

Stationary Mach configurations with an endothermic normal shock wave

© M.V. Chernyshov, M.M. Laptinskaya

Baltic State Technical University „VOENMEKH“ named after Marshal D. F. Ustinov, St. Petersburg, Russia

E-mail: chernyshov_mv@voenmeh.ru

Received February 3, 2026

Revised March 3, 2026

Accepted March 19, 2026

Conditions of existence of triple-shock configurations with normal main (Mach) shock (so-called stationary Mach configurations) in the presence of pulsed energy absorption on that strong main shock are determined analytically and numerically. As formation of stationary Mach configuration corresponds to well-known von Neumann criterion, which describes Mach/regular oblique shock reflection transition, shift of that criterion due to possible flow endothermicity at the main shock is estimated.

Keywords: steady shock wave, triple-shock configuration, regular reflection, Mach reflection, von Neumann criterion.

DOI: 10.61011/TPL.2026.06.63802.20643

Triple configurations formed during the Mach reflection of oblique shocks (for example, shock j_1 in Figure 1, *a*, *b*) separate the gas flow downstream of the triple point T into two regions (above and below the tangential discontinuity τ). As shown in [1,2], many gas-dynamic properties (for example, velocity, Mach number, temperature, density, acoustic impedance, stagnation pressure, dynamic pressure, momentum flux) in these regions downstream of the reflected (j_2) and main or Mach (j_3) shocks differ considerably, sometimes multifold, in particular, at high Mach numbers M of flow upstream of the triple point. Considerable differences in the flow properties on the tangential discontinuity sides shall be considered, for example, for macrobody throwing by cocurrent flow behind a shock wave [3], gas-dynamic design of supersonic air inlets [4,5], detonation engine [6,7] and compound engine design using detonation effects [8,9]. In the last two cases increase in the combustible mixture temperature on the main shock (much more considerable than on the incident and reflected

shocks) facilitates excitation of exothermic reactions behind its surface, forming the stationary detonation wave structure.

According to [10], positive energy release on j_3 noticeably increases the shock dimensions (i. e. height of the Mach reflection triple point above the plane or symmetry axis) compared with the case of reflection of j_1 with the same intensity in the flow with the same properties, but without energy release on the Mach shock. In addition, possibility of energy release moves the known von Neumann criterion of reflection change (regular or Mach) towards weaker incident shocks. This results in the Mach reflection of such shocks, which are reflected only regularly in non-reacting gas. In [11], a generalized von Neumann criterion was obtained — conditions that define the properties of the incident shock j_1 , which forms the stationary Mach configuration (SMC) and corresponds to the change of the type of reflection of this shock, if pulse energy release and change of the gas mixture composition take place on the main shock of possible Mach reflection. It is noted that positive energy release has a more significant influence on

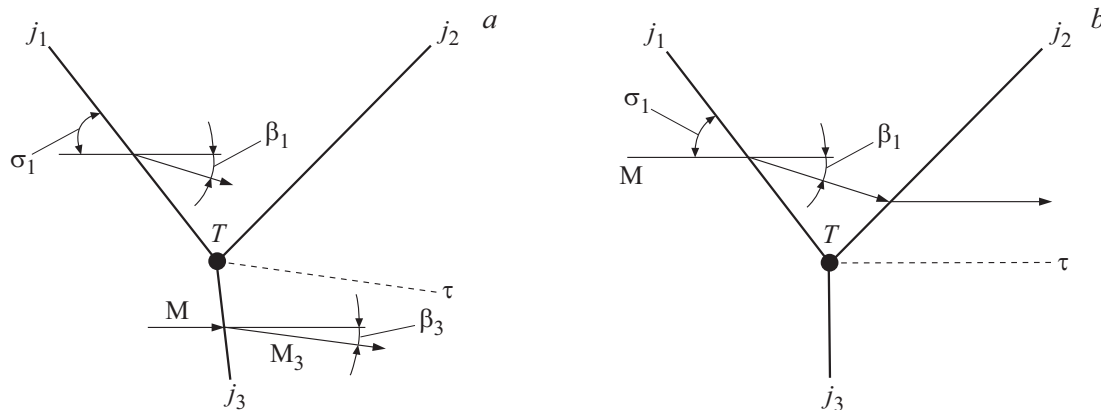


Figure 1. Flow diagram in the neighborhood of the triple point during formation of the typical triple Mach reflection configuration (*a*) and formation of the stationary Mach configuration in the transient case (*b*).

