

Polarization regularities of hologram echo in the $\text{LaF}_3:\text{Pr}^{3+}$ crystal

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Received on May 30, 2025

Revised February 17, 2026

Accepted March 26, 2026

Polarization patterns of stimulated and accumulated echo hologram generation in a $\text{LaF}_3:\text{Pr}^{3+}$ crystal are examined. It is shown that these patterns enable the use of these signals to process and transform information embedded in the spatiotemporal structure of object laser pulses.

Keywords: stimulated echo hologram, pairs of excitation pulses, phase difference, polarization of excitation pulses.

DOI: 10.61011/EOS.2026.04.63759.4398-25

Introduction

The most promising information processing systems today are optical systems using coherent (lasers) sources of light. The areas of practical application of the optical information processing systems: mobile image recognition and processing systems, onboard orientation and guidance systems in the military equipment, faint signal resolvers operating under passive and active noise, radar stations with synthesized aperture, highly efficient computers, metrology, robotics, nondestructive testing. A photon echo processor (PEP) related to a class of multifunctional analog devices is of special interest. Due to the presence of control signals, its pulse characteristic may be programmed in real time mode, and various types of processing may be obtained — from simple memorizing to integral conversions. Besides, same as an optical processor, a PEP may operate in the time, frequency, space and space-time area performing coherent or incoherent processing of information [1]. PEP development assumes development of the methods to perform logical operations using signals of a stimulated echo hologram (SEH), accumulated stimulated echo hologram (ASEH), which makes it possible to process information included both in the images and in the time and frequency structures of exciting laser pulses [2].

This paper considered the opportunity of using SEH and ASEH to implement conversion and processing of information included into the laser pulses of pulse pairs using polarization properties of SEH and ASEH to control the efficiency of its processing. The advantage of such approach is simultaneous reproduction of information included into pulse pair in the ASEH response, which may considerably accelerate the operation of the photon echo processor.

Main equations

Polarization patterns of SEH and ASEH generation in a $\text{LaF}_3:\text{Pr}^{3+}$ crystal enable using their responses to convert information included into the space-time structure of object laser pulses.

The papers [3,4] showed for the first time that despite the fact that Pr^{3+} levels in LaF_3 are simple singlets, polarization and intensity of the response of the stimulated photon echo (SPE) depend on orientations of polarizations of exciting pulses. This is related to various locations (α, β, γ) of Pr^{3+} ions in an elementary cell of a LaF_3 crystal relative to the direction of the local axis of symmetry. In case of linear polarization of exciting laser pulses, the intensity of the electric field of SPE response is proportionate to the following factor [3]:

$$\mathbf{q} = \sum_{\xi=\alpha,\beta,\gamma} \mathbf{e}_{\xi} \sin[\vartheta_1(\mathbf{e}_{\xi} \cdot \mathbf{e}_1)] \times \sin[\vartheta_2(\mathbf{e}_{\xi} \cdot \mathbf{e}_2)] \cdot \sin[\vartheta_3(\mathbf{e}_{\xi} \cdot \mathbf{e}_3)], \quad (1)$$

where $\mathbf{e}_n$ — vector of polarization of n th laser pulse, ϑ_n — area of n th laser pulse, $\mathbf{e}_{\xi}$ — single vector in the direction of crystallographic axis $\xi = \alpha, \beta, \gamma$.

Fig. 1 shows the sequence of the exciting laser pulses in case of ASEH excitation in case of $j = 1, 2$ [5].

Further we will presume that if the exciting laser pulses are carriers of information included into the images, then the polarizations of intensity of the electric fields of flat waves of decomposition in space of wave fronts of object laser pulses in each j th pair will be identical.

