

Numerical calculation of diffraction losses and mode field distribution of a ring three-mirror optical resonator using the eigenvector method

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Received December 12, 2025

Revised March 29, 2026

Accepted April 21, 2026

The problem of finding the field distribution and mode losses of an optical ring three-mirror resonator is considered. A special feature of the problem is the need to solve it in three-dimensional space and apply numerical methods. The eigenvector method is applied to numerically calculate the field distribution and losses of a ring resonator with round and square diaphragms. The convergence of the method is analyzed and the validation of this method is carried out by comparing it with previously obtained analytical and numerical results. Patterns of the electric field distribution in the resonator and the amount of diffraction losses are obtained and analyzed. The method can be applied to the calculation of more complex ring configurations of resonators, including misaligned resonators.

Keywords: optical ring resonator, diaphragm aperture, eigenvector method, electric field distribution, diffraction losses.

DOI: 10.61011/EOS.2026.05.63865.8879-25

Introduction

Optical resonators are an important laser technology element: they form the structure of generated light and ensure multiple light passage through the active medium for higher amplification, therefore cavity field calculation accuracy is important for achievement of the desired laser parameters. Field and diffraction loss calculation data are especially important for ring lasers used in metrology and navigation because a resonator in this case is a sensitive element [1].

Selection of the best optical and physical resonator configuration becomes a key aspect in creating new laser devices. Existing analytical methods for resonator parameter calculation have exact solution only on the assumption of diaphragm or Gaussian diaphragm aperture infinity, and in other cases give only rough estimate of diffraction loss [1,2]. Therefore, numerical methods shall be used. There are several numerical methods for field determination in a resonator [3–5]. This work used an eigen vector method [5] because it doesn't require high computational power and is the most easy-to-implement [6].

For linear resonators, this method is employed in [7] using a rectangular grid for field breakdown on a diaphragm.

Results obtained by this method for a four-mirror ring nonplanar resonator using the Fibonacci grid are reported in [6].

The objective of this work is to calculate diffraction loss and field distribution of a three-mirror resonator using a rectangular grid of field breakdown on a diaphragm which is most illustrative and easy-to-implement. The calculation was used not only to represent a qualitative field

distribution pattern, but also to calculate eigen frequencies and diffraction loss of the resonator for the first transverse modes, and to investigate the method convergence when varying the number of diaphragm breakdown points.

Theoretical fundamentals of the method

The optical resonator of interest is a three-mirror ring system consisting of two flat and one spherical mirror with a round or square diaphragm. Designations of geometrical parameters of the resonator used in calculations are shown in Figure 1.

Light beam propagation in the resonator is simulated. Consider region dS on surface G on one of the resonator components, for which lightwave electric field U_0 distribution is known. In our case, it is a diaphragm plane, in general case it can be any region. Lightwave field U in another point of a simply connected region with homogeneous medium is represented by the Kirchhoff equation [8]:

$$U = \frac{ik}{4\pi} \iint_G U_0 \frac{e^{ikr}}{r} (1 + \cos \varphi) dS, \quad (1)$$

where r is the distance from the surface area element dS to the point of observation, φ is the angle between the outer normal n to surface G at point of dS and direction to the point of observation.

Consider configuration in Figure 1 in the Cartesian coordinate system, where the z axis is oriented along the beam propagation direction in the resonator, and the Ox and Oy axes are in the beam cross-section plane. Diaphragm

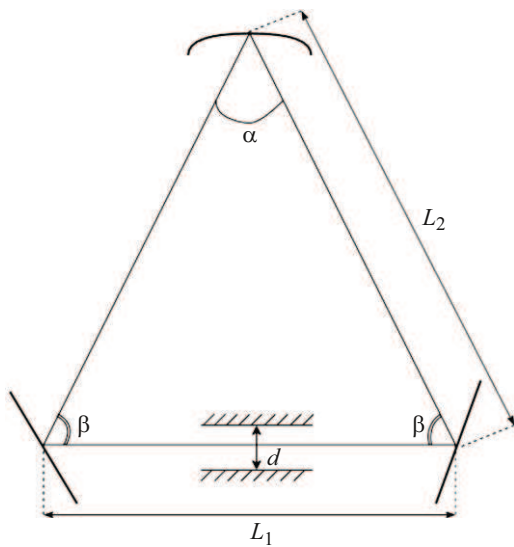


Figure 1. Laser resonator configuration: α — double angle of incidence on a spherical wave front rotation mirror during reflection from the spherical mirror, β — angle during reflection from flat mirrors, d — diaphragm diameter, L_1 and L_2 — resonator dimensions.

cross-section is described by x_0, y_0 . According to [6], $U(x, y)$ on any other resonator element can be found from knowing the distribution $U_0(x_0, y_0)$. For an infinite mirror aperture, intermediate integrals can be omitted, using the ABCD matrix method. Elementary area matrices along the direction of summation are successively multiplied to get a 4×4 ray transfer matrix of the resonator M .

