEI (for a given semiconductor, it is the effective electron  $g$ -factor). Hole gas in Ge quantum wells for investigated  $p$ -Ge/Ge $_{1-x}$ Si $_x$  heterostructures is described by Luttinger Hamiltonian with effective  $g$ -factor highly anisotropic relative to mutual orientation of magnetic field and 2D plane: at the bottom of the ground hole subband  $g_\perp = 6\kappa \approx 20.4$  (where Luttinger parameter  $\kappa \approx 3.4$  for Ge [24]) and  $g_\parallel = 0$  for magnetic fields perpendicular ( $B_\perp$ ) and parallel ( $B_\parallel$ ) to 2D plane, respectively [25,26].

We have found that a dependence of  $\rho$  on magnetic field, namely, on a ratio  $B/T$  (see Fig. 2, *b*), may be quantitatively described only by a combination of both PMR due to Zeeman splitting and NMR due to WL dephasing effect with some predominance of the first one. At  $\omega_c\tau \ll 1$  for



**Figure 3.** *a* — the best fit of Eq. (8) (solid line 4) to experimental data for  $\delta\sigma$  at  $T = 0.4$  K (points) in the weak field limit. Curves 1–3 and 1’–3’ show WL and EEI contributions, respectively: for  $g = 20$ ,  $B_z/B_\phi = 3.9$  (curves 1, 1’);  $g = 14$ ,  $B_z/B_\phi = 3.8$  (curves 2, 2’) and  $g = 10$ ,  $B_z/B_\phi = 3.7$  (curves 3, 3’). *b* — the best fit of Eq. (12) with  $\lambda = 0.69$  and  $\alpha = 0.7$  (solid line) to experimental data for  $\delta\sigma$  at  $T = 0.4$  K (points) in the high field limit. Inset shows the fit of theoretical expression (dashed line) to experimental data (solid line) for the first derivative  $\partial\sigma(z)/\partial z$  ( $z \equiv B/T$ ) for  $g = 14$ .

magnetoresistance  $\Delta\rho_{xx} = \rho_{xx}(B, T) - \rho_{xx}(0, T)$ , we have:

$$\Delta\rho_{xx}(B, T)/\rho_0 = -\delta\sigma(B, T)/\sigma_0$$

and fitting of formulas (8) and (10) to  $\Delta\rho_{xx}(B/T)$  dependence gives an opportunity to estimate both the  $g$ -factor and dephasing time  $\tau_\phi$  (at  $B < 0.3$  T,  $\omega_c\tau < 0.1$ ).

In a weak field limit  $B \ll B_z$  [22,23],  $B \ll B_\phi$  [20], we obtain for  $\delta\sigma(B) = \delta\sigma_z(B) + \delta\sigma_{WL}(B)$  from (8) and (10):

$$\delta\sigma(B, T) = \frac{e^2}{2\pi^2\hbar} \left[ -0.091\gamma_2(1 + \gamma_2) + 0.042\left(\frac{B_z}{B_\phi}\right)^2 \right] \times \left(\frac{g\mu_B}{k}\right)^2 \left(\frac{B}{T}\right)^2, \quad (11)$$

where a ratio  $B_z/B_\phi$  is  $T$ -independent (see Appendix).

A fitting of (11) to experimental data with  $\gamma_2 = 2.2$  and varying values of  $g$ -factor is presented on Fig. 3, *a*.

It gives  $B_z/B_\phi = 3.7, 3.8$  and  $3.9$  for  $g = 10, 14$  and  $20$ , respectively. As seen, the parameter  $B_z/B_\phi$  only weakly varies with variation of  $g$ .

From position of a bending point on the  $\delta\sigma(B/T)$  dependence (a position of the minimum for a first derivative  $\partial\sigma(z)/\partial z$  with  $z \equiv B/T$  on the inset of Fig. 3, *b*), which is very sensitive to the  $g$ -factor value, we have an opportunity to completely define the parameter values. The best fit with experimental data gives  $g = 14$  with accuracy about ten percent. That, bearing in mind the weak field analysis, fixes the value of  $B_z/B_\phi = 3.8$ . Finally, for the known parameters  $g = 14, m/m_0 = 0.08$  and  $\varepsilon_F\tau/\hbar = 0.85$ , we obtain from Eq. (A.1):  $kT\tau_\phi/\hbar \approx 0.65$ .

The obtained value of  $g$ -factor is somewhat lower than the theoretical result for  $\varepsilon_F \rightarrow 0$  ( $g_\perp = 20.4$ ) that may be caused by a nonparabolicity of the ground hole subband in the Ge quantum well. This result is in accordance with the experimental value of effective mass  $m = 0.08m_0$ , which due to nonparabolicity is higher than the theoretical value  $m = 0.056m_0$  on the bottom of the first hole subband in Ge [14*a*].