To estimate the factor $\mathbf{q}$ for ASEH, let us introduce a system of coordinates (XOY), normal to the optical axis, along which the laser pulses spread. Let us introduce angles $\varphi_{\eta x}^{(j)}$ — between the direction of polarization of η th laser pulse in the j th pair and axis X , and also angles α_x ,

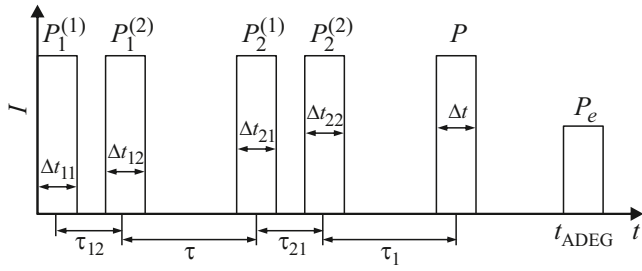


Figure 1. Sequence of exciting laser pulses in formation of ASEH pulses. $\tau_{12}, \tau_{21}, \tau, \tau_1 \ll T_1, T_2$, where T_1, T_2 — times of longitudinal and transverse irreversible relaxation of the resonant system. If there is no second pair of exciting pulses, the SEH mode is implemented.

β_x, γ_x — between crystallographic $\alpha_x, \beta_x, \gamma_x$ axes α, β, γ and axis X . Let us find projections of vector $\mathbf{q}$ in axis X and Y :

$$q_x^{(j)} = q_\alpha^{(j)} \cos(\alpha_x) + q_\beta^{(j)} \cos(\beta_x) + q_\gamma^{(j)} \cos(\gamma_x),$$

$$q_y^{(j)} = q_\alpha^{(j)} \sin(\alpha_x) + q_\beta^{(j)} \sin(\beta_x) + q_\gamma^{(j)} \sin(\gamma_x), \quad (2)$$

where

$$q_\alpha^{(j)} \sin[\vartheta_1^{(j)} \cos(\varphi_{1x}^{(j)} - \alpha_x)] \times \sin[\vartheta_2^{(j)} \cos(\varphi_{2x}^{(j)} - \alpha_x)] \sin[\vartheta_3 \cos(\varphi_{3x} - \alpha_x)],$$

$$q_\beta^{(j)} \sin[\vartheta_1^{(j)} \cos(\varphi_{1x}^{(j)} - \beta_x)] \times \sin[\vartheta_2^{(j)} \cos(\varphi_{2x}^{(j)} - \beta_x)] \sin[\vartheta_3 \cos(\varphi_{3x} - \beta_x)],$$

$$q_\gamma^{(j)} \sin[\vartheta_1^{(j)} \cos(\varphi_{1x}^{(j)} - \gamma_x)] \times \sin[\vartheta_2^{(j)} \cos(\varphi_{2x}^{(j)} - \gamma_x)] \sin[\vartheta_3 \cos(\varphi_{3x} - \gamma_x)],$$

$\vartheta_1^{(j)}$ and $\vartheta_2^{(j)}$ — areas of the first and second exciting laser pulses in the j th pair, ϑ_3 — area of the reading pulse.

Similarly to papers [2,6], the space-time structure of ASEH response with account of polarization will be determined using the equation

$$I \sim \mathbf{E}_{\text{ASEH}} \cdot \mathbf{E}_{\text{ASEH}}^*, \quad (3)$$

$$\mathbf{E}_{\text{ASEH}} = \sum_{j=1}^N \mathbf{E}_j(\mathbf{R}, t) e^{i\Delta\varphi_{1j}}, \quad (4)$$

where N is the number of pairs of exciting laser pulses, and components of vector $\mathbf{E}_j = (\mathbf{r}, t)$ in axes X and Y generally