For a free space with the length L , a translation matrix is written as:

$$T(L) = \begin{pmatrix} 1 & 0 & L & 0 \\ 0 & 1 & 0 & L \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Reflection matrix for a spherical mirror with the radius R with incidence at the angle α is written as:

$$T_{sph}(R, \alpha) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{2}{R \cos \alpha} & 0 & 1 & 0 \\ 0 & -\frac{2 \cos \alpha}{R} & 0 & 1 \end{pmatrix}$$

For a three-mirror resonator (Figure 1), the transfer matrix A is formed by multiplication of matrices of all arms and reflections. Matrix A can be conveniently written in a block form:

$$A = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix}$$

Where A_1, B_1, C_1, D_1 are 2×2 submatrices. Following the ABCD matrix analysis, a field on the initial plane after full transfer is given by the Collins integral — paraxial generalization of the Kirchhoff integral for optical systems

described by ray matrices [4,9]. In the vector-matrix form, denoting $r_0 = (x_0, y_0)^T$, $r = (x, y)^T$, we have

$$U(x, y) = \frac{i e^{-ikL}}{\lambda \sqrt{\det(B_1)}} \times \iint U_0(x_0, y_0) e^{-\frac{ik}{\lambda} (r_0 B_1^{-1} A r_0 + r D_1 B_1^{-1} r - 2 r_0 B_1^{-1} r)} dx_0 dy_0, \quad (2)$$

Here L is the total optical path around the resonator, k is the wavenumber and λ is the wavelength.

In accordance with [6], region G can be represented as a grid consisting of n elements, and by replacing the double integral in (1) and (2) with a sum, the Huygens–Fresnel principle can be used to sum the fields from n point sources. Then, we find a field in each grid node:

$$U_j = \frac{ik}{4\pi} \sum_{i=1}^n \frac{e^{-ikr_{ij}}}{r_{ij}} (1 + \cos \varphi_i) ds_i U_i, \quad (3)$$

where r_{ij} is the distance from the grid node U_i to the grid node U_j , ds is the elementary cell area, φ_i is the angle between the radius vector r_{ij} and the vector of normal to the mirror surface in the i -th node. We rewrite equation (6) in the matrix form:

$$U_j[n] = A_{ij}[n \times n] U_i[n]. \quad (4)$$

Considering that field distribution is reproduced when a ray makes one pass around the resonator, eigen vectors and cavity transfer matrices can be found:

$$\gamma_n U[n] = A U[n]. \quad (5)$$

Here, γ_n is the eigenvalue of A , eigen vector U is to be found for each eigenvalue and is responsible for transverse field distribution in the resonator. Spectrum is characterised by an imaginary part of the eigen vector $\text{Im}(\gamma_n)$. Intermod distance is calculated as follows

$$\Delta \varphi_i = \frac{c(\text{Im}(\gamma_n) - \text{Im}(\gamma_0))}{2\pi L}, \quad (6)$$

where $\text{Im}(\gamma_0)$ is the imaginary part of the eigenvalue with the largest real part (corresponds to TEM_{00}).

Loss is characterized by the real part of the eigenvalue $\text{Re}(\gamma_n)$. Thus, the following algorithm was developed to find the field distribution and loss:

- 1) grid breakdown of the space on the diaphragm;
- 2) transfer matrix calculation using (4) for the flat and spherical matrices;
- 3) calculation of eigen vectors and final transfer matrices (equation (5)) sorted in descending order of the absolute eigenvalue;
- 4) plotting the curve of field distribution on the diaphragm after passing around the resonator.

