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## Relationship between material strength and parameters of pulsed mechanical impact

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As part of the analytical description of the deformation process, it is shown that the process of fracture under pulsed action is determined by the limiting potential energy density and the lifetime of the energy state. A transition to a brittle fracture mode under high-speed loading by a magnetic-pulse method is demonstrated for highly ductile (under static loading) polyethylene and polyethylene terephthalate.

**Keywords:** plastic deformation, high-speed deformation, magnetic-pulse loading, controlled impact.

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Impulse mechanical strength of materials depends strongly not only on their properties (density, Young's modulus, static tensile strength, temperature, etc.), but also on loading modes. It is known that during spallation loading testing of materials in shock wave mode [1–6] and dynamic mode [7–10], and during dynamic fracture toughness testing [11,12], increase in fracture loads with reduction in exposure time [13,14] and fracture delay are observed [15,16].

Two modes are clearly observed on strength dependences: static and impulse, and the fact that fracture loads are greater than static values as the exposure time decreases is typical for the impulse mode.

It is known that the ultimate specific work of deformation corresponding to the fracture energy density is defined as

$$A_{def} = \int_0^{\varepsilon_{fr}} \sigma d\varepsilon = \int_0^{\varepsilon_{fr}} E \varepsilon d\varepsilon, \quad (1)$$

where  $\varepsilon_{fr}$  is the ultimate strain,  $\sigma$  is the stress,  $E$  is the proportionality factor between  $\sigma$  and  $\varepsilon$  (equal to the modulus of elasticity  $E_Y$  in the elastic case). A typical stress-strain curve is shown in Figure 1.

During impulse loading, when deformation parameters also depend on time, the stress-strain curve adjusts to the loading mode and taking into account that  $E(t) : \Leftrightarrow E_t(t) = \sigma_t(t)/\varepsilon_t(t)$  the following can be written

$$A_{def}(t) = \int_0^t E(t) \varepsilon(t) \frac{d\varepsilon(t)}{dt} dt. \quad (2)$$

We assume that fracture is defined by the maximum attainable stress  $\sigma_{fr} = \sigma_{max}$  during exposure. Expression under the integral sign in equation (2) can be transformed

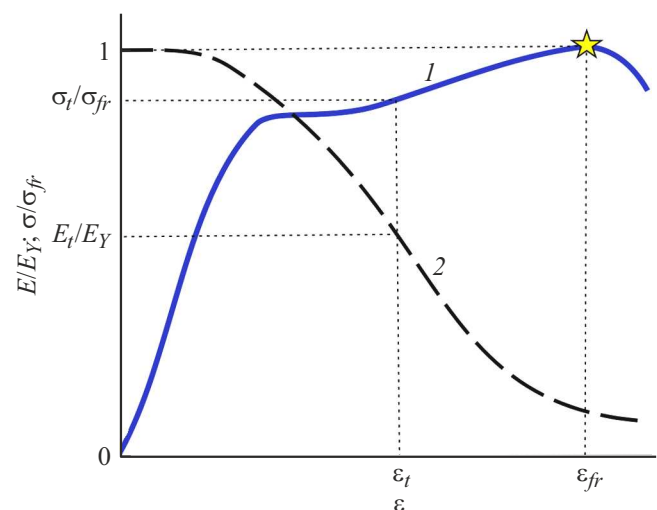
to

$$\begin{aligned} E(t) \varepsilon(t) \frac{d\varepsilon(t)}{dt} &= E(t) \frac{\sigma(t)}{E(t)} \frac{d}{dt} \left( \frac{\sigma(t)}{E(t)} \right) \\ &= \sigma(t) \left[ E(t) \frac{d\sigma(t)}{dt} - \sigma(t) \frac{dE(t)}{dt} \right] \frac{1}{E(t)^2}. \end{aligned} \quad (3)$$

The work of external forces can be represented as the sum of two components

$$\begin{aligned} A_{def}(t) &= \int_0^t \sigma(t) \left[ E(t) \frac{d\sigma(t)}{dt} \right] \frac{1}{E(t)^2} dt - \int_0^t \sigma(t) \left[ \sigma(t) \frac{dE(t)}{dt} \right] \\ &\quad \times \frac{1}{E(t)^2} dt = A_{def}(t) \Big|_I + A_{def}(t) \Big|_{II}. \end{aligned} \quad (4)$$