have the following appearance

$$E_{jx}(\mathbf{r}, t) \sim \sum_{n', n'', n'''} \int_V dV \int_{-\infty}^{+\infty} g(\Delta) d\Delta \cdot q_x^{(j)} \cdot S_1^{(j)*} \times S_2^{(j)}(\Delta) S_3(\Delta) e^{i\Delta(t-t_e^{(j)})} \times \frac{\varepsilon_{1n'}^{*(j)} \varepsilon_{1n''}^{*(j)} \varepsilon_{2n'''}^{(j)} \varepsilon_{3n'''}^{(j)}}{\left| \sum_{n'} \varepsilon_{1n'}^{*(j)} e^{-i\mathbf{k}_{1n'}^{(j)} \cdot \mathbf{r}} \right| \left| \sum_{n''} \varepsilon_{2n''}^{(j)} e^{-i\mathbf{k}_{2n''}^{(j)} \cdot \mathbf{r}} \right| \left| \sum_{n'''} \varepsilon_{3n'''}^{(j)} e^{-i\mathbf{k}_{3n'''}^{(j)} \cdot \mathbf{r}} \right|} \times e^{-i(\mathbf{k}_{en}^{(j)} + \mathbf{k}_{1n'}^{(j)} - \mathbf{k}_{2n''}^{(j)} - \mathbf{k}_{3n'''}^{(j)}) \cdot \mathbf{r}}, \quad (5)$$

$$E_{jy}(\mathbf{r}, t) \sim \sum_{n', n'', n'''} \int_V dV \int_{-\infty}^{+\infty} g(\Delta) d\Delta \cdot q_y^{(j)} \cdot S_1^{(j)*} \times S_2^{(j)}(\Delta) S_3(\Delta) e^{i\Delta(t-t_e^{(j)})} \times \frac{\varepsilon_{1n'}^{*(j)} \varepsilon_{1n''}^{*(j)} \varepsilon_{2n'''}^{(j)} \varepsilon_{3n'''}^{(j)}}{\left| \sum_{n'} \varepsilon_{1n'}^{*(j)} e^{-i\mathbf{k}_{1n'}^{(j)} \cdot \mathbf{r}} \right| \left| \sum_{n''} \varepsilon_{2n''}^{(j)} e^{-i\mathbf{k}_{2n''}^{(j)} \cdot \mathbf{r}} \right| \left| \sum_{n'''} \varepsilon_{3n'''}^{(j)} e^{-i\mathbf{k}_{3n'''}^{(j)} \cdot \mathbf{r}} \right|} \times e^{-i(\mathbf{k}_{en}^{(j)} + \mathbf{k}_{1n'}^{(j)} - \mathbf{k}_{2n''}^{(j)} - \mathbf{k}_{3n'''}^{(j)}) \cdot \mathbf{r}}, \quad (6)$$

where $\mathbf{k}_{in}^{(j)}$ — wave vectors of flat waves of decomposition in space of wave fronts of object laser pulses for each j th pair, V — volume of the excited part of the sample, $g(\Delta)$ — distribution of optical centers by frequencies, $\Delta = \omega - \Omega_0$, ω — frequency of laser emission, Ω_0 — frequency of resonant transition, $S_1^{(j)*}(\Delta)$ and $S_2^{(j)*}(\Delta)$ — spectra of the first and second pulses in the j th pair, $S_3(\Delta)$ — spectrum of the reading pulse,

$$S_\eta^{(j)}(\Delta) = \int_{t_\eta^{(j)} - \frac{\Delta t_\eta^{(j)}}{2}}^{t_\eta^{(j)} + \frac{\Delta t_\eta^{(j)}}{2}} \epsilon_\eta^{(j)}(t) e^{i\hbar^{-1}\Delta \cdot t} dt$$

$\epsilon_\eta^{(j)}(t)$ — temporal shape of the η th pulse in the j th pair, $t_\eta^{(j)}$ — moment of exposure time of the η th pulse in the j th pair, $\Delta t_\eta^{(j)}$ — width of the η th pulse in the j th pair, $\varepsilon_{in}^{(j)}$ — amplitude of intensity of electric fields of flat waves of decomposition in space of wave fronts of object laser pulses in each j th pair, $\Delta\varphi_{1j}$ — phase of the j th pair of pulses relative to the first one, $\Delta\varphi_{11} = 0$, $t_e^{(j)}$ — moment of time of response appearance from the j th pair of exciting laser pulses.

