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IR shielding of window panes

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The possibilities of making coatings on window panes that reduce the transmission of the IR range by several orders of magnitude while maintaining transparency in the optical range are considered. The coatings are supposed to consist of several metallic nanoscale layers alternating with layers SiO_2, as well as in the form of metal nanoparticles embedded in glass. The effect of resonant photon tunneling is supposed to be used for transparency in the optical range.

Keywords: window panes, dielectric constant, homogenization, resonant tunneling, plasmons, metal nanoparticles.

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Introduction

Window panes are currently subject to very high requirements of various kinds [1]. One of the most important requirements is the property of not passing IR radiation and passing the entire optical range. This is due to the fact that the walls of buildings practically do not transmit IR radiation, but ordinary window panes transmit it very well. The implementation of IR-shielding window panes allows you to save from 10 to 30 percent or more of the energy needed for heating or cooling buildings, which, with relatively low costs for glass production technology, allows saving huge amounts of money on the scale of a house, city or country. At room temperature T_0 of the order of 300 K inside the building, the thermal radiation in it can be considered to be Planck equilibrium with a fairly good approximation, i.e. equivalent to the radiation of an absolutely black body. Let the room temperature be $T_0 = 27\,^{\circ}\mathrm{C}$. If the outdoor temperature is $T = -23\,^{\circ}\mathrm{C}$, which is often the case in winter, then with a temperature difference of about $50\,^{\circ}$ the heat flow through the windows of the building to the outside (excluding small reflection and absorption in the glasses) is very high (394 W per square meter). If the outdoor temperature in summer is $T = 37\,^{\circ}\mathrm{C}$ (which is also often the case), then the temperature difference will be ten degrees, and the corresponding flow inside is six times less, but also significant. The surface area of the windows can be on the order of 20 % of the building surface or even more. If there are no other heat leaks in the building, then the main heat flow goes through the windows. The maximum density of thermal radiation

$$u_p = \hbar\omega^3 n/(\pi^2 c^3),$$

where the Bose-Einstein function is introduced

$$f_{FD}(\omega, T) = n = [\exp(\hbar\omega/(k_B T)) - 1]^{-1}$$

(the chemical potential for photons is zero). If the window area is S, then the energy flow per second through almost transparent window panes is $P = \sigma_{SB} S(T^4 - T_0^4)$ W. Here σ_{SB} is the Stefan-Boltzmann constant. The maximum of the flow at temperatures 300 ± 50K lies in the wavelength range from 8.27 to 11.59 μm, i.e., in the middle of the IR range. Thus, the task is to shield the IR range with a maximum of shielding for wavelengths of the order of 10 μm with sufficiently good transmission in the optical region. To achieve optical transparency, the glass structure must be symmetrical with respect to the center (Fig. 1, *a*, *b*), while providing resonant transmission at the central frequency of the optical range of the order of 550 nm. The structure on Fig. 1, *a* is preferable because its transparency characteristic is less rugged. However, the problem is also solved by asymmetric structures (Fig. 1, *c*, *d*), including quasi-periodic ones. They are technologically more difficult to manufacture. The structure of Fig. 1, *a* can be obtained by applying a reflective layer on two halves of glass and then connecting them. Metals reflect electromagnetic radiation well. In optical technology, thin metal films with a thickness of tens of nm are translucent. In the microwave range, the penetration depth of the field is significantly greater than such thicknesses and increases with increasing wavelength, while the reflection coefficient decreases. Multilayer screens in Ref. [2,3] with such films and alternating layers of dielectric, for example SiO_2, located on window glass, are considered. They can be made on glass several mm thick. Then the screening task under consideration is to synthesize the optimal structure of such a coating on glass. At the same time, strong suppression of IR transmission is possible while maintaining good transparency in the optical range [2,3]. In the microwave range, transparency also improves with increasing wavelength, which, for example, is important for cellular communications. However, it is technologically difficult and expensive to produce a multilayer coating. The simplest, cheapest, and most reflective coating can consist of two or three layers.

The purpose of this study is to obtain simple ratios that make it possible to calculate three-layer coatings with two

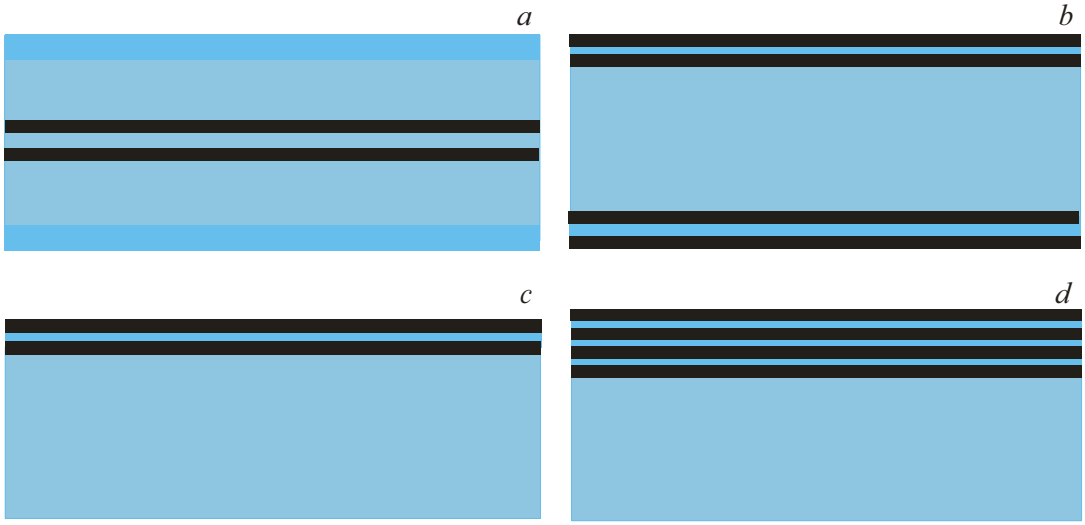


Figure 1. Schematic representation of asymmetrical (a), symmetrical (b) and (c), as well as quasi-periodic (d) with four periods of structures on window glass: dark layers — metallic, light blue — glass dielectric.

thin metal films, to strictly evaluate the effectiveness of such coatings, and to substantiate the possibility of obtaining coatings by embedding metal nanoparticles in glasses (see, for example, [1]).