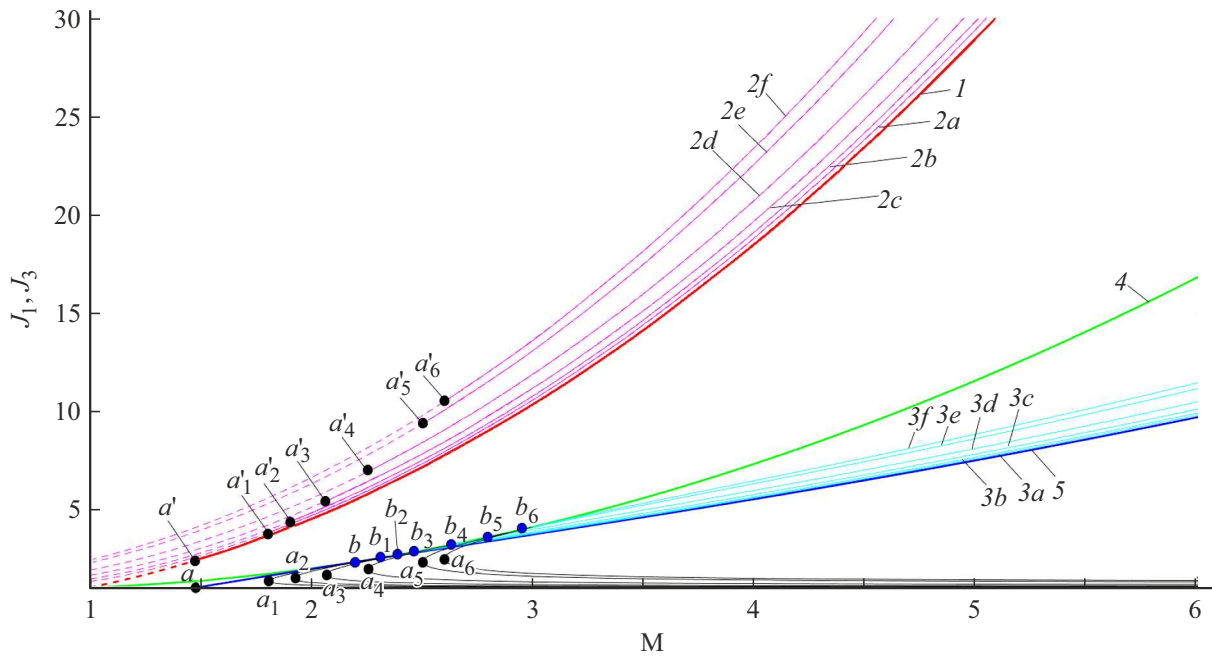


Figure 2. Shock intensities corresponding to the normal shock in perfect non-reacting gas (1), normal shock with absorption of 5% (2a), 10% (2b), 20% (2c), 40% (2d), 80% (2e) and 100% (2f) of the total flow enthalpy, oblique incident shocks that form SMC with the above-listed energy absorptions on the main shock (3a–3f), maximum flow deflection criterion (4) and von Neumann criterion for perfect non-reacting gas (5).

the shift of the generalized von Neumann criterion with respect to the classical one (for perfect non-reacting gas) than the concomitant variation of the chemical composition (change of specific heat ratio).

If the triple configuration with a strong Mach shock is formed in non-reacting gas, then negative energy release (energy absorption) is possible on the Mach shock due to the energy consumption for excitation of vibrational degrees of freedom, molecule dissociation and other high-temperature effects (especially in high-speed flows). Therefore, it is necessary to define the SMC existence conditions corresponding to the generalized von Neumann criterion in the case when there is pulsed energy absorption on the main shock, and to evaluate the shift of these conditions compared with the classical case.

Compatibility conditions on the oblique (Figure 1, a) or normal (Figure 1, b) shock j_3 with positive or negative energy release and change of gas composition lead to the following relation between the shock intensity $J_3 = p_3/p$ and flow deflection angle β_3 on the surface [12]:

$$\beta_3 = \arctg \left[\frac{(J_3 - 1)\sqrt{F - 1}}{\gamma M^2 - (J_3 - 1)} \right],$$

$$F = \frac{2\gamma M^2[(\gamma - \gamma_3) + (\gamma - 1)((J_3 - 1) - (\gamma_3 - 1)\bar{\varphi})]}{(\gamma - 1)(J_3 - 1)[(\gamma_3 + 1)(J_3 - 1) + 2\gamma_3]}.$$
(1)