Note that if exciting laser pulses are carriers of information included into the images, then the space phase synchronism in formation of the ASEH response will have the following appearance $\mathbf{k}_{en}^{(j)} = -\mathbf{k}_{1n'}^{(j)} + \mathbf{k}_{2n''}^{(j)} + \mathbf{k}_{3n'''}^{(j)}$. It

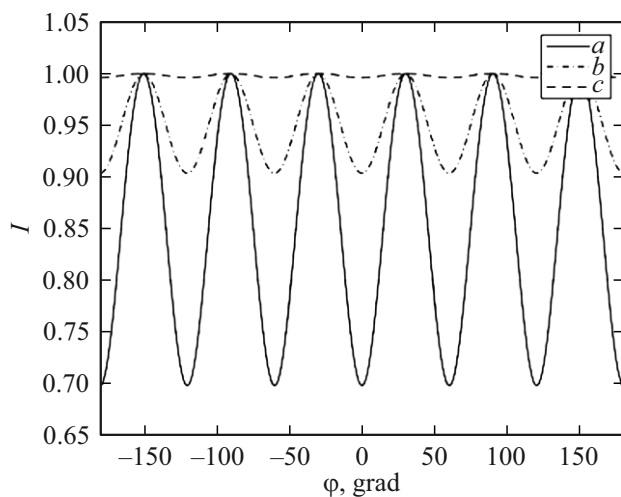


Figure 2. Dependence of SEH response intensity from areas of exciting pulses and orientation of their polarizations relative to crystallographic axes: $\vartheta = \pi/2$ (solid line), $\pi/4$ (dash-dot line), approximation of the small area $\vartheta = \pi/20$ (dashed line).

should be noted that only those components of decomposition of the response field will exist in the ASEH response, for which the values are non-zero for the amplitudes of decomposition of the fields of exciting pulses corresponding to the directions of the wave factors. Pulses that carry no images must be formed by transparent masks with matt structures, to create the sufficient set of flat (spherical) waves necessary to perform the space phase synchronism.

In paper [7] it was shown that the development of the phase difference $\Delta\varphi_{1j}$ between pairs of exciting pulses may cause in reduction or disappearance of frequency modulations of populations upon response formation, which results in its disappearance. Therefore, if the phases between pairs of exciting pulses differ from each other by π , the intensity of the ASEH signal decreases significantly for the elements of images identical in the transparent masks in the first and second pairs of exciting pulses (information erasure effect). Variation of phases and polarizations of exciting pulses enable control of the space-time structure of the ASEH response.

Polarization patterns of SEH and ASEH in a crystal $\text{LaF}_3:\text{Pr}^{3+}$

Consider the case when in (4) $N=1$, i.e. SEH is formed with three-pulse excitation. Let polarizations of all three pulses and their areas be the same, then it follows from equations (4)–(6) that the intensity of the SEH response depends on the areas of the exciting pulses and orientation of their polarizations relative to the crystallographic axes (Fig. 2).

In the estimation, the crystallographic axes were selected at angles to axis X : $\alpha = 0$, $\beta = 2\pi/3$, $\gamma = 4\pi/3$, and areas

of pulses $\vartheta_1 = \vartheta_2 = \vartheta_3 = \vartheta = (\pi/2, \pi/4, \pi/20)$. From Fig. 2 it follows that the dependence of the response intensity on the orientation of the polarizations of exciting pulses relative to the crystallographic axes disappears with the decrease in the areas of pulses. The largest change in the intensity of the SEH response $k = I_{\max}/I_{\min}$ with the change of the orientation of polarizations of exciting pulses happens at large areas of pulses and $k \approx 1.4$.

Consider the case when polarizations of two pulses coincide $\varphi_{2x} = \varphi_{3x} = \varphi$, and $\varphi_{1x} = 0$ (Fig. 3).