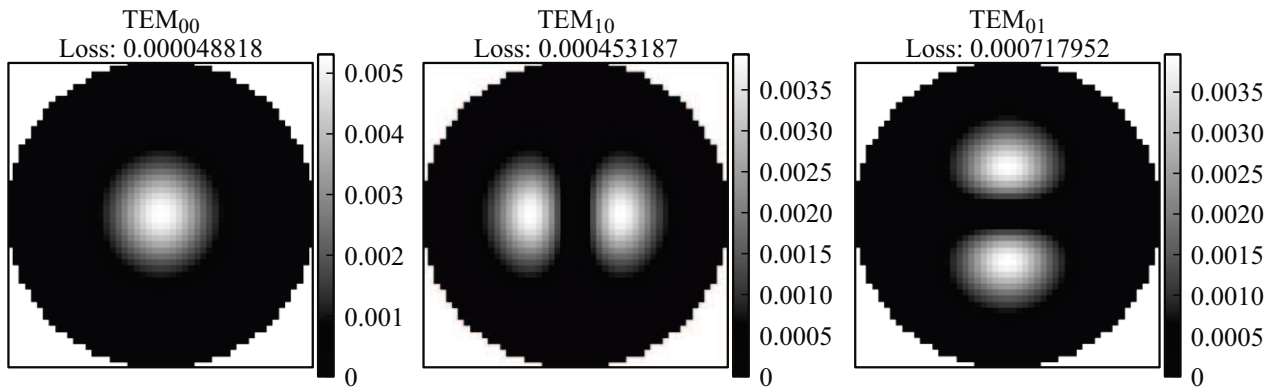


Figure 2. Field distribution and loss for TEM_{00} , TEM_{01} and TEM_{10} modes in a round-diaphragm optical resonator.

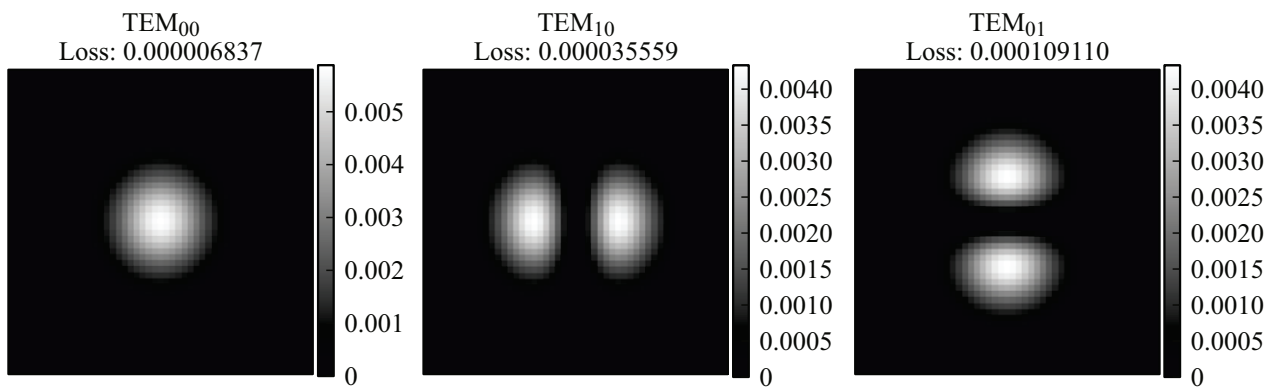


Figure 3. Field distribution and loss for TEM_{00} , TEM_{01} and TEM_{10} modes in a square-diaphragm optical resonator.

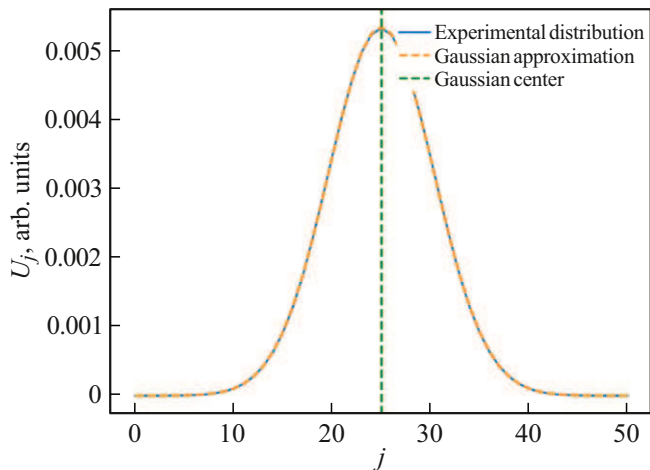


Figure 4. Distribution of the electromagnetic field U_j of TEM_{00} from the breakdown point number j for $n = 50$.