Metal layered structures

We will model the dielectric constant of a metal using the Drude-Lorentz formula

$$\varepsilon_m = \varepsilon_L - \omega_p^2/(\omega^2 - i\omega\omega_c), \qquad (1)$$

In this case, the Lorentz term ε_L, which determines the optical properties of the metal, will be considered a real constant of the order of 10, and the plasma frequency $\omega_p = 2\pi c/\lambda_p$ and the collision frequency ω_c will be set, determining them from experimental data. The consideration of resonant frequencies in the Lorentz term is important in the UV domain, where electronic interband transitions and transitions between atomic levels should be taken into account. For our purposes, it is quite sufficient to consider this member permanent. So, for silver, you can take $\varepsilon_L = 9.3$, $\omega_p = 1.57 \cdot 10^{16}$ Hz, $\omega_c = 3.56 \cdot 10^{13}$ Hz, and for copper, $\varepsilon_L = 13.09$, $\omega_p = 1.65 \cdot 10^{16}$ Hz, $\omega_c = 5.41 \cdot 10^{13}$ Hz. These parameters are determined by knowing the concentration of free electrons in the conduction band, the DC conductivity $\sigma_0 = \varepsilon_0\omega_p^2/\omega_c$, and the experimental frequency of the $\mathrm{Re}(\varepsilon_m(\omega))$ value passing through zero. To determine the optical properties of metals, we use the results of study in Ref. [4,5]. Silver is expensive to use, so acceptable metals are copper, aluminum, nickel, tungsten, chromium, cobalt, lead, iron, and some others. We will consider a three-layer structure applied to glass in the form of two metal layers of thickness t_m, separated by a dielectric layer of thickness t_d (Fig. 1, a). The dielectric layer is considered as SiO_2 with a dielectric permeability (DP) of $\varepsilon_d = 3.2$. If the DP of the glass is different, which is possible with changes in density, temperature, and the addition of impurities, then the dimensions of the structure can be adjusted by changing them in proportion to the square root of the permeability ratio. The structure of Fig. 1, b better shields the IR range, but is technologically more difficult to manufacture and more expensive. The same applies to the quasi-periodic structure (d). A rigorous calculation can be performed using the transfer matrix method [2], which allows you to determine the coefficients of amplitude reflection R and transmission $\tilde{T}$. In this case, it is possible to consider the radiation incidence from different angles, as well as with different polarizations. A simpler method based on impedance transformation is also applicable. It allows determining the reflection coefficient if dissipation is neglected. The losses in the structure under consideration are small, therefore, $|\tilde{T}(\omega)| = \sqrt{1 - |R(\omega)|^2}$ can be approximated for the transmission coefficient. Normalized impedances of the E-wave and H-wave in a medium with a ε DP are

$$\rho_d^e(\omega) = \sqrt{k_0^2\varepsilon - k_\perp^2}/(k_0\varepsilon), \quad \rho_d^h(\omega) = k_0/\sqrt{k_0^2\varepsilon - k_\perp^2}.$$

, respectively

Here $k_\perp^2 = k_x^2 + k_y^2$, the screen is positioned normally to the z axis, in a dielectric medium $k_0^2\varepsilon = k_x^2 + k_y^2 + k_{zd}^2$, and in a vacuum (air) we have a bond of components: $k_0^2 = k_x^2 + k_y^2 + k_{z0}^2$. In this case, $\varepsilon = 1$. We will denote such impedances by zero instead of one: $\rho_0^{e,h}(\omega)$, the impedances for metal will be denoted as $\rho_m^{e,h}(\omega)$, and as $\rho_d^{e,h}(\omega)$ for dielectric. For the incident wave

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_0 \exp(ik_0 t/c - i(k_x x + k_y x + k_{z0} z))$$

the components define the guiding cosines. We have an input (surface) impedance on the glass surface in the area

of the structure location

$$Z_1^{e,h}(\omega) = \rho_d^{e,h}(\omega)\frac{\rho_0^{e,h}(\omega) + i\rho_d^{e,h}(\omega)\tan(k_{zd}t_0)}{\rho_d^{e,h}(\omega) + i\rho_0^{e,h}(\omega)\tan(k_{zd}t_0)}. \quad (2)$$

Here t_0 is the thickness of the window glass. The impedance at the boundary of the metal layer closest to the glass is

$$Z_2^{e,h}(\omega) = \rho_m^{e,h}(\omega)\frac{Z_1^{e,h}(\omega) + i\rho_m^{e,h}(\omega)\tan(k_{zm}t_m)}{\rho_m^{e,h}(\omega) + iZ_1^{e,h}(\omega)\tan(k_{zm}t_m)}. \quad (3)$$

The impedance at the boundary of the layer SiO_2 is given as

$$Z_3^{e,h}(\omega) = \rho_d^{e,h}(\omega)\frac{Z_2^{e,h}(\omega) + i\rho_d^{e,h}(\omega)\tan(k_{zd}t_d)}{\rho_d^{e,h}(\omega) + iZ_2^{e,h}(\omega)\tan(k_{zd}t_d)}. \quad (4)$$

