

Generalized scattering matrix of a radar target

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A two-vibrator model of a radar target with non-orthogonally oriented vibrators is considered. Within the framework of this model, a generalized scattering matrix for a stationary radar target is derived in parametric form using invariant polarization parameters that have a clear physical interpretation.

Keywords: two-vibrator model, non-orthogonally oriented vibrators, rotation operator, invariant polarization parameters.

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The scattering properties of a radar object (RO) in polarization radar scanning are most fully described by a scattering matrix (SM) in a linear polarization basis as the three complex numbers $\dot{S}_{XX}$, $\dot{S}_{YY}$ and $\dot{S}_{XY}$ [1–3]. In its parametric form the scattering matrix of a stable RO in the linear polarization basis is expressed as [3–5]:

$$[S] = \lambda_1 \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} + \lambda_2 \exp(j\Delta\varphi) \begin{bmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ -\sin \theta \cos \theta & \cos^2 \theta \end{bmatrix}, \quad (1)$$

where λ_1 and λ_2 — moduli of eigen values of SM, $\Delta\varphi$ — difference of arguments of SM eigen values, θ sets the orientation of SM eigen coordinate system.

This SM (1) is obtained within a two-vibrator model with orthogonally mutually located vibrators [3,4]. From (1) it follows that a stable RO can be represented by the sum of two orthogonally oriented vibrators, which is true for the class of axisymmetric ROs [3,4]. The physical meaning of SM parameters (1) becomes clear if we turn to the two-vibrator model of RO, where the eigen values of SM λ_1 and λ_2 have the meaning of the vibrators' effective length, and their squares are the effective scattering surface of the object. The difference in the arguments of the eigen values of SM $\Delta\varphi$ is equal to the phase length of the path between the vibrators $\Delta R = \Delta\varphi\lambda/4\pi$ [3,4]. The orientation of the SM's own coordinate system coincides with the orientation of the vibrators for the degenerate object. In general, the parameter θ is determined by configuration of the object, its orientation in space and electrophysical characteristics [3,4].

If we assume that $\lambda_1 > \lambda_2$, then, taking out from SM (1) the modulus of the greater eigen value λ_1 , we'll get SM in

the parameter form

$$[S] = \sqrt{\sigma_m} \left\{ \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} + \rho \exp(j\Delta\varphi) \begin{bmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ -\sin \theta \cos \theta & \cos^2 \theta \end{bmatrix} \right\}, \quad (2)$$

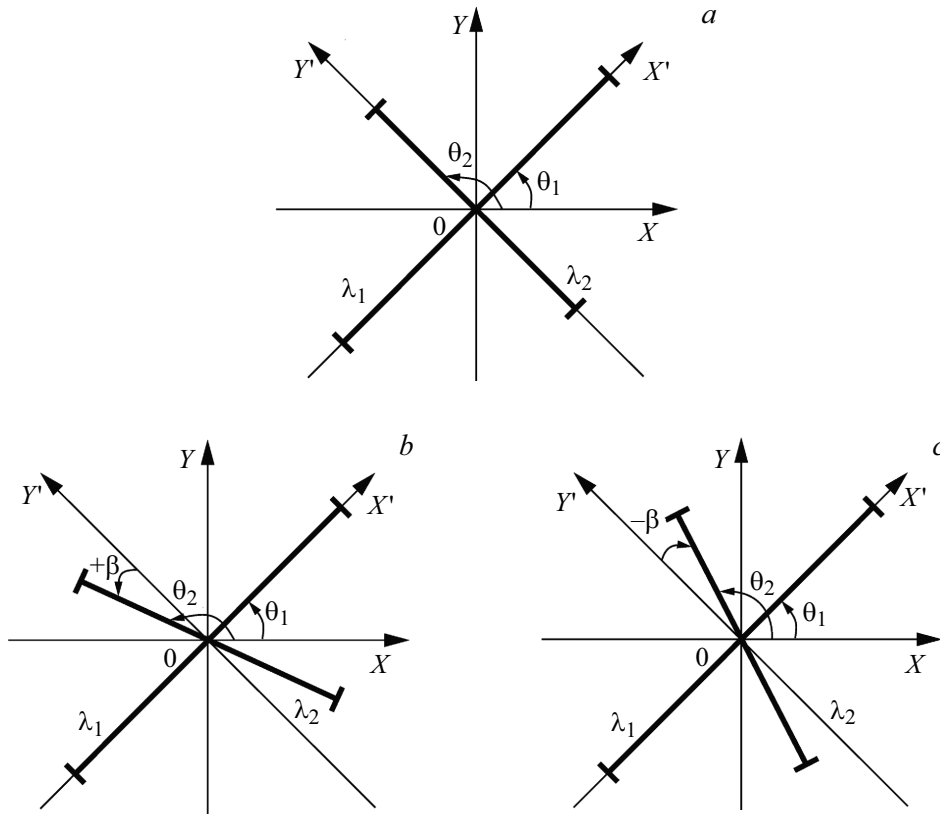
where $\sigma_m = \lambda_1^2$ is the maximum possible effective scattering surface of an object when irradiated by a linearly polarized wave, $\rho = \lambda_2/\lambda_1$ is the shape factor of the object.