Here, $\bar{\varphi} = \varphi/(p/\rho)$ is the dimensionless energy release on j_3 ; φ is the specific energy release (positive or negative); p , ρ and γ are static pressure, density and specific heat

ratio of gas before the shock, p_3 and γ_3 are static pressure and specific heat ratio after the shock; M is the Mach number of the incoming flow. Normal shock intensity J_3 in the stationary Mach configuration is calculated from (1) at $\beta_3 = 0$

$$J_3 = \frac{\gamma + 1}{\gamma_3 + 1} + \frac{\sqrt{\gamma^2 M^4 + \gamma_3^2 - 2\gamma M^2[(\gamma_3^2 - 1)\bar{\varphi} + (\gamma_3^2 - \gamma)/(\gamma - 1)]}}{\gamma_3 + 1},$$

$$J_3 = \frac{J_m(M) + 1}{2} + \frac{\sqrt{\gamma^2(M^2 - 1)^2 - 2\gamma(\gamma^2 - 1)M^2\bar{\varphi}}}{\gamma + 1}$$
(2)

at $\gamma_3 \neq \gamma$ and equal specific heat ratios behind and before the shock, respectively. Here, $J_m = (1 + \varepsilon)M^2 - \varepsilon$ is the normal shock intensity in the non-reacting perfect gas flow with the same Mach number (curve 1 in Figure 2), and $\varepsilon = (\gamma - 1)/(\gamma + 1)$.

Energy release or absorption is interconnected with various physical and chemical transformations that change the specific heat ratio of the gas mixture. However, since the main objective of the study is to qualitatively evaluate the link between energy absorption on the main shock and the shift of the von Neumann criterion (corresponding to SMC formation), and the effect of change of the specific heat ratio on this shift is secondary [11,13], the second of relations (2) is further used for the Mach shock intensity. All calculations correspond to $\gamma = \gamma_3 = 1.4$ and can be generalized to flows with other values of γ .

Curves 2a–2f in Figure 2 show $J_3(M)$ with energy absorption $\bar{\varphi}$, constituting a part $K = \bar{\varphi}/\bar{\varphi}_{\text{lim}}$ of a theoretically

limiting, but hardly practicable value

$$|\bar{\varphi}_{\text{lim}}| = \frac{\gamma}{\gamma - 1} \left(1 + \frac{\gamma - 1}{2} M^2 \right),$$

corresponding to the total absorption of flow stagnation enthalpy. Obviously, occurrence and growth of energy absorption steadily increase the main shock intensity from $J_m(M)$ to $J_3 = \gamma M^2 + 1$ corresponding to $\bar{\varphi} = \bar{\varphi}_{\text{lim}}$ (curve 2f). At high Mach numbers ($M \rightarrow \infty$) within the mathematical perfect gas model, intensity of j_3 is described by the asymptotic dependence

$$\frac{J_3}{M^2} \rightarrow \frac{1 + \varepsilon}{2} \left(1 + \sqrt{1 + (\gamma^2 - 1)K} \right)$$

(in particular, $J_3/M^2 \rightarrow \gamma$ at $\bar{\varphi} = \bar{\varphi}_{\text{lim}}$).

Intensity J_1 of the incident shock that forms SMC is defined as the root of equation of the generalized von Neumann criterion [11], where J_3 is found from (2):

$$\sum_{n=0}^5 H_n J_1^n = 0, \quad (3)$$

$$H_5 = 1 - \varepsilon,$$

$$H_4 = -[2(1 + \varepsilon - \varepsilon^2)M^2 + (1 - \varepsilon)(2 - 2\varepsilon - \varepsilon J_3)],$$

$$H_3 = (1 + \varepsilon)(1 + 2\varepsilon)M^4 + [2 - 6\varepsilon^2 + \varepsilon^3 - \varepsilon^2 J_3(2 - \varepsilon)]M^2 + (1 - \varepsilon)(1 - 4\varepsilon + \varepsilon^2 + (1 - \varepsilon)^2 J_3),$$

$$H_2 = -\varepsilon(1 + \varepsilon)(J_3 - 2\varepsilon)M^4 + \varepsilon[\varepsilon J_3^2 - (2 - 3\varepsilon + 2\varepsilon^2)J_3 + 2(1 - \varepsilon)(1 + 2\varepsilon)]M^2 + 2(1 - \varepsilon)[\varepsilon - \varepsilon^2 - J_3(1 - \varepsilon + \varepsilon^2)],$$

$$H_1 = -(1 + \varepsilon)[\varepsilon^2 + (1 + \varepsilon^2)J_3]M^4 + \varepsilon[2J_3^2 + (1 - 2\varepsilon + 2\varepsilon^2)J_3 - 1 + 2\varepsilon^2 B]M^2 + (1 - \varepsilon)[\varepsilon^2 + J_3(1 - \varepsilon)^2],$$

$$H_0 = -J_3[\varepsilon(1 + \varepsilon)M^4 + (1 - 2\varepsilon^2 - J_3)M^2 - \varepsilon(1 - \varepsilon)].$$

