

Propagation of fluctuations in the presence of shear layers in viscous incompressible fluid flows with rotation

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The propagation of periodic and random-in-time velocity fluctuations in flows formed under the rotation of spherical boundaries has been numerically investigated. Disturbances of all types are introduced into the flow by oscillations or fluctuations in the rotation velocity of the inner sphere. Regions with equal values of damping decrements of the flow velocity pulsations along the radius were found for all types of noise. Spectra of flow velocity pulsations similar to the spectra of turbulent flows have been revealed. At a distance from the inner sphere, the amplitudes of the flow velocity pulsations can either decrease or increase depending on the type of noise spectrum.

Keywords: noise, spherical Couette flow.

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Propagation of disturbances in flows generated by external sources, including periodic oscillations and noise, has already been studied for a long time [1,2]. Some cases of practical applications of such research in technology are presented in [2]. In geophysics, the use of noise to parameterize subgrid processes in turbulence calculations is of interest [3,4]. In studying the noise and wave propagation in flows, a number of analytical solutions have been obtained [2,5–7] mainly in one-dimensional formulation. The spherical Couette flow (SCF) considered in this paper, that is, a boundary-rotation-induced flow of viscous incompressible fluid between coaxial spheres, is always three-dimensional. In the case of periodic variations in the speed of inner sphere rotation relative to the state of rest, radius-exponential damping of the pulsation amplitude of velocity azimuthal component is observed [8]. Previously, the noise exponential damping was obtained analytically in one-dimensional and linear approximations in the limit of high-frequency spectra [2]. When in SCF the boundary oscillates with two independent frequencies, the damping character may change due to nonlinear interaction between the high— and low-frequency modes (see, e.g., Fig. 6 from [8]). In [9], integral characteristics of flows generated by boundary rotation with equal angular speeds were obtained by direct calculation with noise added to the rotation speed signal. However, propagation of noise over the spherical layer was not studied in [9]. Experimentally, amplitudes of velocity fluctuations were obtained during rotation of only the inner sphere for two types of noise [10]. Propagation of velocity fluctuations in three-dimensional flows of viscous incompressible fluid with shear layers and rotation has not been studied previously; this was the goal of the present study.

In this study, disturbances stem from periodic and random-in-time zero-mean fluctuations introduced into the

inner sphere rotation velocity. Below the propagation of fluctuations in flows is considered. In natural systems, dependence of the noise power spectrum on frequency f may have the following form: $1/f^\alpha$, $0 < \alpha < 2$ [11]. In this work we have chosen noises with $\alpha = 0.1, 1$ and 2 . We consider two types of flows remaining stable under stationary rotation of boundaries, which are formed under unidirectional (with equal angular speeds) and counter-directional rotation of the spheres. A shear layer gets formed in the flow in both cases. In the first case, it is a cylindrical shear layer parallel to the rotation axis [9,12]. In the other case, the shear layer separates the regions where both the azimuthal speed and meridional circulation are directed oppositely [13]. The flows are considered in a spherical layer of relative thickness $\delta = (r_2 - r_1)/r_1 = 1$ where r_1 and r_2 are the radii of the inner and outer boundaries, respectively. The flows under consideration are described by the Navier–Stokes and continuity equations

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{U} \times \text{rot} \mathbf{U} - \text{grad} \left(\frac{p}{\rho} + \frac{\mathbf{U}^2}{2} \right) - \nu \text{rot} \text{rot} \mathbf{U},$$

$$\text{div} \mathbf{U} = 0,$$

where $\mathbf{U}$ is the velocity field, p is the pressure, ρ is the density, ν is the kinematic viscosity of fluid in the layer. On the boundaries, no-slip and no-flow conditions are set, and a spherical frame of reference with radial (r), polar (θ) and azimuthal (φ) directions is used: $u_\varphi(r = r_k) = \Omega_k(t)r_k \sin \theta$, $u_r(r = r_k) = 0$, $u_\theta(r = r_k) = 0$, $k = 1, 2$, $k = 1$ is the inner sphere, $k = 2$ is the outer sphere, u_φ , u_r , u_θ are the azimuthal, radial and polar velocity components, respectively, Ω_1 and Ω_2 are the average angular speeds of boundary rotation. In calculation, we used a numerical method with a finite-difference