From (2) it is clear that in such a representation the conventional SM of a stable object of three complex numbers can be replaced by a set of four real numbers. The set includes one energy parameter σ_m and three polarization parameters (PP) ρ , $\Delta\varphi$ and θ . These PP completely describe the polarization characteristics of a certain class of symmetric objects, which include solids of revolution [2–5]. Since PP are independant from the SM setting mode (2), these are the invariant PP [3–5]. SM (2) can be represented through these PP in any of the polarization bases.

The disadvantage of the existing representations of SM of a stable RO in parametric form (1) and (2) is that they are valid only within a two-vibrator scattering model with orthogonally mutually oriented vibrators and are limited to the class of axisymmetric objects.

The paper considers a two-vibrator model of RO scattering with non-orthogonally mutually oriented vibrators. A generalized SM of RO was obtained in parametric form using its invariant polarization parameters.

Let the angular position of the first vibrator with an effective length of λ_1 be fixed, form an angle θ_1 with the positive semi-axis OX of the reference coordinate system (RCS) XOY and coincide in direction with the positive semi-axis OX' of the intrinsic coordinate system (ICS) $X'OY'$ of the RO. The angular position of the second vibrator with an effective length of λ_2 in the initial state is orthogonal to the angular position of the



Explanation of the two-vibrator model with non-orthogonally located vibrators λ_1 and λ_2 for $\Delta R = 0$ at $\theta_2 - \theta_1 = 90^\circ$ (a), $90^\circ + \beta$ (b), $90^\circ - \beta$ (c).

first vibrator, forms an angle θ_2 with the positive semi-axis OX of the RCS XOY and coincides with the positive semi-axis OY' of the ICS $X'OY'$ of the RO (see Fig., a). To simplify the figure, let the phase scattering centers of the first and second vibrators coincide, i.e. $\Delta R = 0$.

Let's turn the second vibrator counterclockwise by an angle of $+\beta$. Then, its angular position in the RCS XOY will be equal $\theta_2 = 90^\circ + \theta_1 + \beta$ (see Fig., b). Then, taking into account (1), using the direct and inverse rotation operators known from [3–6], we obtain a generalized SM in a linear polarization basis in the RCS XOY expressed as

$$[S_0] = \lambda_1 \begin{bmatrix} \cos^2 \theta & \sin \theta \cos \theta \\ \sin \theta \cos \theta & \sin^2 \theta \end{bmatrix} + \lambda_2 \exp(j\Delta\varphi) \left[[R(\beta)]^{-1} \begin{bmatrix} \sin^2 \theta & -\sin \theta \cos \theta \\ -\sin \theta \cos \theta & \cos^2 \theta \end{bmatrix} [R(\beta)] \right], \quad (3)$$

where $[R(\beta)] = \begin{bmatrix} \cos \beta & \sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$ — operator of the second vibrator direct rotation at an angle of $+\beta$ counterclockwise [2–4], $[R(\beta)]^{-1} = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$ — operator of the second vibrator backward rotation at an angle of $-\beta$ clockwise [2–4].