Roots of equation (3) are shown by curves 3a–3f in Figure 2. Equation (3) has no physically realizable solutions at low Mach numbers (at $M < M_{a_i}$, where the values of M_{a_i} depend on the degree of energy absorption K and correspond to the left-most points a_i on curves 3a–3f and to points a'_i on curves 2a–2f). Therefore, left parts of curves 2a–2f (up to points a'_i) in Figure 2 are shown as dashed lines: static pressures behind the main shocks with such intensity at small Mach numbers are unattainable at small Mach numbers in the incident and reflected shock system. M_{a_i} at degrees of energy absorption K corresponding to curves 3a–3f have the following values: $M_{a_1} = 1.809$ at $K = 0.05$, $M_{a_2} = 1.921$ at $K = 0.1$, $M_{a_3} = 2.066$ at $K = 0.2$, $M_{a_4} = 2.254$ at $K = 0.4$, $M_{a_5} = 2.504$ at $K = 0.8$, $M_{a_6} = 2.600$ at $K \rightarrow 1$. When the energy absorption decreases and disappears ($\bar{\varphi} \rightarrow 0$), M_{a_i} tends to the limit $M_a = \sqrt{(2 - \varepsilon)/(1 - \varepsilon)} = 1.483$, i.e. to the smallest Mach number, at which there are solutions for SMC in non-reacting gas.

At $M > M_{a_i}$, there are two solutions to equation (3), which have a physical significance. They are shown, respectively, by upper and lower curve branches 3a–3f in Figure 2. Lower parts of curves 3a–3f describe SMC with strong (corresponding to the upper branches of shock polars) reflected shocks, the flow behind which is subsonic. As proved in [14], regular reflection and, according to [15], also Mach reflection with a strong reflected shock is non-resistant to small perturbations (regular reflection of j_1 with „weak“ reflected shock is actually realized).

Left parts of the upper curve branches 3a–3f (between points a_i and b_i) also describe unstable SMC with strong reflected shocks. At points b_i curves 3a–3f are tangent to line 4 corresponding to other known shock reflection change criterion (maximum flow deflection angle criterion or „detachment criterion“ [16]). Mach numbers M_{b_i} take the following values: $M_{b_1} = 2.281$ at $K = 0.05$, $M_{b_2} = 2.348$ at $K = 0.1$, $M_{b_3} = 2.458$ at $K = 0.2$, $M_{b_4} = 2.629$ at $K = 0.4$, $M_{b_5} = 2.888$ at $K = 0.8$, $M_{b_6} = 2.995$ at $K \rightarrow 1$. At $\bar{\varphi} \rightarrow 0$, M_{b_i} tends to $M_b = 2.202$, which corresponds to the coincidence of two best known shock reflection change criteria and shock waves: maximum flow deflection angle criterion (curve 4) and „classical“ von Neumann criterion shown by curve 5 [16,17].

At $M > M_{b_i}$ the upper curve branches 3a–3f correspond to stable Mach reflection structures with a „weak“ reflected shock. Distance between curves 3a–3f and line 5 make it possible to estimate the change of J_1 of the incident shock (Figure 3, a), shock inclination angle σ_1 (Figure 3, b) and flow deflection angle β_1 on the shock surface (Figure 3, c) compared with the parameters set by the „classical“ von Neumann criterion due to possible energy absorption on the main Mach reflection shock. As can be seen from Figure 3, b, c, typical shift of the von Neumann criterion with respect to shock incidence and flow deflection angles is about 1–2°. σ_1 and β_1 were linked to J_1 through well known relations [17] for shocks in perfect gas.

At large Mach numbers ($M \rightarrow \infty$), curves 3a–3f are described by asymptotes $\frac{J_1}{M^2} \rightarrow C$, $\sigma_1 \rightarrow \arcsin \sqrt{\frac{C}{1 + \varepsilon}}$,

$\beta_1 \rightarrow \arctg \frac{\sqrt{C(1 + \varepsilon - C)}}{\gamma - C}$, where C is derived from the following equation

$$\sum_{n=0}^6 A_n C^n = 0,$$

$$A_6 = (1 - \varepsilon)^6, \quad A_5 = -(1 - \varepsilon)^5(4 + 3\varepsilon - 4\varepsilon^2 + \varepsilon^3),$$

$$A_4 = (1 - \varepsilon)^4[6 + 10\varepsilon - 10\varepsilon^2 - 7\varepsilon^3 + 8\varepsilon^4 - 3\varepsilon^5 - \varepsilon^3(1 + \varepsilon)^2 K],$$