From Fig. 3 it follows that under no orientations of pulse polarizations the intensity of the SEH response is not equal to zero, and the largest change in the intensity of the SEH

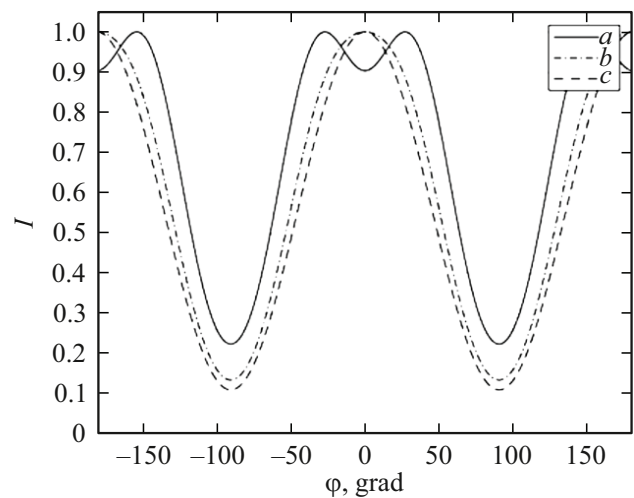


Figure 3. Dependence of the intensity of the SEH response on the areas of exciting pulses and orientation of their polarizations relative to the crystallographic axes at $\varphi_{2x} = \varphi_{3x} = \varphi$ and $\varphi_{1x} = 0$: $\vartheta = \pi/2$ (solid line), $\pi/4$ (dash-dot line), approximation of the small area $\vartheta = \pi/20$ (dashed line).

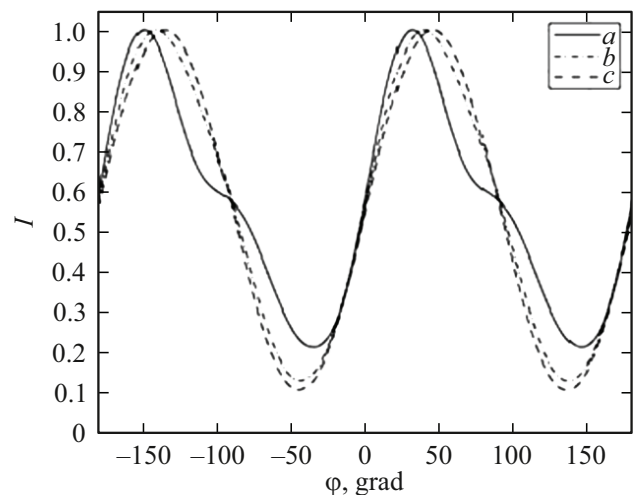


Figure 4. Dependence of the intensity of the SEH response on the areas of exciting pulses and orientation of their polarizations relative to the crystallographic axes at $\varphi_{2x} = \varphi_{3x} = \varphi$ and $\varphi_{1x} = \pi/4$: $\vartheta = \pi/2$ (solid line), $\pi/4$ (dash-dot line), approximation of the small area $\vartheta = \pi/20$ (dashed line).

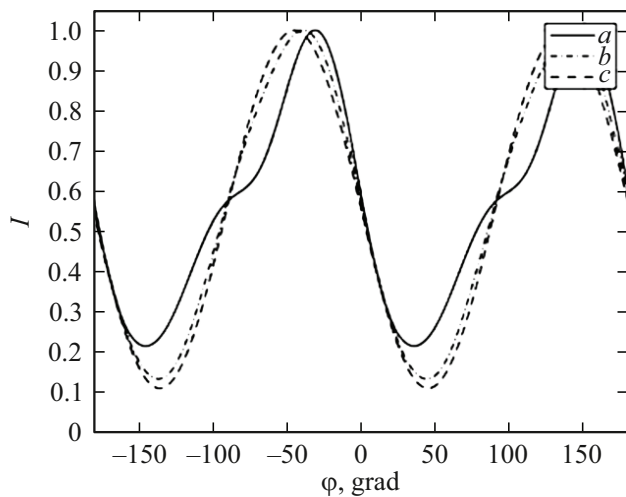


Figure 5. Dependence of SEH response intensity on areas of exciting pulses and orientation of their polarizations relative to crystallographic axes at $\varphi_{2x} = \varphi_{3x} = \varphi$ and $\varphi_{1x} = \pi/4$: $\vartheta = \pi/2$ (solid line), $\pi/4$ (dash-dot line), approximation of small area $\vartheta = \pi/20$ (dashed line).