After matrix calculations we'll get a generalized SM expressed as

$$[S_0] = \lambda_1 \begin{bmatrix} \cos^2 \theta_1 & \sin \theta_1 \cos \theta_1 \\ \sin \theta_1 \cos \theta_1 & \sin^2 \theta_1 \end{bmatrix} + \lambda_2 e^{j\Delta\varphi} \begin{bmatrix} \sin^2(\theta_1 + \beta) & -\sin(\theta_1 + \beta) \cos(\theta_1 + \beta) \\ -\sin(\theta_1 + \beta) \cos(\theta_1 + \beta) & \cos^2(\theta_1 + \beta) \end{bmatrix}. \quad (4)$$

The generalized SM (4) is valid for the case when the vibrators are oriented mutually non-orthogonally with an angle difference of $\theta_2 - \theta_1 = 90^\circ + \beta$ (see figure, b).

By analogy, if the second vibrator is rotated relative to the first vibrator by an angle of $-\beta$ clockwise, the difference in angles will be $\theta_2 - \theta_1 = 90^\circ - \beta$ (see figure, c).

Ultimately, we'll get a generalized SM

$$[S_0] = \lambda_1 \begin{bmatrix} \cos^2 \theta_1 & \sin \theta_1 \cos \theta_1 \\ \sin \theta_1 \cos \theta_1 & \sin^2 \theta_1 \end{bmatrix} + \lambda_2 e^{j\Delta\varphi} \begin{bmatrix} \sin^2(\theta_1 \pm \beta) & -\sin(\theta_1 \pm \beta) \cos(\theta_1 \pm \beta) \\ -\sin(\theta_1 \pm \beta) \cos(\theta_1 \pm \beta) & \cos^2(\theta_1 \pm \beta) \end{bmatrix}. \quad (5)$$

Expression (5) represents a generalized SM of a RO obtained using a two-vibrator model with non-orthogonally mutually oriented vibrators in parametric form. Indeed,

substituting $\beta = 0^\circ$ into (5), we obtain a special case of orthogonally oriented vibrators (1).

If we assume that $\lambda_1 > \lambda_2$, then, taking out of the generalized SM (5) the modulus of the larger eigen term λ_1 , we obtain it in parametric form

$$[S_0] = \sqrt{\sigma_m} \left\{ \begin{bmatrix} \cos^2 \theta_1 & \sin \theta_1 \cos \theta_1 \\ \sin \theta_1 \cos \theta_1 & \sin^2 \theta_1 \end{bmatrix} + \rho e^{j\Delta\varphi} \begin{bmatrix} \sin^2(\theta_1 \pm \beta) & -\sin(\theta_1 \pm \beta) \cos(\theta_1 \pm \beta) \\ -\sin(\theta_1 \pm \beta) \cos(\theta_1 \pm \beta) & \cos^2(\theta_1 \pm \beta) \end{bmatrix} \right\}, \quad (6)$$

where β — parameter characterizing the value of mutual non-orthogonality of vibrators.

The magnitude of mutual non-orthogonality β in (6) is a measure of vibrators deviation from the state of orthogonality (perpendicularity).

We see that in such a representation in the generalized SM (6) an additional polarization parameter β appears apart from the existing three ρ , $\Delta\varphi$, θ_1 . These PP completely describe the polarization characteristics of the object scattering within a two-vibrator model with a non-orthogonal mutual arrangement of vibrators and have a clear physical interpretation. Since the listed parameters do not depend on the method of specifying the generalized SM (6), they are its invariant PP. In any of the polarization bases, the generalized SM can be expressed through these parameters, and its elements in (6) are themselves the functions of the parameters of the proposed two-vibrator model with non-orthogonal vibrators.