The work of external forces up to time  $t = t_{fr}$  is denoted as  $A_{def}^{fr} = A_{def}(t_{fr})$ . After transformation in accordance with



**Figure 1.** Material stress-strain curve with nonlinear mechanical properties (1) and proportionality factor variation between  $\sigma$  and  $\varepsilon$  (2).

the first mean value theorem and integration up to the fracture time, we get

$$A_{def}^{fr}|_I = \left( \frac{1}{E(t)} \right) \frac{(\sigma|_{t_{fr}})^2}{2}, \quad (5)$$

and taking into account that  $\sigma(t)^2 dt = d(\sigma(t)^2 t) - 2t\sigma(t)d\sigma(t)$ , we obtain

$$\begin{aligned} A_{def}^{fr}|_{II} &= \frac{d}{dt} \left( \frac{1}{E(t)} \right) \int_0^{t_{fr}} \sigma(t)^2 dt = \frac{d}{dt} \left( \frac{1}{E(t)} \right) \left( \int_0^{t_{fr}} d(t\sigma(t)^2) \right. \\ &\quad \left. - \int_0^{t_{fr}} 2t\sigma(t)d\sigma(t) \right) = (\sigma|_{t_{fr}})^2 (t_{fr} - \hat{t}) \frac{d}{dt} \left( \frac{1}{E(t)} \right) \\ &= (\sigma|_{t_{fr}})^2 t_{fr} \left( 1 - \frac{\hat{t}}{t_{fr}} \right) \frac{d}{dt} \left( \frac{1}{E(t)} \right). \end{aligned} \quad (6)$$

$\left( \frac{1}{E(t)} \right)$  and  $\frac{d}{dt} \left( \frac{1}{E(t)} \right)$  define the average value of the inverse modulus and its derivative, respectively, over the integration interval, and  $\hat{t}$  characterizes the influence of the type of loading- time dependence on the plastic deformation process, whose multifactority shows on various space-time scales (see, for example, [17,18]). Analysis of plastic deformation mode, which is investigated in numerous works, is beyond the scope of this study.

Thus, the work of external forces expended on fracture is defined by the maximum attainable stress  $\sigma_{fr}$  and is dissipated through two main channels: energy dissipation in plastic deformation and accumulation of potential elastic strain energy. Energy flux distribution between the channels is defined by the material properties  $E(\sigma, t)$  and loading history

$$\begin{aligned} A_{def}(t) &= \frac{(\sigma|_{t_{fr}})^2}{2E_Y} \left( \left( \frac{1}{E(t)} \right) E_Y + (t_{fr} - \hat{t}) \frac{d}{dt} \left( \frac{1}{E(t)} \right) 2E_Y \right) \\ &= \frac{(\sigma|_{t_{fr}})^2}{2E_Y} \left( \Psi(t)|_{el} + \Psi(t)|_{pl} \right) = \frac{(\sigma|_{t_{fr}})^2}{2E_Y} \Psi(t)|_{t_{fr}}. \end{aligned} \quad (7)$$

mean energy during the deformation-to-fracture time

$$W_{def}^{fr} = \frac{1}{t_{fr}} \int_0^{t_{fr}} A_{def}(t) dt = \frac{1}{t_{fr}} \int_0^{t_{fr}} \frac{(\sigma(t))^2}{2E_Y} \left( \Psi(t)|_{el} + \Psi(t)|_{pl} \right) dt. \quad (8)$$

As it follows from (7), the type of loading considerably influences the external force work distribution between the energy consumption channels (see the table).