Calculation and parameters

The above-mentioned algorithms was used to write a program in Python to implement the eigen vector method. The program uses the numpy library containing all required mathematical tools for dealing with matrices and finding

their eigen vectors and values. The matplotlib library was used for graphic imaging of the field distribution. The scipy library was used as the basis for curve approximation.

The following system parameters are input into the program: λ is the wavelength in mm, r is the radius of the round diaphragm in mm, n is the number of elementary cells during breakdown, L is the resonator length in mm, R is the radius of spherical mirror curvature in mm, α is the angle at the resonator vertex in degrees. The program outputs a field distribution pattern in the xOy axes for different modes with the specified loss for each mode, the resonator frequency spectrum is calculated and field distribution curves are displayed in the same way for the 00-mode in the yOz and xOz axes. The obtained curves are approximated by a Gaussian, and the size of the resultant beam is calculated by the Gaussian parameters.

The program was started at different n to clarify the loss. Taking into account quadratic dependence of the computational algorithm complexity $O(n^2)$ on the grid size, the maximum value of n was limited by the available RAM capacity.

Analysis was performed during the study with varying n from 30 to 120 at 5 intervals. Such approach made it possible to study convergence of the calculated loss as the

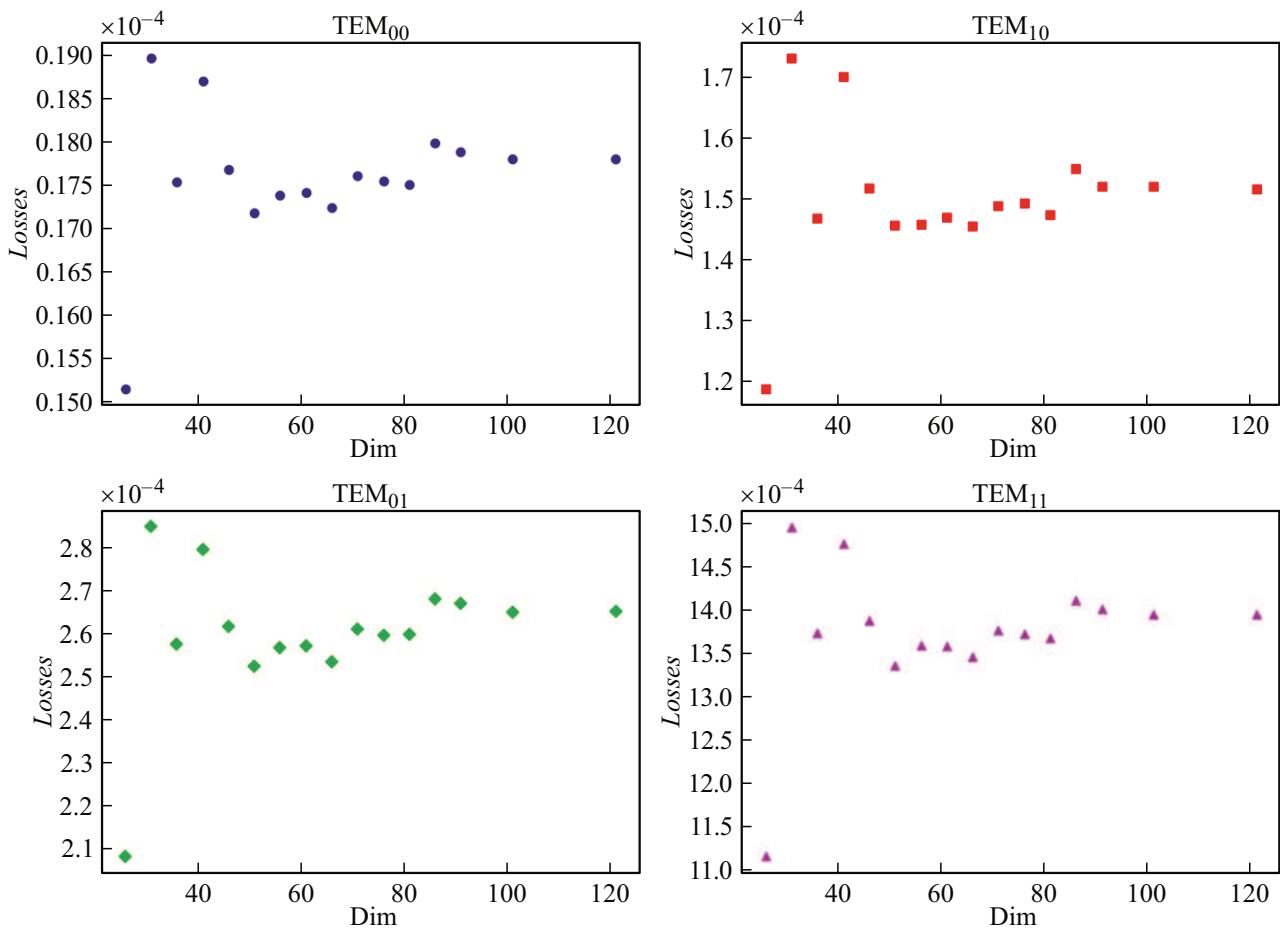


Figure 5. Loss convergence curves for TEM₀₀, TEM₀₁, TEM₁₀ and TEM₁₁ in the round-diaphragm optical resonator.

Comparison of experimental and theoretical data of the calculated resonator frequency spectrum.

Source	$\Delta\varphi_{01}$, MHz	$\Delta\varphi_{10}$, MHz
Theoretical value	53	47
Experimental value	54	49

grid resolution increased, and to determine the best relation between computational accuracy and cost.

Results

As a result, electric field distribution patterns and diffraction loss were obtained for modes shown in Figure 2 and 3. The obtained field distribution patterns correspond to theoretical and experimental data. Distribution view coincides with distribution patterns shown in [10]. The fundamental mode diameter (0.507 mm) measured by the Gaussian parameters using the program output in Figure 4 is close to the beam diameter estimated by the ABCD matrix analysis (0.493 mm) with an accuracy of 3 %.

The obtained loss values were compared with the analytical estimates from [11] for Gaussian beam modes in a linear resonator for a square diaphragm. For correct comparison, a linear resonator equivalent to a ring one was used. The Fresnel number was calculated from the system geometry. For this Fresnel number from [11], theoretical diffraction loss values were determined that coincided in an order of magnitude with the eigen vector analysis results.

