12

Evaluation of the metrological characteristics of quartz control plates

© A.A. Golopolosov

All-Russian Research Institute for Optical and Physical Measurements, Moscow, Russia

e-mail: golopolosov@vniiofi.ru

Received October 06, 2025
Revised February 28, 2026
Accepted February 28, 2026

This paper describes a mathematical model of quartz control plates used in the calibration and verification of polarimeters. The Berreman method was used to the passage of radiation through the plate. Using this model, we assessed the influence of certain technical requirements to plate manufacturing on the rotation angle of polarization plane. Based on the modeling results, recommendations have been made to reduce measurement errors.

Keywords: quartz, polarimetry, gyrotropy, optical activity, optical rotation, angle of rotation of the plane of polarization.

DOI: 10.61011/0000000000

Introduction

Crystalline α-quartz (hereinafter — quartz) is an anisotropic and optically active (gyrotropic) material. Its structure belongs to a trigonal crystal system, which is a type of hexagonal symmetry and is characterized by the presence of a third-order axis of symmetry. This crystal lattice determines its unique optical properties. Due to these properties, quartz finds wide applications in those fields of science and industry where light polarization is used. Quartz plays a special role in polarimetry, since plane-parallel quartz plates (hereinafter referred to as quartz control plates) are used for calibration and verification of polarimeters that measure the rotation angle the polarization plane (optical rotation angle) [1].

Despite the high quality requirements of quartz polarimetric plates [2], the assessment of their actual metrological characteristics is fraught with a number of difficulties. The accuracy and stability of the measurement results of the optical rotation angle value of the plates strongly depends on various factors. Experimental studies of the metrological characteristics of quartz control plates can be found in Ref. [3–5]. Theoretical studies are mainly devoted to the description of the optical properties of quartz [6–9]. At the same time, Russian researchers have made significant progress in the theoretical description of the optical properties of crystals of various crystallographic groups [10–17].

In this paper, a theoretical description of the process of radiation passing through a quartz control plate, which is used in the calibration of polarimeters, is considered. The Berreman method was used to describe the radiation transmission process [18].

Research object

The use of quartz in polarimeter calibration is explained by the good stability and reproducibility of the optical rotation angle value. Another substance used in calibration and calibration is a sugar solution. However, unlike quartz, the sugar solution is susceptible to degradation (mutarotation) [19], concentration changes due to evaporation and bacterial exposure. Another advantage of quartz is that, depending on its modification (chirality), the plate can be either positive (right-handed) or negative (left-handed).

The optical activity of quartz is maximal along its optical axis [20], i.e., where there are no anisotropy effects. Therefore, control plates are cut out of quartz in such a way that the optical axis of the crystal is perpendicular to the working edge of the plate. By changing the thickness of the plate, different nominal values of the optical rotation angle are achieved. In practice, when manufacturing a plate, the optical axis will not be strictly perpendicular and will form a certain angle with a normal to the working edge.

For use as a calibration and verification tool, the quartz plate is mounted in a special cylindrical tube so that its plane is oriented perpendicular to the axis of the tube. If the plate is poorly installed in the tube, then when it is placed in the measuring compartment of the polarimeter, the radiation will fall on the plate at a certain angle. Thus, there are two reasons why the quartz control plate deviates from the ideal one:

— a plate is incorrectly cut from a crystal quartz blank with a deviation of its optical axis from the normal to the plate surface;

— the cylindrical tube is poorly made, as a result of which the normal to the surface of the plate will not coincide with the axis of the tube.

These deviations in the manufacture of polarimetric plates can have a strong effect on the polarization state of the transmitted radiation and, as a result, on the measurement results of optical rotation angle. In Russia, the requirements for the manufacture of polarimetric plates are regulated by GOST 22409-77. However, the requirements of this standard apply to plates reproducing the unit of optical rotation angle in the range from minus 45° to plus 50°

at a wavelength of 546.07 nm. According to the State Verification Scheme for measuring the rotation angle of the polarization plane, approved by Rosstandart Order No. 2652 dated 20.10.2022, plates reproducing the optical rotation angle in the range from minus 90° to plus 90° at a wavelength in a vacuum of 546.23 nm can be used as working standards of the first category. Therefore, the urgent task is to study quartz control plates with nominal optical rotation angle values close to minus 90° and plus 90°, in order to use them as working standards. The theoretical model considered below makes it possible to numerically evaluate the effect of the above deviations on the measurement result of optical rotation angle.