Since, regardless of deformation modes, fracture is associated with discontinuity, i.e. with breaking of atomic bonds, the ultimate elastic energy density required for fracture is presumably independent of the loading time down to typical atomic times. We introduce  $S_{fr} = W_{def}^{fr} t_{fr}$ ,

a parameter which links the mean work of external forces during deformation with the time to fracture. During fracture in near-brittle mode  $\Psi(t)|_{el} \rightarrow 1$ ,  $\Psi(t)|_{pl} \rightarrow 0$ , and  $W_{def}^{fr} = \widehat{\sigma_{fr}}^2 / 2E_Y$ , where  $\widehat{\sigma_{fr}}$  is defined by  $\sigma(t)$ . Then the following can be written for different  $\sigma_{fr}$  for one type of loading

$$\frac{S_{fr1}}{S_{fr2}} : \Leftrightarrow D_W = 1 = \frac{(\widehat{\sigma_{fr1}})^2 (\Psi(t))|_1(t_{fr1})}{(\widehat{\sigma_{fr2}})^2 (\Psi(t))|_2(t_{fr2})} \approx \frac{(\widehat{\sigma_{fr1}})^2 (t_{fr1})}{(\widehat{\sigma_{fr2}})^2 (t_{fr2})}. \quad (9)$$

In practice, it is important to compare fracture loads in static and impulse modes, energy distribution in which differs insignificantly only in brittle materials. However, as follows from (7), in other cases, fracture is defined by the maximum attainable potential energy  $W_{def}^{fr}$ , which can be associated with the Landau potential, and  $t_{fr}$  defines the lifetime of the stress state necessary for fracture.

As it follows from (9),  $(\sigma_{fr1})^2 / (\sigma_{fr2})^2 = t_{fr2} / t_{fr1}$ , i.e. the increase of ultimate loads in the brittle fracture mode is independent of material properties, which reflects the fracture process generality. Individual material properties are exhibited when static characteristics are considered. And if the static material strength  $\sigma_0$  is determined, then the time interval  $\tau_L$ , which defines the lifetime of the static stress state  $(\sigma_0)^2$ , can be formally calculated from

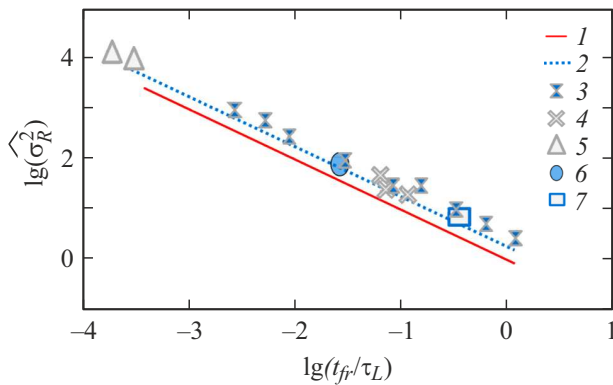
$$\frac{(\sigma_{fr1})^2}{(\sigma_0)^2} = \frac{\tau_L}{t_{fr1}}. \quad (10)$$

Figure 2 shows experimental data processing results for impulse fracture of various materials. These results are representative enough for identification of the type and parameters of loading, in the  $(t_{fr}/\tau_L$  and  $\widehat{\sigma_R}^2)$  coordinates, where  $\widehat{\sigma_R}^2 \equiv (\sigma_{fr}/\sigma_0)^2$ , by assuming that Young's modulus is independent of the loading rate. This dependence shows the data on behavior of some engineering materials [16,19–21] and natural materials: gabbro-diabase, sandstone, and granite [13].

The scatter of the data observed in Figure 2 may result both from insufficient accuracy in determining the loading

Influence of the type of loading on the main parameters

| No<br>Item No.            | Type of loading                                                                                             | $\widehat{\sigma_{fr}}/\sigma_{fr}$ | $\hat{t}/t_{fr}$ |
|---------------------------|-------------------------------------------------------------------------------------------------------------|-------------------------------------|------------------|
| 1                         | $\sigma(t) = \sigma_{fr} t$                                                                                 | $1/\sqrt{3}$                        | 2/3              |
| 2                         | $\sigma(t) = \sigma_{fr} t^2$                                                                               | $1/\sqrt{5}$                        | 4/5              |
| 3                         | $\sigma(t) = \sigma_{fr} \sin(\omega t)$                                                                    | $1/\sqrt{2}$                        | 1/2              |
| 4                         | $\sigma(t) = \sigma_{fr} (\sin(\omega t))^2$                                                                | $\sqrt{0.375}$                      | 5/8              |
| 5                         | $\sigma(t) = \sigma_{fr} \left( \frac{2}{1+e^{-2kt}} - 1 \right)$<br>at $k = 1/t_{fr}$<br>at $k = 2/t_{fr}$ | $\sqrt{0.434}$<br>$\sqrt{0.662}$    | 0.761<br>0.482   |
| Parameter variation range |                                                                                                             | $0.636 \pm 30\%$                    | $0.639 \pm 25\%$ |