The estimation for  $\tau_\phi$  is in a rather good accordance with numerical solution of Eq. (A.2): for  $\gamma_2 = 2.2$  and  $\varepsilon_F\tau/\hbar = 0.85$  we have  $x = 1.3$ . The main result of fitting is that we obtain a right order of magnitudes for both the  $g$ -factor and  $\tau_\phi$ .

The value of prefactor  $\alpha$  may be estimated from a high field data, as according to Eqs. (8) and (10) at  $B \gg B_z$  [22,23],  $B \gg B_\phi$  [20], the expression for  $\delta\sigma$  reduces to

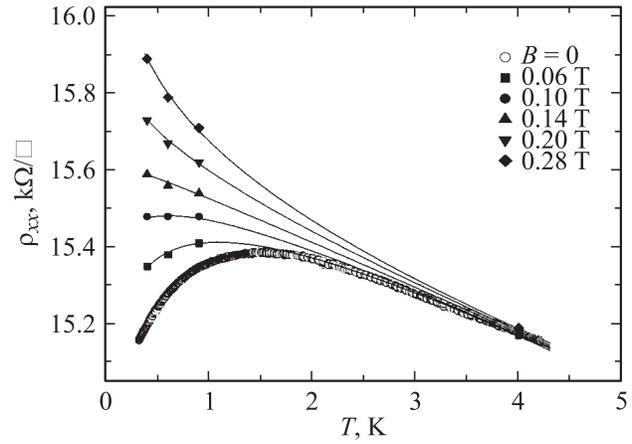
$$\frac{2\pi^2\hbar}{e^2} \delta\sigma(B) = \text{const} + (\alpha - 2\lambda) \ln \frac{B}{T}. \quad (12)$$

From the fit of Eq. 12 to experimental data in the high-field limit (see Fig. 3, *b*) we obtain  $\alpha = 0.7$ , in a rather good accordance with the theoretical value (9) for our  $\sigma = 1.7e^2/h$ :  $\alpha = 0.63$ .

## 2.2. Magnetic field induced metal–insulator transition

In Fig. 4 shown is the resistivity of investigated sample as a function of temperature at several fixed *perpendicular* to the 2D-plane magnetic fields between 0 and 0.3 T. It is seen that the effect of magnetic field is mainly observed for  $T < T_{\text{max}}$  where the conducting („metallic“) phase to insulating phase transition takes place at  $B \approx 0.1$  T. We believe that the transition is induced by Zeeman splitting in the electron spectrum that leads to effective suppression of antilocating triplet channel in favor of localizing exchange channel in the total interaction correction  $\delta\sigma_{\text{ce}}$  [21–23].

Suppression of the low-temperature conducting phase by a *parallel* to the 2D-plane magnetic field  $B_\parallel$  has been first observed in high-mobility Si-MOSFET for electron densities near the zero-field conductor–insulator transition [11,27]. Also,  $B_\parallel/T$  scaling of the magnetoconductance has been found [12] and such a behavior attributed just to the Zeeman



**Figure 4.** Magnetoresistivity as a function of temperature in different magnetic fields perpendicular to the 2D-plane.

splitting, with  $\delta\sigma_z(B_\parallel, T)$  dependence being fitted to the form suggested in [22] [see Eq. (11)] for  $\gamma_2 \approx 1.3$ . Effect of Zeeman splitting on the in-plane magnetoconductivity of high-mobility Si-MOSFET in ballistic regime has been investigated by Pudalov et al. [8] and Vitkalov et al. [28].

In an electron system, the Zeeman splitting suppresses a conducting phase independently of the angle between the field and the 2D-plane (see, for example, [27]). But for a *hole* system with highly anisotropic  $g$ -factor ( $g_\parallel \ll g_\perp$ ) this effect in parallel magnetic field should be weakened considerably. In the *perpendicular* to the 2D-plane magnetic field, we investigate a situation where a WL effect should be present equally with interaction correction, and the usual negative WL magnetoresistance should be observed.

Magnetoresistance in *perpendicular* fields for  $p$ -SiGe samples on the metallic side of the  $B = 0$  metal–insulator transition has been investigated by Coleridge et al. [13]. Magnetoresistance shows clear evidence of both quantum interference and Zeeman interaction effects. The initial NMR attributed to dephasing by the magnetic field due to the WL term is followed by a PMR due to the term identified with the Zeeman interaction effects. The Zeeman term, which scales as  $B/T$ , could not be quantitatively described by a conventional theory for a weakly interacting 2D-system [21]. The best fit to the data has been obtained using low- and high-field limits of Castellani et al. theory [22] with the value of  $\gamma_2$  up to 2.6.

In a recent work of Gao et al. [29] on the  $p$ -GaAs system that is metallic at  $T \leq 0.3$  K, only a negative low-field MR in perpendicular fields shows up, so that the WL effect overwhelms the effect of Zeeman splitting observed in  $p$ -GaAs in parallel magnetic field [30,31].