We investigate the properties of the generalized SM (6) by specifying different values of the parameters of the two-vibrator model with non-orthogonal vibrators. To do this, we will lower the degree of the terms responsible for the SM of the first and second vibrators included in (6), and make the transition to trigonometric functions of double arguments. Then, (6) will be expressed as

$$[S_0] = 0.5\sqrt{\sigma_m} \left\{ \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \cos 2\theta_1 & \sin 2\theta_1 \\ \sin 2\theta_1 & -\cos 2\theta_1 \end{bmatrix} \right) + \rho e^{j\Delta\varphi} \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \cos 2(\theta_1 \pm \beta) & \sin 2(\theta_1 \pm \beta) \\ \sin 2(\theta_1 \pm \beta) & -\cos 2(\theta_1 \pm \beta) \end{bmatrix} \right) \right\}. \quad (7)$$

Let the phase centers of the vibrators scattering coincide, i.e. $\Delta\varphi = 0$, and the effective lengths of vibrators $\lambda_1 = \lambda_2 = \lambda$. Then, (7) are transformed to

$$[S_0] = \sqrt{\sigma_m} \left\{ \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 0.5 \begin{bmatrix} \cos 2\theta_1 & \sin 2\theta_1 \\ \sin 2\theta_1 & -\cos 2\theta_1 \end{bmatrix} \right) - 0.5 \begin{bmatrix} \cos 2(\theta_1 \pm \beta) & \sin 2(\theta_1 \pm \beta) \\ \sin 2(\theta_1 \pm \beta) & -\cos 2(\theta_1 \pm \beta) \end{bmatrix} \right\}, \quad (8)$$

where $\sigma_m = \lambda^2$.

The generalized SM (8) is a set of three SM: the first and second SM correspond to the sum of a polarization-isotropic object (a trihedral corner reflector, a sphere) and an anisotropic object (a dihedral corner reflector), and the third SM characterizes the anisotropic part of the object, accounting for the magnitude of the vibrators non-orthogonality.

Let the magnitude of vibrators non-orthogonality $\beta = 0$, i.e. the vibrators are mutually orthogonal, then (8) will have the form

$$[S] = \sqrt{\sigma_m} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (9)$$

From (9) it follows that the generalized SM (8) is transformed into SM $[S]$, which corresponds to a polarization-isotropic object (sphere, metal sheet, triangular corner reflector) and does not depend on the orientation of the ICS $X'OY'$ of the RO, which is completely consistent with the well-known conventional SM (2), presented within a special case of a two-vibrator model with orthogonal vibrators.

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Conflict of interest

The authors declare that they have no conflict of interest.

References

- [1] S.R. Cloude, *Polarisation: applications in remote sensing* (OUP Oxford, 2010).
- [2] Y. Yamaguchi, *Polarimetric SAR imaging: theory and applications* (CRC Press, Boca Raton, 2020).
- [3] A.I. Kozlov, A.I. Logvin, V.A. Sarychev, *Polyarizatsionnaya struktura radiolokatsionnykh signalov* (Radiotekhnika, M., 2005).
- [4] V.N. Tatarinov, S.V. Tatarinov, L.P. Lighthart *Vvedeniye v sovremennuyu teoriyu polyarizatsii radiolokatsionnykh signalov* (Published in Tomsk University, Tomsk, 2012), v. 1.
- [5] V.L. Gulko, A.A. Meshcheryakov, *Instrum. Exp. Tech.*, **65** (6), 1000 (2022). DOI: 10.1134/S0020441222050165
- [6] V.L. Gulko, A.A. Meshcheryakov, V.M. Rulevsky, *Metody i tekhnika polyarimetrii v radiomayachnykh sistemakh navigatsii* (Tomsk State University of Control Systems and Radioelectronics, Tomsk, 2025), p. 64–75.

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