The impedance at the boundary of the entire structure is

$$Z_4^{e,h}(\omega) = \rho_m^{e,h}(\omega)\frac{Z_3^{e,h}(\omega) + i\rho_m^{e,h}(\omega)\tan(k_{zm}t_m)}{\rho_m^{e,h}(\omega) + iZ_3^{e,h}(\omega)\tan(k_{zm}t_m)}. \quad (5)$$

Now we have

$$R^{e,h} = (Z_4^{e,h}(\omega) - \rho_0^{e,h}(\omega))/(Z_4^{e,h}(\omega) + \rho_0^{e,h}(\omega)).$$

These coefficients correspond to the location of a thin three-layer structure on a relatively thick window pane. The main values that we are considering are — the squares of the $|R^{e,h}|^2$ modules, i.e., the energy coefficients of reflection. They should be minimal at the average frequency of the optical range. These values should be maximum at the average frequency of the considered IR range. There are two independent parameters t_d and t_m. As the central wavelengths of these ranges, we take $\lambda_1 = 0.55\,\mu$m, $\lambda_2 = 10\,\mu$m (for a temperature of 300 K). It is possible to optimize the values of $|R^{e,h}|^2$ separately, and then we have four equations for two unknowns. However, we will optimize the $|A(\omega)|^2\left(|R^e(\omega)|^2 + |R^h(\omega)|^2\right)$ value at two frequencies using two parameters t_d and t_m. Strictly speaking, this value must be integrated in frequency, where the square of the amplitude can be related to the density of thermal Planck radiation $u_p = \hbar\omega^3 n(\omega, T)/(\pi^2 c^3)$ in the form of $|A(\omega)|^2 = u_p(\omega, T)/2$. Here n is the photon density. The heat flow in the direction of z now looks like

$$P(T_0) = \frac{S}{2}\int_0^\infty u_p(\omega, T_0)\left(|\tilde{T}^e(\omega, T_0)|^2 + |\tilde{T}^h(\omega, T_0)|^2\right)d\omega. \quad (6)$$

This value can be replaced with good accuracy by

$$P(\omega, T_0) = \frac{S}{2} \times \int_0^\infty u_p(\omega, T_0)\left(2 - |R^e(\omega, T_0)|^2 - |R^h(\omega, T_0)|^2\right)d\omega. \quad (7)$$

Here, a multiplier of 1/2 means that a stream falling on the glass is being considered. For the difference flow, calculate (6) for the temperature T and take the difference. If the structure is well transparent, we obtain

$$\Delta P = P(T_0) - P(T) = S\sigma_{SB}(T_0^4 - T^4).$$

Let us consider the case of a heat flux falling on an infinite half-space from a dielectric. We have

$$R^e(\omega, k_\perp) = (k_{zd}(\omega, k_\perp) - \varepsilon k_{z0}(\omega, k_\perp)) / (k_{zd}(\omega, k_\perp) + \varepsilon k_{z0}(\omega, k_\perp)),$$

$$R^h(\omega, k_\perp) = (k_{z0}(\omega, k_\perp) - k_{zd}(\omega, k_\perp)) / (k_{z0}(\omega, k_\perp) + k_{zd}(\omega, k_\perp)).$$

The main contribution to the reflection of the heat flow is made by modes incident at small angles. The same can be stated for a complex structure with screens. Therefore, it is sufficient to consider the normal radiation drop. The task is further simplified if we consider the structure from the point of view of resonant tunneling [6,7]. Such tunneling in a non-dissipative structure assumes that the reflection coefficients are equal to zero. This effect, for example, occurs in quantum mechanics. In a dissipative electrodynamic structure, the reflection coefficient may be small, but not zero. It is necessary to make the structure transparent in the middle of the optical range. Strict calculation of the dimensions that ensure the minimum condition $|R^e|^2 + |R^h|^2$ is difficult. As a first approximation, let's take the half-wave thickness of the t_d dielectric layer. In our case, it is 159 nm. The size t_m for the metal inclusion is selected from the condition of maximum reflection at the wavelength of the maximum Planck radiation for $T = 300\,$K ($\lambda = 10\,\nu$m). These dimensions are approximate, and they should be further optimized based on a rigorous approach. The thicknesses of window panes of the order 3–5 mm are very large compared to the wavelengths of the IR range, therefore, the square of the reflection coefficient modulus varies in the vicinity of the frequency under consideration from 0 to a value of $\left(\sqrt{\varepsilon_d} - 1\right)^2/\left(\sqrt{\varepsilon_d} + 1\right)^2 = 0.07$ and is equal at an arbitrary frequency.

$$|R(\omega)|^2\frac{(\varepsilon_d - 1)\tan\left(k_0\sqrt{\varepsilon_d}t_0\right)}{\sqrt{4\varepsilon_d + (\varepsilon_d + 1)^2\tan^2\left(k_0\sqrt{\varepsilon_d}t_0\right)}}.$$

This value is small compared to the reflection from the structure of the IR screen, so the glass can be ignored in the approximate calculation. Considering the above, we obtain an approximate energy reflection coefficient of the structure $|R_0|^2$. Since the collision frequency of ω_c is significantly less than the considered central frequency of the IR range, ε_m can be considered a large negative value, and the values of

$$\rho_m = -i/\sqrt{|\varepsilon_m|}, \quad k_z = -ik_0/\sqrt{|\varepsilon_m|},$$

$$\tan(k_z t_m) = -i \tan h\left(k_0 t_m \sqrt{|\varepsilon_m|}\right) = -i\alpha$$