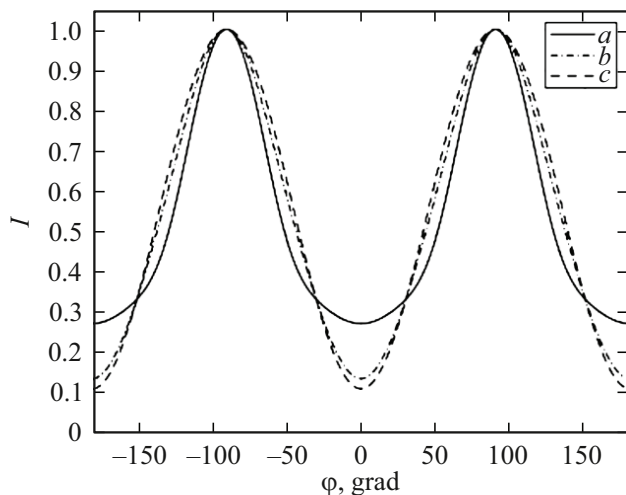


Figure 6. Dependence of SEH response intensity on areas of exciting pulses and orientation of their polarizations relative to crystallographic axes at $\varphi_{2x} = \varphi_{3x} = \varphi$ and $\varphi_{1x} = -\pi/4$: $\vartheta = \pi/2$ (solid line), $\pi/4$ (dash-dot line), approximation of small area $\vartheta = \pi/20$ (dashed line).

response with the change in the orientation of polarizations of exciting pulses happens at small areas of pulses and $k \approx 8.9$. At large areas of pulses $k \approx 4.4$.

Upon polarization of the first pulse the dependence of the SPE response intensity on orientation of polarizations of exciting pulses is given in Fig. 4 and 5.

From Fig. 4 and 5 it follows that in the considered case the values change for the angles of orientation of the polarizations of pulses corresponding to the maximum and minimum values of SPE intensity.

Upon polarization of the first pulse $\phi_{1x} = \pm\pi/2$ the dependence of the SEH response intensity on orientation of polarizations of the exciting pulses is shown in Fig. 6.

Compared to Fig. 3 ($\phi_{1x} = 0$), in Fig. 6 the values changed mutually for the angles of orientation of polarizations corresponding to the maximum and minimum values of SEH intensity, and in case of large areas of pulses the value $k \approx 3.5$ decreased.

Note that from equation (4) it follows that the ASEH response is a sum of SEH responses. Therefore the obtained polarization properties of SEH may be spread on the ASEH response, too.

Conclusions

Polarization patterns of SEH and ASEH generation in $\text{LaF}_3:\text{Pr}^{3+}$ crystal enable using signals to convert information included in the space-time structure of object laser pulses. If polarizations of all pulses are identical, in the ASEH response they are summed up at zero phase difference and are subtracted at phase difference equal to π . If exciting laser pulses in different pairs are somewhat shifted in time, in the first case the response will contain a broad pulse, and in the second case two narrow pulses will be generated.

If pulse polarizations differ, the response polarization will be same as that of the reading pulse, and intensities of the response components depend on all angles of polarizations. For example, if the polarization of the second pair pulses forms angle $\pi/2$ with the first pair, for all components of responses to be reproduced, the reading pulse must have the polarization at angle $\pi/4$. Therefore, the analyzer must have the same. The analyzer in this case may change the total intensity of the response. Selection of polarizations at the phase difference equal to π makes it possible to change the duration of the response pulse, besides, angles of polarization directions may vary within $0 - \pi/2$.

Funding

The paper was completed at the expense of the funds of the Program of Strategic Academic Leadership of Kazan (Privolzhsky) Federal University („PRIORITY-2030“).

Conflict of interest

The authors declare no conflict of interest.

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Translated by M.Verenikina