$$A_3 = -(1 + \varepsilon)(1 - \varepsilon)^3[(1 - \varepsilon)(2 + 5\varepsilon - 2\varepsilon^2)(2 + \varepsilon - 2\varepsilon^2 + \varepsilon^3) - \varepsilon^3(2 + 6\varepsilon - 2\varepsilon^2 + \varepsilon^3)K],$$

$$A_2 = (1 - \varepsilon^2)^2[(1 - \varepsilon)^2(1 + 6\varepsilon + 2\varepsilon^2 - 7\varepsilon^3 + 4\varepsilon^4) - \varepsilon^3(2 + 5\varepsilon + 2\varepsilon^2 - 5\varepsilon^3 + \varepsilon^4)K],$$

$$A_1 = -\varepsilon(1 + \varepsilon)^3(1 - \varepsilon)^2[(1 - \varepsilon)^3(1 + 2\varepsilon) - \varepsilon^2(2 + 4\varepsilon^2 - \varepsilon^3)K],$$

$$A_0 = -K\varepsilon^3(1 + \varepsilon)^4[1 - 3\varepsilon + 3\varepsilon^2 - \varepsilon^3(1 + K)]$$

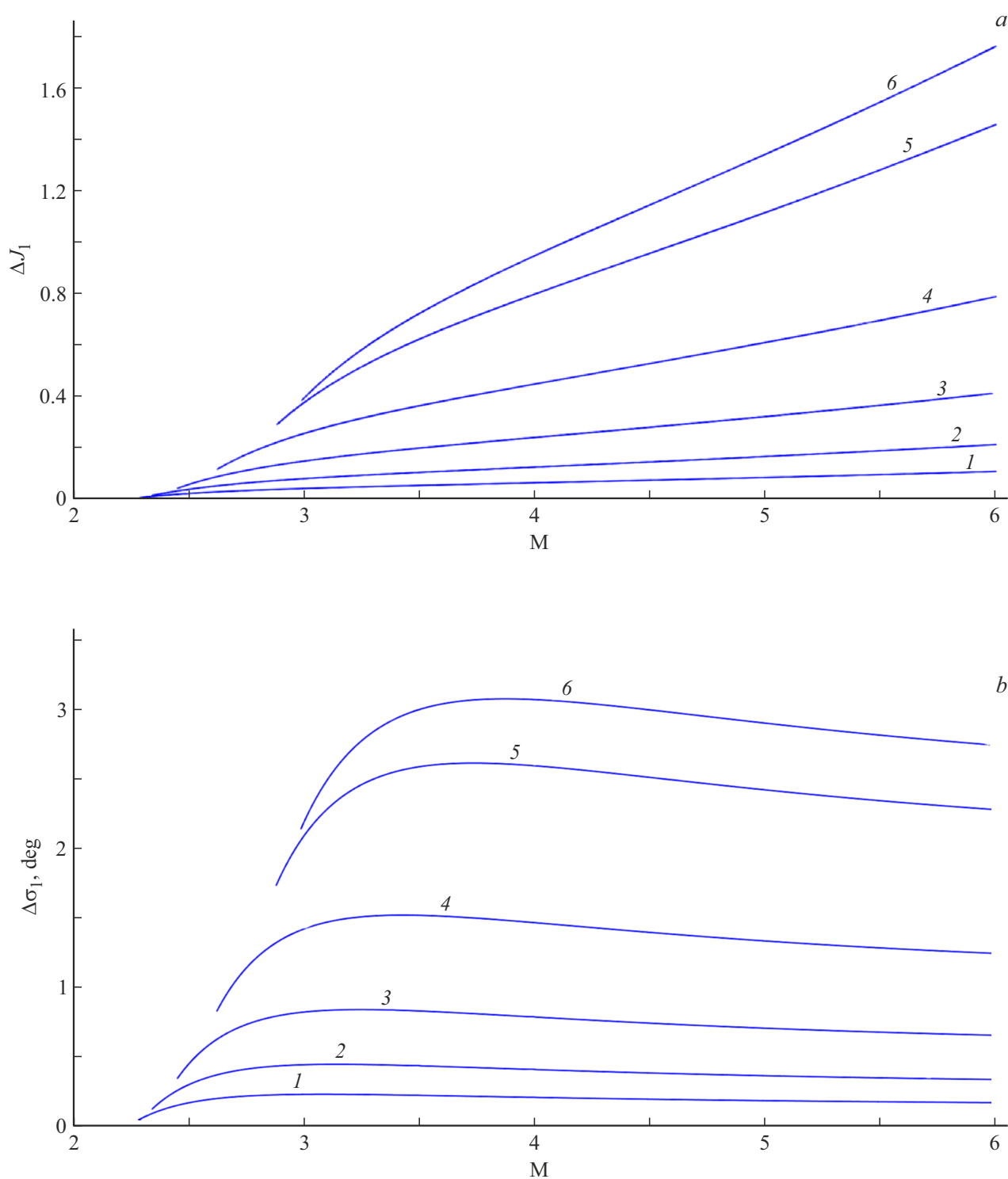


Figure 3. Increase in the intensity (*a*) of the incident shock that forms SMC, shock inclination angle (*b*) and flow deflection angle on the shock surface (*c*) with energy absorption on the Mach shock constituting 5% (1), 10% (2), 20% (3), 40% (4), 80% (5) and 100% (6) of the total flow enthalpy.