scheme for spatial discretization of the Navier–Stokes equations and semi-implicit third-order-accuracy Runge–Kutta scheme for time integration. The algorithm was tested in [14]; that for the case of a three-dimensional problem involving non-uniform r and θ grids was considered in [15]. The scheme of time integration involves estimation of local error and automatic selection of time step [14]. Being discretized in space, the Navier–Stokes equations preserve a number of their properties: e.g., kinetic energy conservation by nonlinear and pressure-gradient terms [14]. When both noise and periodic fluctuations are introduced, all the flow velocity components depend on time, but symmetries with respect to the rotation axis and equator, which are inherent to the case of stationary rotating boundaries, are preserved. For the case when noise is introduced into the co-directional rotation of spheres at $\text{Re}_1 = (\Omega_1 r_1^2)/\nu = 500$, $\Omega_2 = \Omega_1$, this is confirmed in [9] by three-dimensional calculations; in this study, the same is confirmed for counter-rotation at $\text{Re}_1 = (\Omega_1 r_1^2)/\nu = 280$ and $\text{Re}_2 = (\Omega_2 r_2^2)/\nu = -700$. The presence or absence of the flow axial symmetry was established based on parameter A_{3D} proposed in [14]:

$$A_{3D}(t) = \frac{1}{V} \sum_{\beta} \int_V (u_{\beta}(r_i, \theta_j, \varphi_k, t) - u_{\beta av}^{\varphi}(r_i, \theta_j, t))^2 dv_{ijk},$$

$$\beta = r, \theta, \varphi,$$

$$u_{\beta av}^{\varphi}(r_i, \theta_j, t) = \frac{1}{K_{\varphi}} \sum_1^{K_{\varphi}} u_{\beta}(r_i, \theta_j, \varphi_k, t).$$

Here $u_{\beta}(r_i, \theta_j, \varphi_k, t)$ and $u_{\beta av}^{\varphi}(r_i, \theta_j, t)$ are one of the velocity components and its value averaged over angle φ , K_{φ} is the number of nodes in the azimuthal direction, V is the spherical layer volume. Calculations showed that in the case of noise-free counter-rotating boundaries $A_{3D} < 5 \cdot 10^{-17}$, while in the presence of noise $A_{3D} < 8 \cdot 10^{-17}$; this indicates preservation of axial symmetry of the noisy flow. Below we present the results obtained in calculations for the cases of symmetry with respect to the rotation axis and equatorial plane. Spatial discretization was performed with decreasing the near-boundary cell size (r) so that the ratio of maximum to minimum cell size was 2. The results' dependence on the computational grid parameters for co-directional rotation was considered in [9] where the chosen computational grid was identical to that used in this study. In the case of counter-rotation, the results' convergence to the same values with increasing number of grid nodes is demonstrated in the Table by using the example of azimuthal E_{φ} and meridional E_{ψ} components of the flow kinetic energy. E_{φ} and E_{ψ} are determined by integrating relevant flow velocity components over the entire spherical layer volume: $E_{\varphi} = \int u_{\varphi}^2(r, \theta, t)$, $E_{\psi} = \int (u_r^2(r, \theta, t) + u_{\theta}^2(r, \theta, t))$. In the case of co-directional rotation, the number of computational grid nodes was $3.864 \cdot 10^4$, while in the case of counter-rotation it was $1.8 \cdot 10^4$. The calculations were performed with a constant time step: $dt = 6 \cdot 10^{-4}$ s in the first case and $dt = 1 \cdot 10^{-2}$ s in

Dependence of the meridional E_{ψ} and azimuthal E_{φ} components of the flow kinetic energy on the number of computational grid nodes G in the case of counter-rotating spherical boundaries