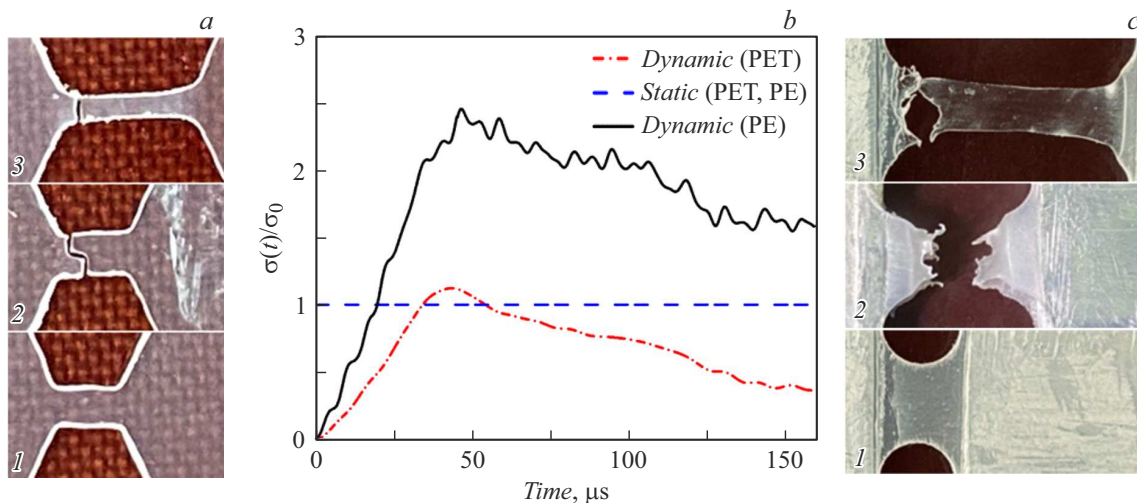


**Figure 2.** Correlation between the fracture impulse time and ultimate potential energy. 1 — calculation using equation (10), 2 — calculation using equation (10) for ramp function, 3 — data [13], 4 — 09G2S steel [16], 5 — Ti [19], 6 — Al [20], 7 — PMMA (polymethylmethacrylate) [21].

parameters and from some influence of loading rate on the modulus of elasticity  $\left(\frac{1}{E(t)}\right)E_Y \simeq 1 \neq 1$ .

Higher accuracy in determining the loading parameters can be achieved using a magnetic-pulse method for generating controlled pressure pulses [22,23]. Figure 3 shows the results of high-speed loading of polymer samples by the magnetic-pulse method. Stresses were recorded using a technique described in [24].

As shown in Figure 3, materials demonstrating the static plastic deformation mode switch to a near-brittle fracture mode, i.e. without substantial plastic strains, already at loading rates of  $\sim 6$  and  $1 \text{ MPa}/\mu\text{s}$  for PET and PE, respectively. Moreover, there is an increase in the fracture stress with respect to the static stress  $\sigma_0$ : for PET — increase by 12 % ( $\sigma_0$  — 210 MPa), for PE — increase by 133 % ( $\sigma_0$  — 12 MPa).



**Figure 3.** *a, c* — view of polyethyleneterephthalate (PET) and polyethylene (PE) samples, respectively: 1 — in initial state, 2 — after dynamic loading, 3 — after static loading. *b* — time dependence of loading.

Impulse deformation analysis has identified that the stress form and amplitude affect fracture parameters, with the potential elastic deformation energy being the determining factor. Ultimate pulsed impact on a material for a known time dependence of stresses is possible with known material properties. They include the ultimate static fracture stress and stress state lifetime, which is responsible for the transient nature of material response to the impact.

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## Conflict of interest

The authors declare no conflict of interest.

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