imaginary. At the same time, ρ_m is small in modulus, and $\alpha = 1 - \delta$. This means that it should be $t_m \geq 20$ nm, $\sqrt{|\varepsilon_m|} \sim \lambda_2/\lambda_p \approx 100$, $\lambda_p \approx 100$ nm. At the same time, $\alpha = \tan h(2\pi t_m/100) = 1 - \delta$, $\delta \ll 1$. So, we consider the value $|\varepsilon_m|$ large, $k_z \approx -ik_0\sqrt{|\varepsilon_m|}$ in the IR range, while the thickness of the metal layer is such that $\tan(k_z t_m) = -i(1-\delta)$, $\delta \ll 1$, $|\rho_m| = 1/\sqrt{|\varepsilon_m|} \ll 1$. Then the input normalized impedance at the interface of the metal and dielectric layers is equal to

$$Z_1 = \rho_m \frac{Z_0 + i\rho_m \tan(k_z t_m)}{\rho_m + iZ_0 \tan(k_z t_m)} \approx -i|\rho_m|\Big(1 + \delta(1 + i|\rho_m|)\Big). \tag{8}$$

It has a normalized vacuum impedance of $Z_0 = 1$. It is possible to ignore the small real part and take $Z_1 \approx -i|\rho_m|(1+\delta)$. In dielectric $k_z t_d \ll 1$, and you can take $k_z \approx k_0\sqrt{\varepsilon_d}$. Then the input impedance at the boundary of the second metal layer is

$$Z_2 = \rho_d \frac{Z_1 + i\rho_d k_0 \sqrt{\varepsilon_d} t_d}{\rho_d + iZ_1 k_0 t_d \sqrt{\varepsilon_d}}$$

$$\approx -i\Big(\rho_d \beta \sqrt{\varepsilon_d} - |\rho_m|(1+\delta)\Big) + \delta|\rho_m|^2,$$

where $\beta = k_0 t_d$. Neglecting a very small real term, it can also be considered imaginary: $Z_2 = i\tilde{Z}_2$. The input impedance of the entire Z_3 structure is given by the left part of the formula (8), in which $Z_0 = 1$ should be replaced by Z_2, while it is also almost imaginary: $Z_3 \approx -i\tilde{Z}_3$, and

$$\tilde{Z}_3 \approx |\rho_m| \frac{\Big(\rho_d \beta \sqrt{\varepsilon_d} - |\rho_m|(1+\delta)\Big) - |\rho_m|(1-\delta)}{\Big(\rho_d \beta \sqrt{\varepsilon_d} - |\rho_m|(1+\delta)\Big)(1-\delta) - |\rho_m|} \approx |\rho_m|.$$

Therefore

$$R_0 = (Z_3 - Z_0)/(Z_3 + Z_0) \approx -(1 - i\tilde{Z}_3)^2/(1 + \tilde{Z}_3^2)$$

and $|R_0|^2 \approx 1$. Due to the dissipation $|R_0|^2$ is slightly less than one. Using (8), we obtain

$$Z_3 \approx -i|\rho_m| + |\rho_m|^2\delta/\Big[|\rho_m| - \tilde{Z}_2(1-\delta)\Big],$$

and the energy reflection coefficient of

$$|R_0|^2 = \frac{\Big(|\rho_m| - \tilde{Z}_2(1-\delta) - |\rho_m|^2\delta\Big)^2 + |\rho_m|^2}{\Big(|\rho_m| - \tilde{Z}_2(1-\delta) + |\rho_m|^2\delta\Big)^2 + |\rho_m|^2}$$

$$\approx 1 - \frac{4|\rho_m|^2\delta}{|\rho_m| - \tilde{Z}_2(1-\delta)}.$$

$\rho_m \approx 1/\sqrt{\varepsilon_m}$ in it, and the value of δ depends on the thickness of the metal layer. At optical frequencies $|\varepsilon_m| \sim 1$, and the impedance can no longer be considered small, and the hyperbolic tangent is small. In this case,

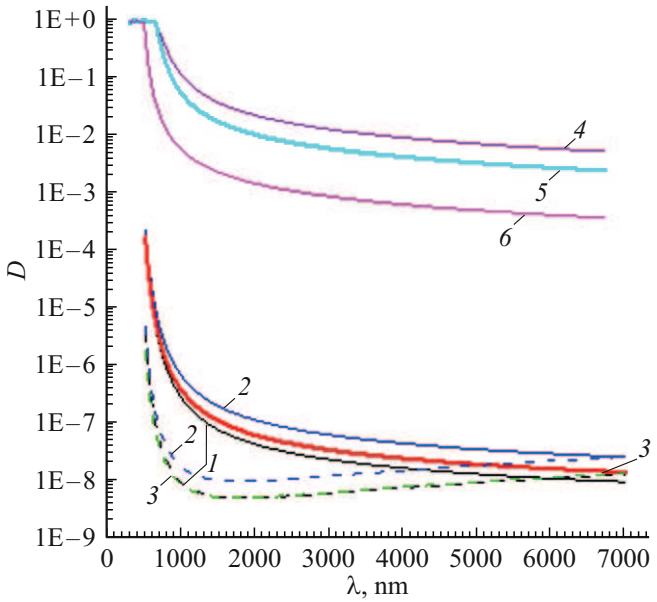


Figure 2. Modules of transmission coefficients depending on wavelength (nm) based on homogenization (dashed curves) and based on a strict model (solid curves) for the structure $t_m = t_d = 10$ nm with a number of periods of 22 and a structure thickness of 440 nm. Curves *1* — normal drop, curves *2* — E-mode drop, curves *3* — H-mode drop (angle of incidence $\pi/4$). The curves *4*–*6* are constructed for 5 periods: $t_m = 5$ nm, $t_d = 25$ nm (*4*, *5*) and $t_m = 10$ nm, $t_d = 50$ nm (*6*). The curve *5* is constructed with 5 dielectric and 6 metal layers.