$G, 10^4$	$E_{\psi}, 10^{-5} \text{ m}^2/\text{s}^2$	$E_{\varphi}, 10^{-3} \text{ m}^2/\text{s}^2$
0.29	2.452	8.927
0.45	2.443	8.924
0.88	2.435	8.922
1.46	2.432	8.921
1.80	2.431	8.920
2.00	2.431	8.920

the second one. We used the following dimensional parameters meeting the experimental conditions [13]: $\nu = 5 \cdot 10^{-5} \text{ m}^2/\text{s}$, $r_2 = 0.15 \text{ m}$, $r_1 = 0.075 \text{ m}$. In the case of periodic disturbances, the inner boundary rotation speed varied as $\Omega(t_j) = \Omega_1 + A \sin(2\pi f t_j)$, A and f are the disturbance amplitude and frequency. In the presence of noise $\Omega(t_j) = \Omega_1 + N r n(j)$, where

$$N = \frac{1}{\Omega_1} \sqrt{\frac{1}{K-1} \sum_{j=1}^K (\Omega(t_j) - \Omega_1)^2}$$

is the noise amplitude, $r n(j)$ is the pseudo-random number from the sequence K in length with the standard normal distribution and zero mean, and j is the time step counter. The sequence of random numbers was calculated for each type of noise in advance. The calculations were performed at $N = 0.04$, $K = 3 \cdot 10^4$ for counter-rotating boundaries and $K = 7 \cdot 10^5$ for unidirectional rotation.

First consider the flows in the case of unidirectional rotation of spheres. Figs. 1, *a* and 2 show the dependences of normalized velocity pulsation standard deviations (amplitude) D on the normalized inter-sphere distance R . Here $D = (\text{rms } u_{\varphi})/(\Omega_1 r_1)$, $R = (r - r_1)/(r_2 - r_1)$, r is the distance from the sphere

centers, $\text{rms } u_{\varphi} = \sqrt{\frac{1}{K-1} \sum_{i=1}^K (u_{\varphi N}(t_i) - \bar{u}_{\varphi N})^2}$, $u_{\varphi N}(t_i)$ is the

instantaneous azimuthal velocity in the presence of applied disturbances, the right term is averaged velocity $\bar{u}_{\varphi N}(t_i)$. If both periodic and random-in-time fluctuations are introduced, sections with rapid exponential damping of the velocity pulsation amplitude D are visible at all the latitudes near the inner sphere (Figs. 1, *a* and 2). At the equator (Fig. 1, *a*) near $R = 0.14$, damping decrements decrease abruptly for all the types of noise. The change in the noise damping character is associated with the flow structure: $R = 0.14$ at the equator corresponds to the shear layer center (the middle of the distance between the inner sphere and circulation separation line in Fig. 2, *a* from [9]). In the $0 < R < 0.14$ range, the fastest D damping with distance from the inner sphere (Fig. 1, *a*) takes place in the case of noise with spectrum slope $\alpha = 0.1$, while the slowest one is observed at $\alpha = 2$. In addition, Fig. 1, *a* presents the dependences of D variation on R for the case of periodic fluctuations in the inner boundary rotation speed. The fluc-