Calculation procedure

Consider the case of incident plane monochromatic wave with wavelength. (with a frequency $\omega = 2\pi c/\lambda$, c — the speed of light in vacuum) onto a plane-parallel plate with a thickness of d made of an anisotropic, uniaxial, optically active material. Let the radiation fall on the first surface of the plate at an angle γ_i with respect to its normal. The coordinate system of the xyz incidence plane is chosen so that the xz plane coincides with the incidence plane, and the z axis is directed along the normal towards the second face. The Berreman method [18] is based on the representation of Maxwell's equations and material equations (constitutive relation) in matrix form. If we assume that the parameters of the medium do not depend on z, then for a plate with a thickness of d, we can write

$$\mathbf{\Psi}(z+d) = \mathbf{L}(d)\mathbf{\Psi}(z), \tag{1}$$

where $\mathbf{\Psi}(z)$ — generalized field vector at the point z; $\mathbf{L}(d)$ — radiation propagation operator in the medium, defined by the expression

$$\mathbf{L}(d) = \exp \mathrm{M}(\mathbf{\Delta}) i\omega d/c,$$

where $\exp \mathrm{M}(\ldots)$ is a matrix exponent; $\mathbf{\Delta}$ is a 4×4 matrix characterizing the properties of the medium in a given direction. There are certain difficulties associated with calculating the matrix exponential [21], however, modern mathematical programs have ready-made solutions that allow it to be calculated with sufficiently high speed and accuracy. The components of the matrix $\mathbf{\Delta}$ are expressed in terms of the components of the block matrix $\mathbf{M}$ 6×6, which is part of the matrix form of the system of material equations. The material equations complement Maxwell's equations and make it possible to connect electric and magnetic field strength vectors with induction vectors in a single form, taking into account specific electromagnetic properties of the medium. For an anisotropic and optically active medium, the material equations can be given in various forms. General expressions for determining the elements of the matrix $\mathbf{\Delta}$ in terms of the matrix $\mathbf{M}$ are given in Ref. [18].

Since in this paper we are interested in changes with small deviations from the optical axis, we will adhere to the exact theory and use the Condon–Fedorov equations (Condon model) [13]:

$$\mathbf{D} = \boldsymbol{\varepsilon}\mathbf{E} + i\boldsymbol{\alpha}\mathbf{H},$$

$$\mathbf{B} = \boldsymbol{\mu}\mathbf{H} - i\boldsymbol{\alpha}\mathbf{H} - i\boldsymbol{\alpha}^T\mathbf{E}, \tag{2}$$

where $\boldsymbol{\varepsilon}$ is the dielectric constant tensor, $\boldsymbol{\mu}$ is the magneticsusceptibility tensor, $\boldsymbol{\alpha}$ is the gyration tensor. The „T" index means transpose. In general, tensors are defined so that their main axes coincide with the axes of the coordinate system xyz. To set another specific position of the axes of the tensor, you can use the rotation matrix determined by the Euler angles. In this paper, we will use the w matrix, which defines successive rotations by an angle φ around the z axis, by an angle θ around the y axis, and by an angle φ. around the z' axis:

$$\mathbf{w} = \begin{pmatrix} -\sin\varphi\sin\psi + \cos\varphi\cos\theta\cos\psi & -\sin\varphi\cos\psi - \cos\varphi\cos\theta\sin\psi & \cos\varphi\sin\theta \\ \cos\varphi\sin\psi + \sin\varphi\cos\theta\cos\psi & -\cos\varphi\cos\psi + \sin\varphi\cos\theta\sin\psi & -\sin\varphi\sin\theta \\ -\sin\theta\cos\varphi & \sin\theta\sin\varphi & \cos\theta \end{pmatrix}. \tag{3}$$

The prime in this case means that the axis of rotation belongs to the coordinate system transformed by the previous rotation.