$\tan(k_z t_m) = -i|k_z|t_m$, and, as it is easy to see, the thickness of the metal layers can be chosen so that in the optical range $|R_0|^2 \approx 0$. At the same time, in the IR range $|R_0|^2 \approx 1$. The indicated thicknesses for silver range from 20 to 80 nm.

In this case, very thin (on the order of several nm) layers are not enough to ensure good reflection in the IR range of two metal layers, so it is necessary to perform a structure of several periods of alternating layers of metal and dielectric (Fig. 1, *d*). The estimate gives the value $1 - |R_0|^2 \sim 10^{-3}$ in the IR range at $t_m = 20$ nm. A further increase in the thickness of the metal layers is not very advisable.

Figure 2 shows rigorous calculations $D = |\tilde{T}|$ for three structures with 22 metal layers (440 nm thick) and 5 layers (180 and 310 nm thick) by calculating the complete transfer matrix [2]. The corresponding results (dashed curves) for 440 nm thick structures homogenized according to Rytov jcite8 are also shown there. Note that energy transparency is determined by D^2. Adding an additional, sixth metal layer to the 5-period structure creates a symmetrical structure and greatly increases the barrier (curves *4* and *5*). The structure with 22 periods (44 layers) is transparent in the UV range, and the other two are transparent in the optical range. The shift of the transparency range to low frequencies is achieved by increasing the thickness of the layers. Although using two layers leads to a result several orders of magnitude worse [2], this is practically

quite enough to almost completely cut off the IR flow, while it is the cheapest technology for applying layers.

Using a half-wavelength glass thickness for the central wavelength of the optical range is not entirely correct. We will find the exact frequency of the photon when the symmetrical metal/dielectric/metal structure is completely transparent (ignoring losses). Let's take the wavelength of $\lambda_0 = 550\,\text{nm}$ and neglect the DP dispersion in its vicinity. Let's rewrite the equation (1) in wavelengths and neglect the dissipation: $\varepsilon_m = \varepsilon_L - \lambda^2/\lambda_p^2$. Hence, for silver, we have $\varepsilon_m = -11.7$. The normalized impedance transformed by the first layer of silver is

$$Z_1 = -i|\rho_m| \frac{1 - i|\rho_m| \tan h(k_m t_m)}{-i|\rho_m| + \tan h(k_m t_m)} = 0.2352 - i0.4783.$$

The thickness of the metal is 20 nm, $\tan h(k_m t_m) = 0.66$, $|\rho_m| = 0.292$. If we take a thickness of 100 nm, then $\tan h(k_m t_m) \approx 1$, and then $Z_1 = -i|\rho_m|$. In the center of the dielectric layer, the input impedance can be either infinite or zero. In the first case

$$1 + iZ_1\sqrt{\varepsilon_d} \tan\left(k_0\sqrt{\varepsilon_d} t_d/2\right) = 0,$$

and in the second case

$$Z_1\sqrt{\varepsilon_d} + i \tan\left(k_0\sqrt{\varepsilon_d} t_d/2\right) = 0.$$

Both of these conditions determine the complex resonant frequencies $\omega_{1,2} = \omega'_{1,2} + i\omega'_{1,2}$ of a dielectric layer overlaid on both sides with metal layers. The task is simplified for thick metal layers. In this case, for the magnetic wall in the center, we have

$$t_d/\lambda_0 = \left[1 - \arctan(\rho_d/|\rho_m|)/\pi\right]/\sqrt{\varepsilon_d},$$

and for the electric wall

$$t_d/\lambda_0 = \left(1 + \arctan(|\rho_m|/\rho_d)/\pi\right)/\sqrt{\varepsilon_d}.$$

In the case of a magnetic wall $t_d/\lambda_0 = 0.375$, which corresponds to a thickness of $t_d = 206\,\text{nm}$, and in the case of an electric wall $t_d = 365\,\text{nm}$. Since the impedance Z_1 is not zero, it is possible to realize the case of an electric wall without field variations at $t_d = 48\,\text{nm}$. Fig. 2 shows, in particular, the calculation of these cases, taking into account the dispersion of the DP metal. Examples of calculating structures with two and four periods for metal layers with thicknesses from 15 to 30 nm and a dielectric layer with thicknesses from 50 to 300 nm are given in Ref. [2]. So, it is possible to carry out an approximate synthesis of metallic layered structures with subsequent clarifying optimization according to a strict model.

Creation of coatings by embedding metal particles

Two metal layers can be replaced by two dielectric layers with embedded metal nanoparticles. It is technologically quite simple to introduce metal particles into the molten glass layer. After such an introduction, it is necessary to apply a layer of glass on top, into a part of which metal particles are again embedded. For the calculation, we will use mixing formulas such as Garnett or Bruggeman. It is convenient to embed spherical metal particles with a diameter of about 20 nm or less into a layer with a thickness of about 100 nm. In this case, all the depolarization coefficients are the same and equal to 1/3, the effective DP is a scalar and in the Garnett model is given by the well-known formula

$$\varepsilon_{\text{ef}} = \varepsilon_d \left(1 + 2\delta_m \frac{\varepsilon_m - \varepsilon_d}{\varepsilon_m + 2\varepsilon_d}\right) \Big/ \left(1 - \delta_m \frac{\varepsilon_m - \varepsilon_d}{\varepsilon_m + 2\varepsilon_d}\right),$$