Numerical simulation results show that the calculated resonator frequency spectrum (table) meets the data shown in [2]. $\Delta\varphi_{01}$ and $\Delta\varphi_{10}$ in the table are intermode distances for TEM₀₁ and TEM₁₀ calculated using equation (6). Maximum deviation of frequencies from the experimental values for an identical resonator configuration was 3 % max.

The parametric analysis of aperture discretization effect (n is the number of field breakdown elements on the diaphragm) on the calculation accuracy provided the dependences of diffraction loss on the degree of discretization. The curves in Figure 5 show that the diffraction loss tends to steady-state values as n increases, which is indicative of the numerical method convergence.

According to the loss convergence curves in Figure 5, two typical regions can be distinguished.

1. At $n < 50$, there is a significant spread of loss values, which is associated with insufficient grid breakdown where coarse breakdown cannot adequately describe the mode structure. This causes a significant discretization error and, consequently, large fraction of mode power is dropped at the dimension grid boundaries.

2. At $n > 50$, the spread of values decreases significantly, loss values are stabilized. Further increase in n leads to increased computational cost rather than to a significant change of loss.

Conclusion

The work shows that the eigen vector method can be used for numerical solution of an integral equation defining natural frequencies, spatial field distribution and diffraction loss of modes of the three-mirror optical resonator with a round and square diaphragm. A program was developed for eigen vector analysis of field distribution and diffraction loss of the three-mirror resonator. It was found that the method convergence depends on the breakdown of a region where the field is calculated. The optimum number of breakdown elements was determined.

The method was validated by comparing with the known work of Boyd and Gordon [11] and also with the analytical solution for field distribution in case of infinite aperture. Calculated field distribution and diffraction loss data are provided for one of the resonator configurations.

The findings confirm the method efficiency for optical resonator design applications, including resonator geometry optimization and consideration of diffraction effects. Of particular value is the suitability of the developed algorithm for analysis of resonators with an arbitrary aperture geometry and for simulating systems with misaligned resonators, which expands its scope of application compared with analytical methods.

Conflict of interest

The authors declare no conflict of interest.

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Translated by E.Ilyinskaya

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