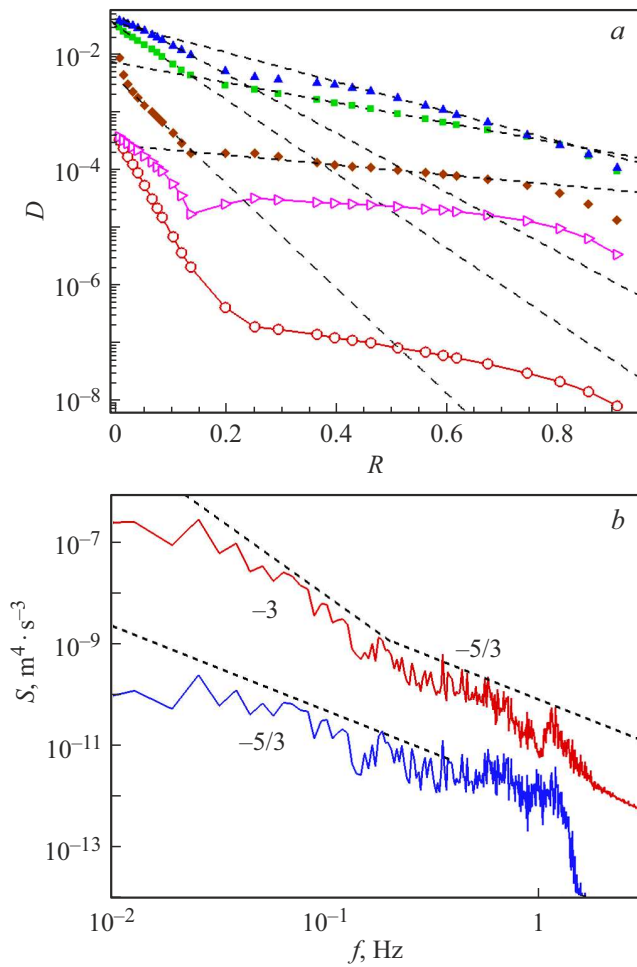


Figure 1. *a* — amplitude D dependence on the inter-sphere distance R in the equatorial plane ($\theta = \pi/2$) in the case of unidirectionally rotating boundaries. Solid symbols represent noise, open symbols are periodic fluctuations; for clarity, the latter are shifted down along the ordinate axis (with 50-fold reduction). Blue triangles are for $\alpha = 2$, green squares are for $\alpha = 1$, brown diamonds are for $\alpha = 0.1$, purple triangles are for $f = 0.7$ Hz ($2\pi f < 2\Omega_1$), red circles are for $f = 4$ Hz ($2\pi f > 2\Omega_1$). Dashed lines represent approximation of sections with constant damping decrements. *b* — azimuthal velocity spectra in the case of unidirectional rotation of boundaries. The upper spectrum (red line) is for $\alpha = 1$, $R = 0.51$, $\theta = 0.3\pi$, the lower one (blue line) is for $\alpha = 0.1$, $R = 0.56$, $\theta = 0.12\pi$. The dashed lines represent approximation of sections with constant spectral slopes. The color version of figures is presented in the electronic version of the article.

tuations were considered at frequency $f = 0.7$ Hz belonging to the range where inertial waves exist ($2\pi f < 2\Omega_1$) and at frequency $f = 4$ Hz (beyond this range). Near $R = 0.14$, damping decrements of periodic disturbances change; especially strong variations take place at frequency $f = 0.7$ Hz. Fluctuations at frequency $f = 4$ Hz exhibit the lowest D .

Investigation of the spectra transformation depending on the introduced noise propagation has started long ago [2]

and continues at present [16]. Fig. 1, *b* shows the spectra of azimuthal velocity pulsations in the middle of the layer. The spectrum slopes in the frequency range not exceeding the inner sphere rotation frequency were found to be equal to the slopes of turbulent flow spectra and depend on α , R and θ . For instance, the noise spectrum with $\alpha = 0.1$ near the rotation axis transforms into a spectrum with the constant slope of $-5/3$ corresponding to the spectra of three-dimensional turbulence with a direct energy-transfer cascade (the lower curve in Fig. 1, *b*). The upper curve in Fig. 1, *b* with the spectrum slope of -3 at low frequencies and with the slope of $-5/3$ at higher frequencies is qualitatively similar to the spectra of upper atmosphere turbulence; it was obtained by introducing noise with $\alpha = 1$. Spectra of both types have been observed previously both in experiments and in direct numerical simulation of turbulence in SCF [13]. While qualitative characters of the