in which δ_m is the volume concentration of metal particles. If it is negative (for $\varepsilon''_m \ll |\varepsilon'_m|$) and $\varepsilon_d \ll |\varepsilon'_m|$, then according to this formula, $\varepsilon_{\text{ef}} \approx \varepsilon_d(1 + 2\delta_m)/(1 - \delta_m)$, i.e., the model does not give a negative DP. Obviously, a negative value with a not too low concentration of δ_m is obtained with $\varepsilon'_m \approx -2\varepsilon_d$. In this case, $\varepsilon_{\text{ef}} \approx -2\varepsilon_d = \varepsilon'_m$. This condition means $\varepsilon_L - \omega_p^2/\omega^2 = -\varepsilon_d$ or $\omega = \omega_p/\sqrt{\varepsilon_L + \varepsilon_d}$, i.e. this is nothing more than a plasmon resonance condition for localized plasmons. For silver particles in glass, this corresponds to a frequency of $\omega = 4.48 \cdot 10^{15}\,\text{Hz}$, i.e., a wavelength of 420 nm. According to this formula, there is an acute resonance with large opposite values of the effective DP. However, this result is incorrect [9]. It is Garnett's formula that is inapplicable in this case. The Bruggeman approximation is more convenient here, which for a three-component medium we write as

$$\sum_{j=1}^{3} \delta_j \frac{\varepsilon_{\text{ef}} - \varepsilon_j}{\varepsilon_j + 2\varepsilon_{\text{ef}}} = 0,$$

$$\varepsilon_{\text{ef}}\varepsilon_{\text{ef}} - B = 2\varepsilon_{\text{ef}} \frac{\left[\left(2 - \frac{\delta_m}{\delta_d}\right)\varepsilon_d - \varepsilon_m\left(1 - 2\frac{\delta_m}{\delta_d}\right)\right]}{4\left(1 + \frac{\delta_m}{\delta_d}\right)} - C$$

$$= \varepsilon_d\varepsilon_m/2 = 0,$$

$$-\varepsilon_d = (2 - \delta_d/C)\varepsilon_{\text{ef}},$$

$$\varepsilon_{\text{ef}} = \frac{\delta_m\varepsilon_m - C\varepsilon_m}{(C2 + \delta_m)} = -\frac{\varepsilon_d}{(2 - \delta_d/C)}.$$

$\delta_1 + \delta_2 + \delta_3 = 1$ in it. The disadvantage of this formula is that it is implicit. However, its solution can be sought iteratively. For example, taking $\delta_1 = \delta_d$, $\delta_2 = \delta_m$, we have the formula for iterations

$$\varepsilon_{\text{ef}} = \varepsilon_d - \frac{\delta_m}{\delta_d} \frac{\varepsilon_d + 2\varepsilon_{\text{ef}}}{\varepsilon_m + 2\varepsilon_{\text{ef}}} (\varepsilon_{\text{ef}} - \varepsilon_m)$$

$$-\frac{\delta_3}{\delta_d} \frac{\varepsilon_d + 2\varepsilon_{\text{ef}}}{\varepsilon_3 + 2\varepsilon_{\text{ef}}} (\varepsilon_{\text{ef}} - \varepsilon_3),$$

the direct iteration method for it does not converge for all values, so it is convenient to use the method with correction

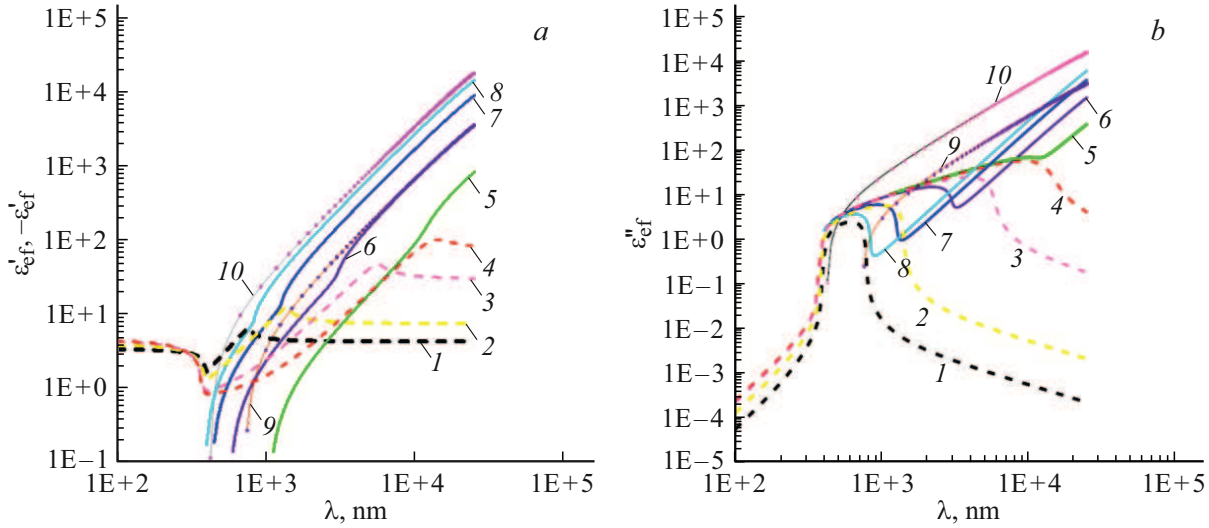


Figure 3. Dependences of the real parts (*a*) $\varepsilon'_{\text{ef}}(\lambda)$ (curves *1–4*) and $-\varepsilon'_{\text{ef}}(\lambda)$ (curves *5–10*) and the negative imaginary part $\varepsilon''_{\text{ef}}(\lambda)$ of the effective permittivity on the wavelength (nm) at different δ: 0.1 (curves *1*, *9*), 0.2 (*2*), 0.3 (*3*), 0.32 (*4*), 0.35 (*5*), 0.4 (*6*), 0.5 (*7*, *10*). 0.6 (*8*). The curves *1–8* correspond to the Bruggeman formula, the curves *9*, *10* are constructed for the formula (10).