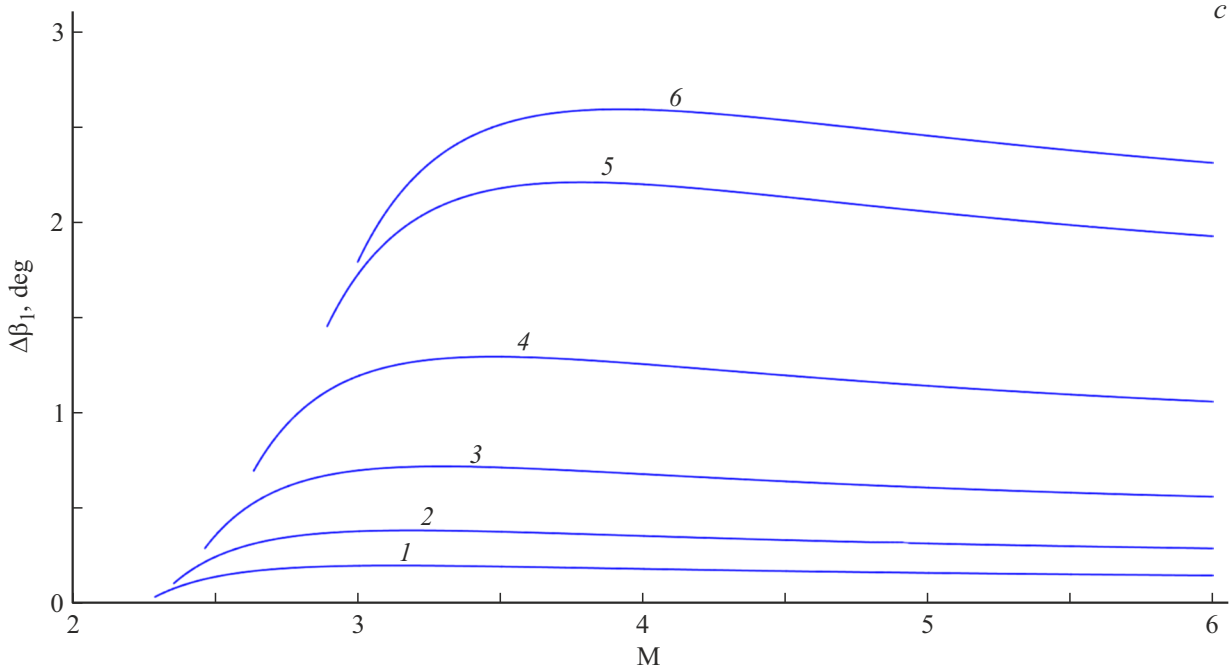


Fig. 3 (Contd.).

(in particular, $C = 0.163$, $\sigma_1 = 21.919^\circ$ and $\beta_1 = 18.082^\circ$ at $K = 0.05$; $C = 0.165$, $\sigma_1 = 22.065^\circ$ and $\beta_1 = 18.200^\circ$ at $K = 0.1$; $C = 0.176$, $\sigma_1 = 22.875^\circ$ and $\beta_1 = 18.853^\circ$ at $K = 0.4$; $C = 0.197$, $\sigma_1 = 24.251^\circ$ and $\beta_1 = 19.957^\circ$ at $K \rightarrow 1$).

Comparison of the above-listed values with asymptotes describing the „classical“ von Neumann criterion ($C = 0.160$, $\sigma_1 = 21.769^\circ$, $\beta_1 = 17.961^\circ$) makes it possible to estimate the shift of this criterion at high supersonic speeds with respect to incidence and flow deflection angles on the shock surface. In particular, as applicable to σ_1 of j_1 , equation (3) of the generalized von Neumann criterion is written as follows:

$$\sum_{n=0}^5 h_n \sin^{2n} \sigma_1 = 0, \quad (4)$$

$$h_5 = (1 - \varepsilon)(1 + \varepsilon)^3 M^8,$$

$$h_4 = -(1 + \varepsilon)^2 M^6 [2(1 + \varepsilon - \varepsilon^2) M^2 - (1 - \varepsilon)(\varepsilon J_3 - 2 - 3\varepsilon)],$$