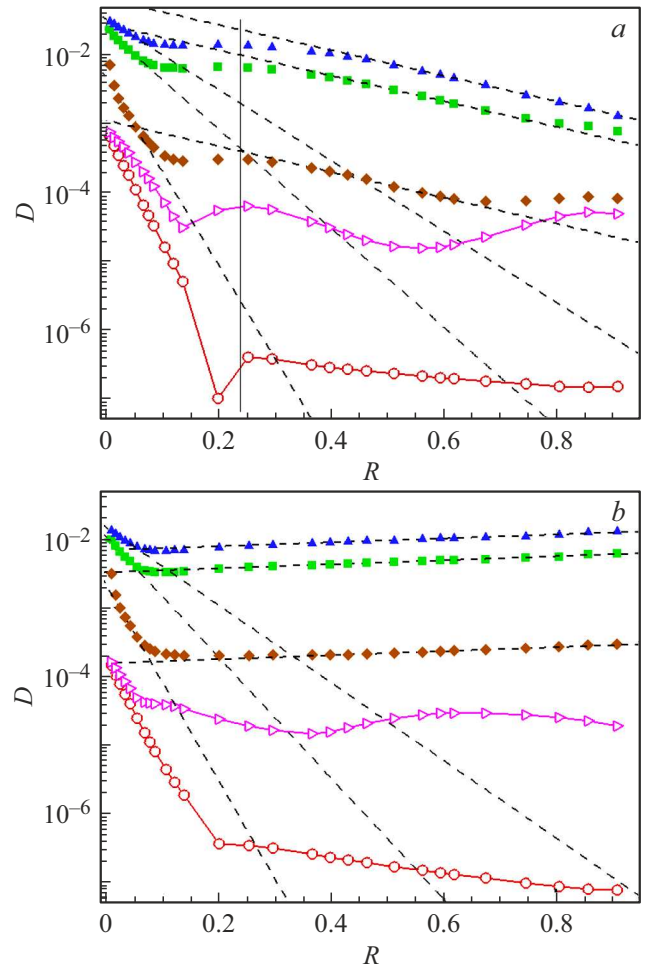


Figure 2. The amplitude D dependence on the inter-sphere distance R for the case of unidirectional rotation of boundaries. *a* — at mid-latitudes ($\theta = 0.3\pi$), *b* — near the rotation axis ($\theta = 0.12\pi$). The vertical line in panel *a* corresponds to the shear layer position at $\text{Re} \rightarrow \infty$. The designations are the same as in Fig. 1, *a*. For clarity, curves corresponding to periodic fluctuations (purple and red lines) are shifted down the ordinate axis (*a* — reduced 20-fold, *b* — reduced 40-fold).

D damping during propagation in the equatorial plane are the same for noise and periodic fluctuations (Fig. 1, *a*), those outside the equator appear to be somewhat different (Fig. 2). For all the types of noise, at mid-latitudes (Fig. 2, *a*) there are observed flow sections between the shear layer and outer sphere, where damping decrements are identical: $\ln D \approx -BR$, where $B = 4.29 \pm 0.02$. Lengths of such sections depend on the type of noise (the shortest one is at $\alpha = 0.1$). The character of propagation of periodic medium-frequency fluctuations is more complex. Near the rotation axis (Fig. 2, *b*), sections where D increases with the same increments exist for all the types of noise. In these sections $\ln D \approx AR$, where $A = 6.72 \pm 0.05$. In the case of periodic fluctuations at frequency $f = 4$ Hz, near the rotation axis only damping of the velocity pulsation amplitude D is observed. In all the cases, deviation from average noise decrements/increments does not exceed 0.75 %. In almost all flow regions, condition $D_2 > D_1 > D_{0.1}$ is satisfied, where subscripts represent the α values (Figs. 1, *a* and 2), i. e., velocity pulsation amplitude D increases with increasing α . This allows us to explain the integral results obtained earlier in [9], which show that, while the noise amplitude remains the same, standard deviations of pulsations in the flows' kinetic energy azimuthal component are more than an order of magnitude higher at $\alpha = 1$ than at $\alpha = 0.1$.

Contrary to the case of unidirectional boundary rotation, in the case of counter-rotation either decrease and increase in amplitude is observed in the flow's equatorial plane depending on the periodic fluctuation frequency and the type of noise spectrum (Fig. 3, *a*). In the region between the inner sphere and line of zero azimuthal velocity ($0 < R < 0.15$), in the presence of low-frequency periodic fluctuations ($f = 0.05$ Hz) and noise with spectral slopes $\alpha = 1$ and 2, amplitude D increases. In the presence of high-frequency fluctuations ($f = 0.3$ and 1 Hz) and noise with $\alpha = 0.1$, a decrease in D is observed. The source of difference in behaviors of D may be assumed to be variations in intensity of mean flows generated by disturbances. Different mean-flow intensities were found out earlier in [9] in the case of unidirectional rotation at $\alpha = 0.1$ and 1. Consider the normalized difference between the radial velocity components: $U = (u_{r\alpha} - u_{r0.1})/\Omega_1 r_1$, where $u_{r\alpha}$ are the time-averaged radial velocities at $\alpha = 1$ and 2, and $u_{r0.1}$ is the time-averaged radial velocity at $\alpha = 0.1$ (Fig. 3, *b*). Note that the positions of U maxima are close to those of maximum velocity pulsation amplitudes D in Fig. 3, *a*. Thus, one of the probable sources of different characters of variations in the velocity pulsation amplitudes at different α is the radial velocity increase with increasing α .