of the iteration step for calculations [10]. For two components, the Bruggeman formula gives an explicit solution

$$\varepsilon_{\text{ef}} = \varepsilon'_{\text{ef}} - i\varepsilon''_{\text{ef}} = B \pm \sqrt{B^2 + C}, \tag{9}$$

where

$$B = [(2 - \delta_m/\delta_d)\varepsilon_d/4 - \varepsilon_m(1 - 2\delta_m/\delta_d)/4]/(1 + \delta_m/\delta_d),$$

$$C = \varepsilon_d\varepsilon_m/2.$$

One of the roots should be discarded, exactly the root for which $\varepsilon''_{\text{ef}} < 0$ is used. The formula (9) was converted for calculations with large $|\varepsilon_m|$. So, for $\delta = 0$, we have two roots $\varepsilon_{\text{ef}} = \varepsilon_d$ and $\varepsilon_{\text{ef}} = -\varepsilon_m/2$. $\varepsilon''_{\text{ef}} = -\varepsilon''_m/2$ for the second root, and it should be discarded. For a large modulo DP of metal, consider the case of the percolation threshold: $\delta_m = 1/3$ and

$$\varepsilon_{\text{ef}} = \varepsilon_d(1 \pm \sqrt{1 + 8\varepsilon_m/\varepsilon_d})/4.$$

If the value of ε'_m is negative and $|8\varepsilon_m/\varepsilon_d| \gg 1$, then

$$\varepsilon_{\text{ef}} \approx \varepsilon_d/2 - \sqrt{\varepsilon_m\varepsilon_d/2}.$$

For the wavelength $\lambda = 550$ nm, strict results are shown in Fig. 3. As can be seen, $\varepsilon'_{\text{ef}} > 0$ at $\delta_m < 1/3$ and $\varepsilon'_{\text{ef}} < 0$ at $\delta_m < 1/3$. When a wave is applied to a metal nanoparticle, its size is much smaller than the depth of penetration into the metal at all frequencies. Therefore, the field in such a particle is homogeneous. The current density of polarization in it

$$\mathbf{J}^p = i\omega\varepsilon_0(\varepsilon_m - \varepsilon_d)\mathbf{E}.$$

The dipole moment of the unit volume for all particles

$$\mathbf{P} = -i\delta_m\mathbf{J}^2/\omega.$$

Hence

$$\varepsilon_{\text{ef}} = \delta_m\varepsilon_m + (1 - \delta_m)\varepsilon_d. \tag{10}$$

The same result is obtained by the Rytov homogenization formula for the transverse DP component for a finely layered medium of metallic and dielectric layers [8]. The formula for the longitudinal component is written in the same way, but for the inverse values of the DP layers and the effective DP component. At normal incidence, the polarization is determined by the transverse component DP. Since in our case the normal drop is significant and the interaction with the transverse component of the electric field is important, formula (10) should be considered preferable compared to the Bruggeman model, as shown in Fig. 3, where the frequency characteristics of — $\varepsilon'_{\text{ef}}(\lambda)$ and $\varepsilon''_{\text{ef}}(\lambda)$ are given for different δ_m according to both formulas. It can be seen that for metal particles, the Bruggeman formula works well for $\delta_m < 0.02$ and $\delta_m > 0.8$. In addition, the Bruggeman formula gives an overestimated value of ε'_{ef} outside these regions and especially overestimates the losses of $\varepsilon''_{\text{ef}}$ (Fig. 3, *b*). An SiO_2 layer with a thickness of 100 nm with the introduction of metal balls with a concentration of about 1/2 has approximately the same reflective effect as a metal layer with a thickness of about 30–50 nm, but it is easier to perform such a coating. It should be noted that particle sizes are not critical, but their concentration is important. The particles only need to be significantly smaller than the thickness of the layer. The depth of field penetration into the metal is always significantly greater than the diameters of the nanoparticles, and due to the negative DP in the optical and IR ranges, the field inside the particles is constant in amplitude, which makes homogenization (10) a fairly accurate procedure, especially in the IR range.

The dependences of ε'_{ef} and ε''_{ef} on the concentration of δ_m for (10) and the Bruggeman formula are shown in Fig. 4. They once again show the limited applicability of the latter. The effect of the field leads to the excitation of localized plasmons in the particles, while the effective DP of the layer with a negative real part occurs due to a phase shift between the field and the polarization current. When waves fall at different angles in the coatings under consideration, it is possible to excite surface plasmons. This means that when these surfaces are viewed from different angles, glare of different colors (frequencies) should be observed in the reflected light in the optical and IR ranges.

The simplest symmetrical structures on thick glass

Symmetrical structures exhibit the best transparent properties in the optical domain and the best barrier in the IR range. For a quasi-periodic structure, symmetry is achieved by performing it on both sides of the glass. It is more convenient to perform a symmetrical structure inside, i.e., to connect two halves of glass into one double-glazed unit. In this case, the roughness of the transparency curve in the optics decreases. It is technologically easier to produce particle-metallized layers. Apparently, the cheapest way to obtain a reflective layer on the glass surface is to embed metal particles into the melt to a certain depth. Such a layer with particles can have a thickness of the order of hundreds of nm. As a result, the two halves of the glass form the structure of Fig.1, *c*. Fig. 5 shows the calculation