Taking into account „rotation" tensors and material equations (2) the block matrix $\mathbf{M}$ can be written as

$$\mathbf{M} = \begin{bmatrix} \mathbf{w}\boldsymbol{\varepsilon}\mathbf{w}^T & \mathbf{w}(i\boldsymbol{\alpha})\mathbf{w}^T \\ \mathbf{w}\left(\mathbf{i}\boldsymbol{\alpha}^T\right)\mathbf{w}^T & \mathbf{w}\boldsymbol{\mu}\mathbf{w}^T \end{bmatrix}.$$

The components of the field at the plate outlet can be determined from (1) using the following expression:

$$\mathbf{Q} = (\mathbf{T}_m - \mathbf{L}(d)\mathbf{R}_m)^{-1}\mathbf{L}(d)\mathbf{\Psi}_i, \tag{4}$$

where $\mathbf{Q}$ — vector of unknown components of the transmitted and reflected waves, $\mathbf{\Psi}_i$ — generalized vector of the incident wave, $\mathbf{T}_m$ and $\mathbf{R}_m$ — auxiliary matrices connecting the corresponding generalized vectors of the transmitted and reflected waves with the vector $\mathbf{Q}$. Let the indices p and s mean the components of the polarized wave parallel and perpendicular to the plane of incidence, respectively, and the indices i, r, t —the incident, reflected and transmitted waves, respectively. Then we can write

$$\mathbf{Q} = \begin{pmatrix} E_{tp} \\ E_{ts} \\ E_{rp} \\ E_{ts} \end{pmatrix}; \quad \mathbf{\Psi}_i = \begin{pmatrix} E_{ip}\cos\gamma_i \\ E_{ip}n_i \\ E_{is} \\ E_{is}m_i\cos\gamma_i \end{pmatrix},$$

$$\mathbf{T}_m = \begin{pmatrix} \cos\gamma_t & 0 & 0 & 0 \\ n_t & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & n_t\cos\gamma_t & 0 & 0 \end{pmatrix},$$

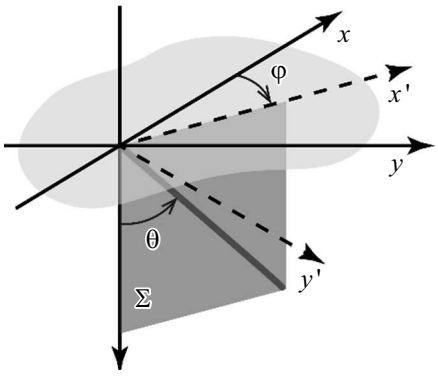


Figure 1. The scheme of determining the main plane relative to the coordinate system of the plane of incidence.

$$\mathbf{R}_m = \begin{pmatrix} 0 & 0 & -\cos\gamma_i & 0 \\ 0 & 0 & n_i & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -n_i\cos\gamma_i \end{pmatrix}.$$

For a linearly polarized incident wave with an azimuth of β_i

$$E_{ip} = \cos\beta_i, \qquad E_{is} = \sin\beta_i.$$

By calculating the vector **Q** using the formula (4), it is possible to estimate the state of polarization at the outlet of a plate made of an anisotropic, optically active material. By changing the parameters of the model, it is possible to estimate the change in the polarization state. The following expressions will be used to calculate the angle of rotation of the plane of polarization of quartz control plate with variations in various parameters.

Model

To better understand the process of radiation passing through a quartz control plate, it is necessary to compare the parameters of the mathematical model with the physical characteristics of a real object. As mentioned earlier, two main deviations may occur in the manufacture of quartz control plates:

— deviation of the optical axis of the crystal from the normal to the surfaces of the plate;

— deviation from the perpendicular of the working surface of the plate to the axis of the incident radiation.

The direction of the optical axis is determined by the position of the main plane Σ, i.e., the plane in which the optical axis and the normal to the first surface of the plates lie [11]. In the rotation matrix **w** the main plane relative to the plane of incidence, the angle of φ is determined, and the inclination of the optical axis relative to the normal is determined by the angle of θ (Fig. 1).

Let the plate rotate freely around the axis of the incident radiation, and the radiation falls on the plate at an angle of γ_i. In this case, when the plate rotates, the plane of incidence will also rotate relative to the initial position, but the angle of incidence will remain constant. Relative to the plane of incidence, the position of the main plane will remain unchanged, but the position of the plane polarization of the incident radiation will change (Fig. 2). If the plate is rotated by an angle of ξ, then the azimuth of polarization of the incident radiation will also change by this angle. In the described calculation method, by changing the azimuth of the plane of polarization, it is possible to simulate the rotation of the plate.