Under the counter-rotation of boundaries (Figs. 3, 4), as well as under unidirectional rotation, minimum D values are observed at the same distances from the inner sphere in the presence of noise with spectrum slope $\alpha = 0.1$. In the presence of periodic fluctuations, D at the same R decreases with increasing frequency. In the case of counter-rotation, sections with equal damping decrements are observed in the flow for all the types of noise (Figs. 3, *a*

and 4). These sections begin beyond the zero azimuthal velocity line (vertical lines in Figs. 3, *a* and 4, *a*), but do not reach the outer boundary. At the equator (Fig. 3, *a*) $\ln D \approx -BR$, where $B = 11.8 \pm 0.1$. Near the axis of rotation (Fig. 4, *a*), the constant damping decrement may be represented as $\ln D \approx -BR$, $B = 3.97 \pm 0.02$. At the same time, in the case of periodic fluctuations, the damping decrement dependences on R are more complex and depend on the disturbance frequency both at the equator and near the rotation axis. In the case of counter-rotation, spectra of azimuthal velocity pulsations at the equator (Fig. 4, *b*) exhibit slopes characteristic of

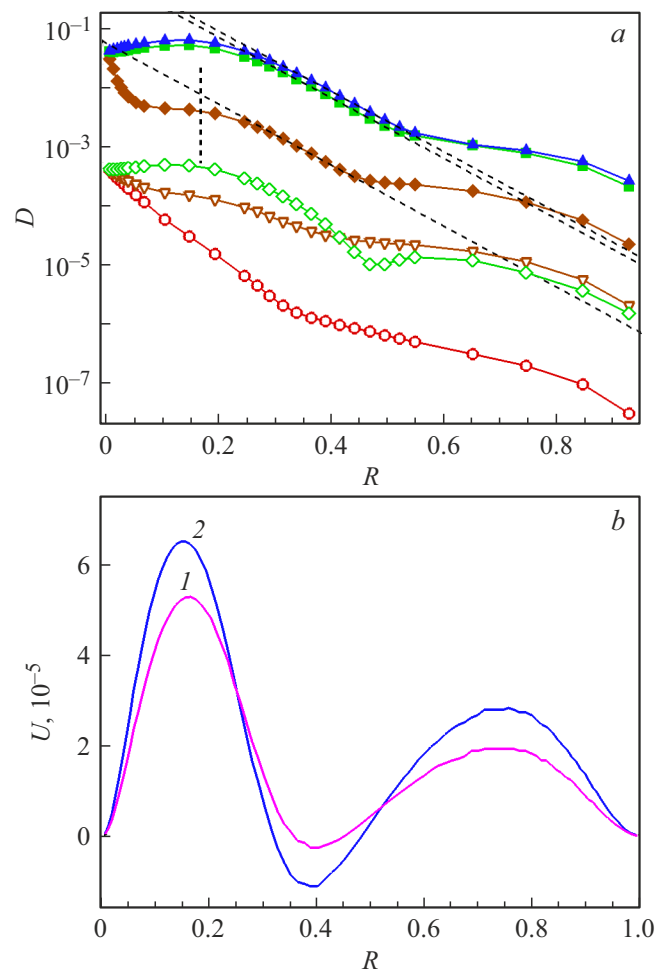


Figure 3. *a* — amplitude D dependence on the inter-sphere distance R in the equatorial plane in the case of counter-rotating boundaries. Solid symbols are the noise, open symbols are periodic fluctuations; for clarity, the latter are shifted down the ordinate axis (and reduced 100-fold). Blue triangles are for $\alpha = 2$, green squares are for $\alpha = 1$, brown diamonds are for $\alpha = 0.1$. The inclined lines represent approximation of sections with constant damping decrements. The vertical line represents the zero azimuthal velocity. Green diamonds are for $f = 0.05$ Hz, brown triangles are for $f = 0.3$ Hz, red circles are for $f = 1$ Hz. *b* — normalized difference in radial velocities $U = (u_{r\alpha} - u_{r0.1})/\Omega_1 r_1$ versus R in the equatorial plane for counter-rotating boundaries. $\alpha = 1$ (1) and 2 (2).

turbulent flows: -3 near the inner sphere and $-11/5$ near the outer sphere. Spectra with the slope of -3 corresponding to two-dimensional turbulence with an inverse energy transfer cascade were obtained in SCF both numerically and experimentally [13]. Spectra with the slope of $-11/5$ corresponding to three-dimensional turbulence with a direct energy transfer cascade were observed in experiments [17].