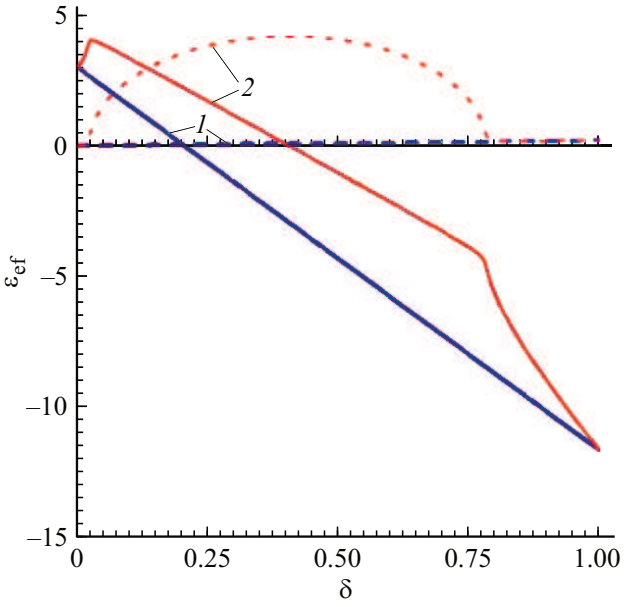


Figure 4. Dependencies of the real $\varepsilon'_{ef}(\lambda)$ (solid curves) and negative imaginary $\varepsilon''_{ef}(\lambda)$ (dashed curves) parts of the effective dielectric constant at a wavelength of 550 nm from the metal concentration δ. Curves *1* — formula (10), curves *2* — the Bruggeman formula.

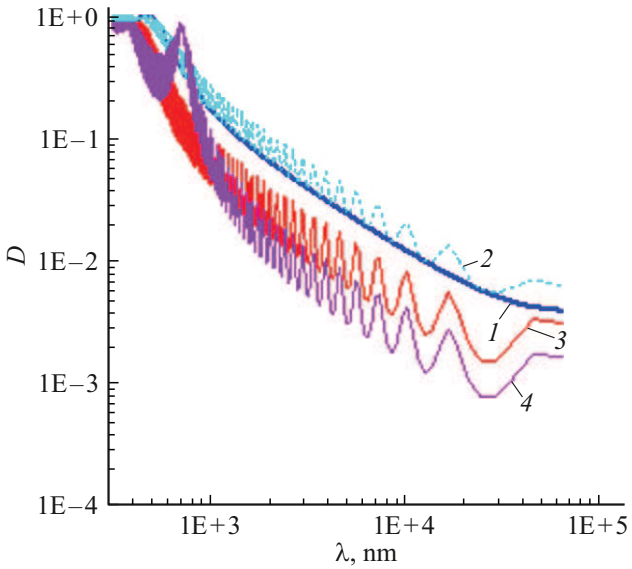


Figure 5. Transparency $D(\lambda) = |T(\lambda)|^2$ of 3mm double glass with a three-layer resonant structure with two layers of metal $t_m = 25$,nm in the center (curves *1*, *2*), single glass with such a structure on the surface (*3*) and a separate resonant structure (*4*) depending on the wavelength at different t_d: 50 (curve *1*), 150 (*2*) and 25 nm (*3*, *4*). The losses were approximated so that in the optical range $\varepsilon''_d/\varepsilon'_d = 10^{-6}$, in the center of the IR range $\varepsilon''_d/\varepsilon'_d = 10^{-4}$.

results of two such structures between two glasses of 3 mm each. The structures differ in the thickness of the dielectric layer between the metal layers. It is quite possible to use a $\lambda = 550$ nm half-wave layer SiO_2 with a thickness of about 150 nm, which is technologically more convenient. Compared to Ref. [11], which uses a thermochromic metal-insulator transition material and a package of two glasses, which reduces the heat flux by 30% and reduces the optical transparency by at least 0.65 in more than 75% of the visible spectrum. The structures considered in this work make it possible to reduce the heat flux by orders of magnitude while maintaining approximately the same transparency in the visible part of the spectrum. This is due to a stronger barrier of multilayer metal structures in the IR range during resonant passage in the optical range.

Conclusion

The passage of IR radiation through glass can be reduced by several orders of magnitude both by using multilayer metal-dielectric coatings and by embedding metal nanoparticles into the glass layers. Three-layer structures with two highly reflective metallic or metallic nanoparticle-containing layers separated by a thin transparent layer SiO_2 are considered as the simplest structures in the optical range. Their use is quite sufficient to reduce IR radiation by 2–3 orders of magnitude. Transmission in the optical range

is based on the principle of resonant photon tunneling, when the DP of a metal in optics has a negative real part. Lowering the frequency to the IR range causes the resonant properties to disappear, and a strong increase in ε''_{ef} leads to an increase in reflections in the IR region. The main effect is achieved precisely due to reflection, while absorption plays a small role and can be omitted in the calculation.

Coating is also possible by applying a thin molten layer of glass containing solid metal nanoparticles in the melt, as well as by introducing a stream of nanoparticles into a heated (molten) surface layer of glass. The depth of penetration of such particles into the melt will depend on their velocities and masses.

The presented results can serve as a basis for the development of „smart windows“ [11], the properties of which adapt to different conditions. For this, semiconductor layers and inclusions are promising, as well as the design of double glasses with an air or gas gap, which leads to a decrease in thermal conductivity.

It is shown that the Garnett formula is of limited use for modeling, and the Bruggeman formula correctly reflects the behavior of the effective DP, but is less accurate than the simple formula for the weighted average contribution from the dielectric and metallic phases (10). The Bruggeman formula is not at all accurate in the vicinity of the percolation threshold. Obtaining formulas for effective DP with metal particles requires an electrodynamic approach [9]. This problem can be solved most simply for periodically embedded small identical metal particles using the method of integral equations for the corresponding photonic crystals and homogenization based on the calculation of the polarization [12]. This approach allows taking into account the shape of the particles. The simplification is achieved because the electric field inside the particles is almost constant and it is necessary to determine three components of the field, i.e. the problem has an algebraic dimension of 3.

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Conflict of interest

The authors declare that they have no conflict of interest.

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