In contrast to electron Si-MOSFET system [27] or hole SiGe [13] and GaAs [29] systems, we do not observe the low-field NMR and believe that this is a consequence of a particular parameter relation characterizing the investigated  $p$ -Ge quantum wells, specifically, of a large value of the

hole band  $g$ -factor. In our systems, mechanisms responsible for both — NMR due to WL and PMR due to Zeeman splitting — coexist in the same range of magnetic fields, the latter being predominant.

### 2.3. Nonmonotonous temperature dependence of resistivity at $B = 0$

There exist at least two approaches in explanation of the nonmonotonous temperature dependence of resistivity in samples with low temperature „metallic“ phase: for pure diffusive [9] and pure ballistic [10] regimes (see review papers [5]).

In the paper of Punnoose and Finkelstein [9] the highly nonmonotonous temperature dependence of resistivity in a Si-MOSFET sample close to the critical region of the metal–insulator transition [ $\rho(T_{\max}) \leq h/e^2$ ] has been well described on a basis of the renormalization group (RG) theory [15,22]. According to Refs. [15,22], the interplay of EEI and disorder leads to such a renormalization of Fermi-liquid parameter  $\gamma_2$  that it increases monotonically as the temperature is lowered. When  $\gamma_2$  increases beyond the value  $\gamma_2^*$ , for which  $(p + 1 - 3\lambda) = 0$  (see Eq. (7)), the resistivity passes through a maximum.

On the other hand, Das Sarma and Hwang [10] have explained a transition from „metallic“ to apparent „insulating“ phase with increasing  $T$  on the basis of quasiclassical theory of the temperature dependent screening of impurity potential [32,33], suggesting that the resistivity maximum is due to a crossover between the Fermi and Boltzmann statistics. In Refs. [6,7] it is argued that this approach has a common physical origin with the EEI effect at  $kT\tau/\hbar \gg 1$ , i.e. in a limit of single-impurity scattering, and that the theory of EEI correction in ballistic regime provides a systematic microscopic of the concept of temperature dependent screening (see Sec. IIIF of Ref. 6 and Sec. IV of Ref. 7).

We believe that in our experiment the crossover between diffusive and ballistic regimes with an assumption of smooth character of random impurity potential may be responsible for the nonmonotonous  $\rho(T)$  dependence. Really, in a smooth disorder (small angle scattering), the EEI contribution in ballistic regime, which is proportional to the return probability after a single-scattering event, vanishes as  $\exp(-k_F d)$ , with  $d$  being a spatial range of random impurity potential [7]. As shown in Refs. [6,7], the crossover between diffusive and ballistic limits should take place at small values of  $kT\tau/\hbar \approx 0.1$ , since the natural dimensionless variable of the theory is  $2\pi kT\tau/\hbar$ . A flat region in  $\rho(B)$  dependencies of a high-mobility  $n$ -GaAs heterostructure at  $T \gtrsim 1.2$  K in fields  $\omega_c\tau < 1$  (after the initial rapid drop of MR due to WL dephasing) is interpreted in [34] as a clear indication that in the ballistic regime the long-range potential suppresses the zero-field interaction correction.

For a range of impurity potential in our structures we have  $k_F d \approx 1$ , where  $d$  is an effective spacer width [25,35], thus the impurity potential is of the long-range nature. So,

Some experimental data for Fermi-liquid interaction parameter in diffusive regime

Semiconductor	$\epsilon_F\tau/\hbar$	$r_s$	$\gamma_2$	$F_0^\sigma$	Reference
Si-MOSFET	1.25	—	3.5	−0.78	[36]
Si-MOSFET	1.3	1.6	3.2	−0.76	[37]
$p$ -SiGe	7.2	4	2.6	−0.72	[13]
$p$ -Ge	0.85	1.75	2.2	−0.69	This work
Si-MOSFET	0.93	5.6	1.3	−0.56	[12]

we believe that a transition to the „insulating“ behavior at  $T > T_{\max}$  (see Fig. 1,  $a$ ) takes place due to a gradual change of regime to ballistic one where the EEI correction at  $B = 0$  is exponentially small [7]:  $\delta\sigma_{ee} \propto \exp[-\text{const}(T\tau)^{1/2}]$  and thus the WL effect becomes predominant.

Note that for *point-like* scatterers the linear-in- $T$  contribution (2), (3):

$$\delta\sigma = \frac{e^2}{2\pi^2\hbar} \left( 1 + \frac{3F_0^\sigma}{1 + F_0^\sigma} \right) \frac{kT\tau}{\hbar} \equiv \frac{e^2}{2\pi^2\hbar} (1 - 3\gamma_2) \frac{kT\tau}{\hbar}$$

will come to light in total  $\delta\sigma_{ee}$  correction with a transition to ballistic regime [6]. For our value of  $\gamma_2$ , we have  $(1 - 3\gamma_2) \approx -5.6$  and this contribution should lead to a steeper increase of  $\rho(T)$  („more metallic“ behavior) with  $T$  increasing, which obviously is not the case in our experiment.