Thus, we have established the following facts for two different flows with different shear layers. During propagation, noise of all the considered types may be transformed into disturbances having all the spectra currently known for turbulence in spherical layers. The possibility of turbulent spectra formation depends on α : the higher is the α value, the larger is the flow part they are observed in. In the case of $\alpha = 0.1$ and unidirectional

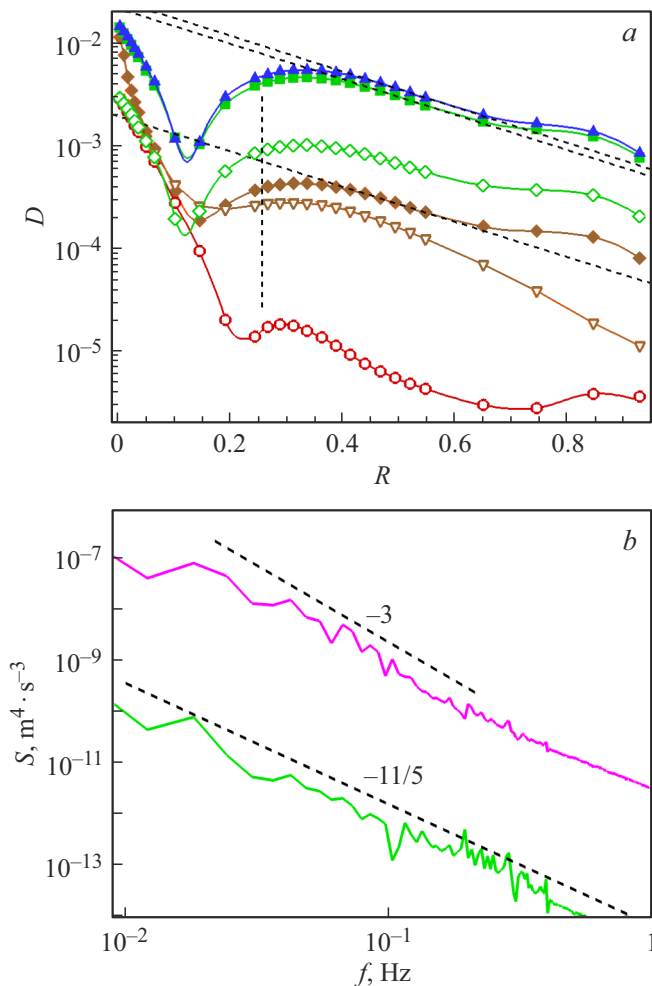


Figure 4. *a* — amplitude D versus inter-sphere distance R near the axis of rotation ($\theta = 0.12\pi$). Designations are the same as in Fig. 3, *a*. For clarity, representations of periodic fluctuations (green, brown and red curves with open symbols) are shifted down the ordinate axis (and reduced 5-fold). *b* — azimuthal velocity spectra in the case of counter-rotating spherical boundaries at $\alpha = 2$ and $\theta = \pi/2$. The upper line (purple) corresponds to $R = 0.145$, the lower line (green) is for $R = 0.93$. The dashed lines represent approximation of sections with constant spectrum slopes.

rotation of the boundaries, such spectra are observed at $R > 0.5$; in the case of counter-rotation, they are observed in most of the layer ($0.93 > R > 0.14$). For the counter-rotating layer boundaries, either an increase or decrease in the azimuthal velocity pulsation amplitude may be observed depending on the type of disturbance. In the region between the shear layer and outer sphere there were found sections with identical decrements of exponential flow velocity pulsation damping with increasing distance from the noise source, that is, inner sphere. The character of the periodic fluctuation propagation changes depending on the frequency and can either coincide with or differ from the noise propagation character.

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Conflict of interests

The author declares that he has no conflict of interests.

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