$$h_3 = (1 + \varepsilon) M^4 [(1 + \varepsilon)(1 + 2\varepsilon) M^4 + h_{31} M^2 - (1 - \varepsilon^2)((1 - 3\varepsilon) J_3 + 1 + 3\varepsilon)],$$

$$h_2 = -M^2 [\varepsilon(1 + \varepsilon)(3 + 4\varepsilon + J_3) M^4 - \varepsilon h_{21} M^2 + (1 + \varepsilon)(1 - \varepsilon^2)((2 - 3\varepsilon) J_3 + \varepsilon)],$$

$$h_1 = -(1 + \varepsilon)((1 - \varepsilon) J_3 - 2\varepsilon^2) M^4$$

$$+ \varepsilon(1 - \varepsilon) h_{11} M^2 + (1 - \varepsilon^2)^2 J_3,$$

$$h_0 = (1 - \varepsilon)^2 (J_3 - 1)(J_3 - \varepsilon^2),$$

$$h_{31} = -\varepsilon^2 J_3 (2 - \varepsilon) + 2 + 8\varepsilon + 2\varepsilon^2 - 7\varepsilon^3,$$

$$h_{21} = \varepsilon J_3^2 - (2 - 3\varepsilon - 4\varepsilon^2 + 3\varepsilon^3) J_3 - 4 - 10\varepsilon + 2\varepsilon^2 + 9\varepsilon^3,$$

$$h_{11} = 2J_3^2 + (1 + 2\varepsilon - 3\varepsilon^2) J_3 - 1 + 2\varepsilon + 5\varepsilon^2.$$

Solutions (4) are shown in Figure 4 by curves 1a–1f. Upper segments of these curves located to the right of points b_i of tangency to curve 2, which displays the maximum flow deflection angle criterion, correspond to the stable Mach reflection. Shift of curves 1a–1f compared with line 3, corresponding to the „classical“ von Neumann criterion, characterizes the shift of transfer from the Mach to regular reflection due to the flow endothermicity on the Mach shock.

Thus, conditions of SMC existence with energy absorption on the main shock were obtained using the generalized von Neumann criterion. Their analysis shows upward shift of the von Neumann criterion by up to 2° with respect to the incident shock inclination and flow deflection angles on the shock surface. This shift is generally compensated only marginally by the inverse effect resulting from concomitant growth of the specific heat ratio of gas mixture.

Funding

The study was supported by the Ministry of Science and Higher Education of the Russian Federation (project No FZWF-2024-0003).

Conflict of interest

The authors declare no conflict of interest.