### 3. Conclusions

We think that a mutual compensation of WL and EEI effects takes place for investigated  $p$ -Ge/Ge $_{1-x}$ Si $_x$  heterostructure with parameter values near a nominal 2D „metal–insulator transition“,  $\rho \approx h/e^2$  ( $\epsilon_F\tau/\hbar \approx 1$ ). In the pure diffusive regime  $kT\tau/\hbar < 0.025$  ( $T < 1$  K), the predominance of the antilocalizing triplet channel contribution into EEI correction leads to an apparent metallic behavior,  $\partial\rho/\partial T > 0$ . But with a crossover to the ballistic regime (at  $kT\tau/\hbar \approx 0.1$ ), the gradual reduction of EEI contribution in favor of WL one ( $\partial\rho/\partial T) < 0$  occurs for a smooth or predominantly smooth disorder.

Due to the high value of the Ge valence band  $g$ -factor, the Zeeman splitting in perpendicular to 2D-plane magnetic field causes an effective suppression of the triplet channel contribution and conduces to the insulating behavior of  $\rho(T)$  in the whole temperature interval.

Finally, we compare our results for Fermi-liquid amplitude  $\gamma_2(F_2^\sigma)$  with those obtained from an analysis of experimental data on other semiconductor systems in *diffusive* regime. The highest values of the parameter  $\gamma_2$  reported for low-mobility [36,37] and high-mobility [12] Si-MOSFET as well as for  $p$ -SiGe [13] and  $p$ -Ge (this work) are presented in Table. Here  $r_s = \pi n^{-1/2}/a_B$  is the usual dimensionless Wigner-Seitz interaction parameter with  $n$  as the density of carriers and  $a_B$  as the effective semiconductor Bohr radius. Note that all the values of  $F_0^\sigma$  shown in Table

are appreciably larger (for similar values of  $r_s$ ) than those obtained from an analysis of transport effects in terms of recent EEI theories in ballistic regime [see Ref. 38].

It is also seen that almost all of the data in Table (with an exception of the result of Ref. 13) correspond to a region of nominal metal–insulator transition with  $\varepsilon_F\tau/\hbar \approx 1$ . Then it may be that such a high  $\gamma_2$  value is a consequence of the renormalization of the Fermi-liquid parameter due to an interplay of interaction and disorder in the diffusive regime, which, according to RG theory [9,15,22], is especially significant just in a proximity of  $\rho = h/e^2$ , i.e. for  $\varepsilon_F\tau/\hbar \approx 1$ . On the other hand, an apparent reduction of the interaction amplitude extracted from the temperature dependence of resistivity in the ballistic regime may be related to a mixed (point-like plus smooth) character of the random impurity potential (see Eq. (2.53) of Ref. 7).

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## Appendix A

From the definitions of  $B_z$  and  $B_\phi$  we have

$$\frac{B_z}{B_\phi} = \frac{4eD}{g\mu_{\text{BC}}} x = \frac{8\varepsilon_F\tau/\hbar}{g(m/m_0)} x, \quad (\text{A.1})$$

where  $x = kT\tau_\phi/\hbar$  and the expressions  $\mu_{\text{B}} = e\hbar/2m_0c$  and  $D = \varepsilon_F\tau/m$  are used. Dephasing time in disordered 2D-system due to EEI in diffusive limit is determined by self-consistent equation [2,19]:

$$\Lambda x \ln x = 2\varepsilon_F\tau/\hbar \quad (\text{A.2})$$

with  $\Lambda = 1$  for weak EEI ( $\gamma_2 \ll 1$ ) [2] and

$$\Lambda = 1 + \frac{3\gamma_2^2}{\gamma_2 + 2}$$

at an arbitrary value of  $\gamma_2$  [19] [considering the relation  $F_0^\sigma = -\gamma_2/(1 + \gamma_2)$ ]. The solution of Eq. (A.2) may be written as

$$x = f(\gamma_2, \varepsilon_F\tau/\hbar) \quad (\text{A.3})$$

and for a ratio  $B_z/B_\phi$  we then have

$$\frac{B_z}{B_\phi} = \frac{8\varepsilon_F\tau/\hbar}{g(m/m_0)} f(\gamma_2, \varepsilon_F\tau/\hbar), \quad (\text{A.4})$$

where the right hand part does not depend on temperature.

So, the relation between WL and EEI contributions to the total magnetoconductivity, and thus the sign of MR, should remain constant during the variation of temperature, at least in the weak field region.

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