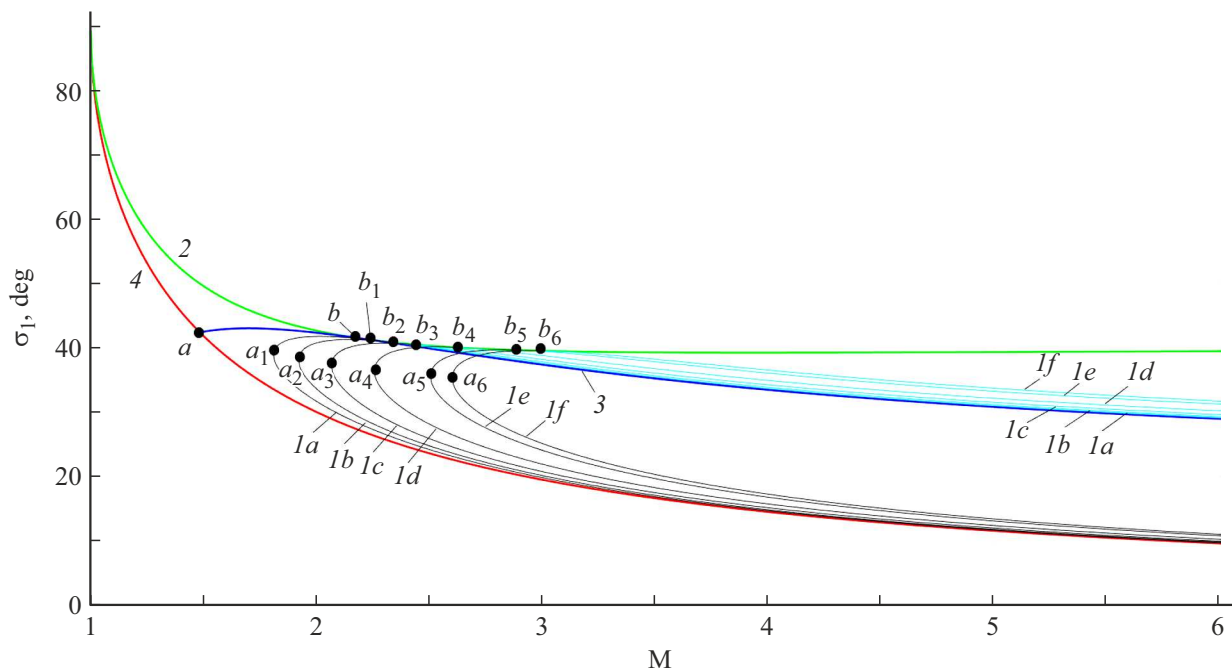


Figure 4. Incident shock inclination angles corresponding to formation of SMC with energy absorption on the Mach shock constituting 5% (1a), 10% (1b), 20% (1c), 40% (1d), 80% (1e) and 100% (1f) of the total flow enthalpy, maximum flow deflection angle criterion (2), von Neumann criterion for perfect non-reacting gas (3) and shock degeneracy into a weak discontinuity $\sigma_1 = \arcsin(1/M)$ (4).

References

- [1] V.N. Uskov, M.V. Chernyshov, J. Appl. Mech. Tech. Phys., **47** (4), 492 (2006). DOI: 10.1007/s10808-006-0081-5
- [2] M.V. Chernyshov, Izv. vuzov. Aviatsionnaya tekhnika, No 2, 82 (2019). [M.V. Chernyshov, Russ. Aeronaut., **62** (2), 259 (2019). (in Russian) DOI: 10.3103/S1068799819020120].
- [3] M.V. Chernyshov, K.E. Savelova, Acta Astron., **226** (1), 876 (2025). DOI: 10.1016/j.actaastro.2024.11.017
- [4] V.I. Zvegintsev, Thermophys. Aeromech., **24** (6), 807 (2017). DOI: 10.1134/S0869864317060014
- [5] D.A. Rakhmanin, E.V. Karpov, E.V. Rakhmanina, Vestn. MAI, **30** (2), 35 (2023). (in Russian) DOI: 10.34759/vst-2023-2-35-45
- [6] D.M. Davidenko, I. Gökalp, A.N. Kudryavtsev, in *15th AIAA Int. Space Planes and Hypersonic Systems and Technologies Conf.* (Dayton, OH, 2008), paper 2008-2680. DOI: 10.2514/6.2008-2680
- [7] P.V. Bulat, P.V. Denisenko, K.N. Volkov, Nauch.-tekhn. vestn. informatsionnykh tekhnologii, mekhaniki i optiki, **16** (1), 1 (2016). (in Russian) DOI: 10.17586/2226-1494-2016-16-1-1-21
- [8] M.S. Ivanov, A.N. Kudryavtsev, A.V. Trotsyuk, V.M. Fomin, *Sposob organizatsii detonatsionnogo rezhima goreniya v kamere sgoraniya sverkhzvukovogo pryamotochnogo vozdušno-reaktivnogo dvigatelya*, patent RU 2285143 C2 (zayavl. 10.12.2004, opubl. 10.10.2006). (in Russian)
- [9] K.E. Savelova, M.M. Alekseeva (Laptinskaya), S.A. Matveev, M.V. Chernyshov, J. Phys.: Conf. Ser., **1959**, 012043 (2021). DOI: 10.1088/1742-6596/1959/1/012043
- [10] M.V. Chernyshov, K.E. Savelova, Fluids, **8** (4), 132 (2023). DOI: 10.3103/S1068799823010087
- [11] M.V. Chernyshov, M.M. Laptinskaya, Tech. Phys. Lett., **52** (1), 6 (2026).]
- [12] A.E. Medvedev, J. Appl. Mech. Tech. Phys., **42** (2), 211 (2001). DOI: 10.1023/A:1018811516116
- [13] K.E. Savelova, *Regulyarnoe i makhovskoe otrazhenie gazo-dinamicheskikh razryvov s energoydeleniem*, kand. dis. (SPbGU, SPb., 2024). (in Russian)
- [14] V.M. Teshukov, J. Appl. Mech. Tech. Phys., **30**, 189 (1989). DOI: 10.1007/BF00852163.
- [15] H.G. Hornung, Shock Waves, **7**, 123 (1997). DOI: 10.1007/s001930050068
- [16] G. Ben-Dor, *Shock wave reflection phenomena* (Springer, Berlin–Heidelberg–N.Y., 2007).
- [17] A.L. Adrianov, A.L. Starykh, V.N. Uskov, *Interferentsiya stacionarnykh gazo-dinamicheskikh razryvov* (Nauka, Novosibirsk, 1995) (in Russian).

Translated by E.Ilyinskaya