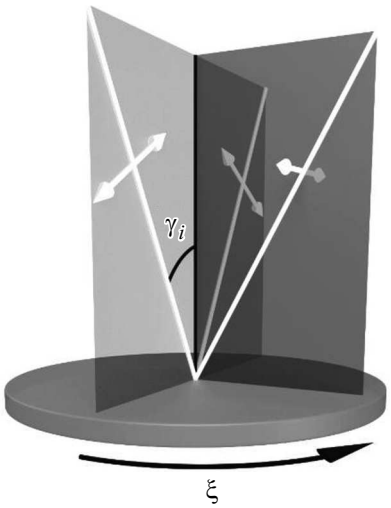


Figure 2. The scheme of changing the direction of the plane of polarization during rotation of the plate in the tube.

Let the plane of polarization in the initial position of the system lie in the plane of incidence, i.e., at $\xi = 0$, the azimuth is $\beta_i = 0$. Then, when rotating the plate, it is possible to write $\beta_i = \xi$. The plate is in the air, i.e., the medium before and after the plate is isotropic, and its refractive index is $n_i = n_t = 1$.

For quartz, when radiation propagates along the optical axis, the main axes of the tensors $\boldsymbol{\varepsilon}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\mu}$ coincide with the axes of the coordinate system of the incidence plane xyz and the tensors take the following form:

$$\boldsymbol{\varepsilon} = \begin{pmatrix} \varepsilon_o & 0 & 0 \\ 0 & \varepsilon_o & 0 \\ 0 & 0 & \varepsilon_e \end{pmatrix},$$

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{pmatrix},$$

$$\boldsymbol{\mu} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

where ε_o and ε_e are the dielectric constant of ordinary and extraordinary waves, which are determined through the corresponding refractive indices:

$$\varepsilon_o = n_o^2, \qquad \varepsilon_e = n_e^2.$$

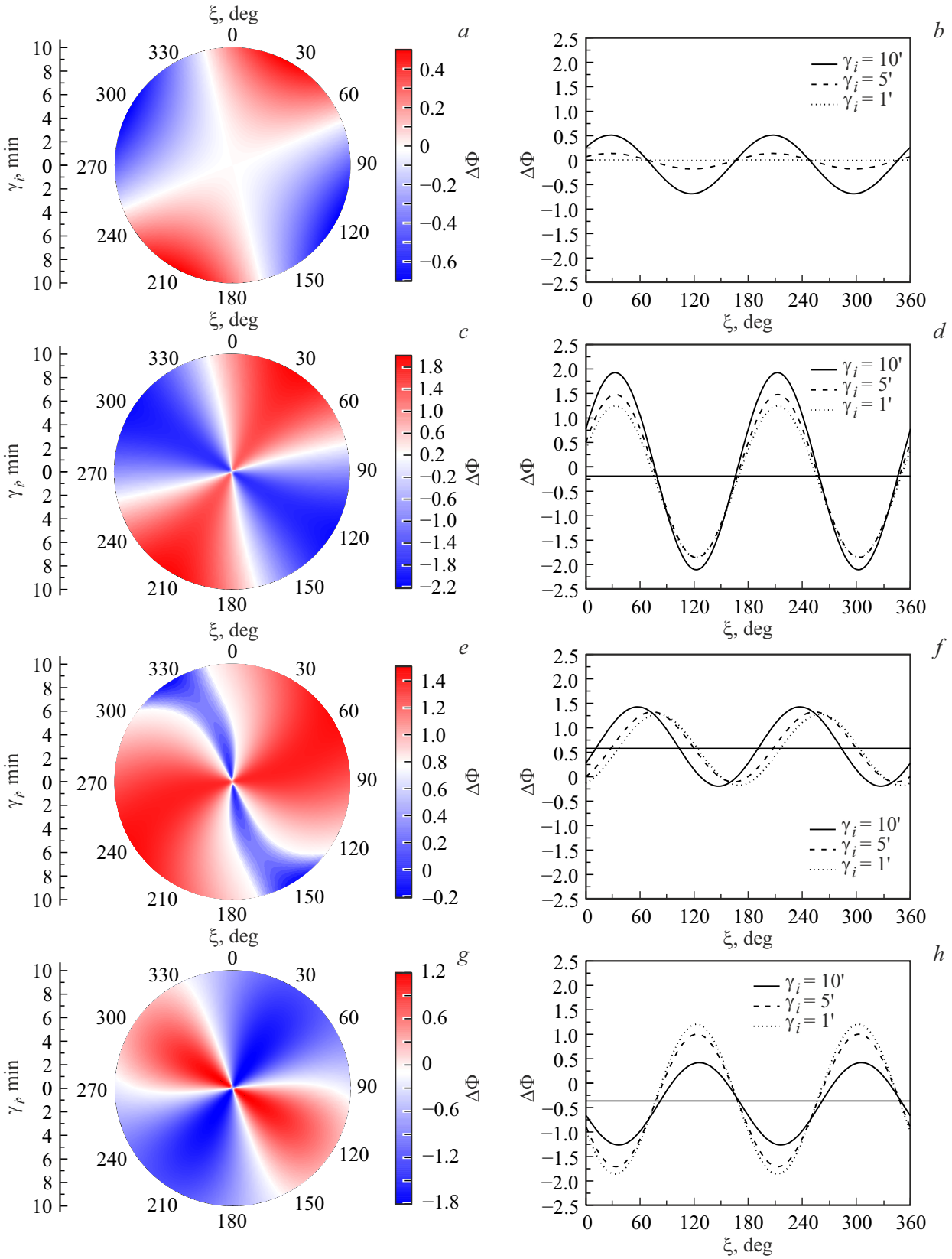


Figure 3. Dependence of the $\Delta\Phi(\xi, \gamma_i$ value (in units of deg·10^{-3}) for the following system parameters: $\theta = 0$ (a, b), $10'$ $(c-h)$; $\varphi = 0$ $(a-d)$, 45° (e,f), 90° (g, h).

The value of the component of the gyration tensor α_{11} is determined from the relation

$$\alpha_{11} = -\frac{\rho_0 \lambda}{2\pi},$$

where ρ_0 is the specific rotation of the polarization plane when radiation propagates along the optical axis, expressed in rad/nm (wavelength λ also expressed in nm). The value of the component α_{33} is more difficult to determine, since birefringence effects interfere with estimating the specific rotation during radiation propagation along the optical axis. It is known that the components of the gyration tensor α in the exact theory are related to the components of the gyration tensor g in the approximate theory as follows:

$$g_{11} = (\alpha_{11} + \alpha_{33})\bar{n}, \qquad g_{33} = 2\alpha_{11}\sqrt{\varepsilon_o},$$

where

$$\bar{n} = \sqrt{\frac{\varepsilon_o + \varepsilon_e}{2}}.$$

The g_{11}/g_{33} ratio is considered constant over a wide range of wavelengths and in various studies [6–9,20] is estimated to range from -0.45 to -0.54. Let's take this ratio to be -0.5. The values of the refractive indices n_o and n_e and the specific rotation ρ_0 are well studied and can be calculated for a wide range of wavelengths using approximating formulas [22,23]. The above expressions fully define all the parameters of the model and can be used to calculate the state of the field at the output of the polarimetric plates.

Simulation results

The value of the optical rotation angle Φ is determined by the azimuth β_t of the transmitted wave:

$$\Phi = \beta_t - \beta_i, \tag{5}$$

where

$$\beta_t = \frac{1}{2}\arctan\left(\frac{2\mathrm{Re}(E_{ts}/E_{tp})}{1 - |E_{ts}/E_{tp}|^2}\right).$$

The value of the optical rotation angle calculated by the formula (5) is a function that depends on the system parameters, i.e. $\Phi = f(\lambda, h, \varphi_0, \theta, \xi, \gamma_i)$, where the values of the refractive indices n_o and n_e and the specific optical rotation ρ_0 are determined by the wavelength λ. Let the $\Phi(p)$ entry imply that the optical rotation angle is considered as a function of the parameter p, and the remaining parameters are considered constant. To assess the effect of the model parameters, we will consider the difference

$$\Delta\Phi(p) = \Phi(p) - \Phi_0,$$

where Φ_0 is the optical rotation angle, calculated for $\theta = 0$, $\gamma_i = 0$, and, in fact, represents the nominal value of the measure.

To begin with, let's consider the case when radiation with a wavelength of $\lambda = 546.23$ nm falls on a plate of positive quartz with a thickness of $d = 1$ mm. For such a $\Phi_0 = 25.535°$ plate, which coincides with the specific rotation of quartz at a given wavelength. For a plate made of negative quartz, the result will be the same up to the optical rotation angle sign.

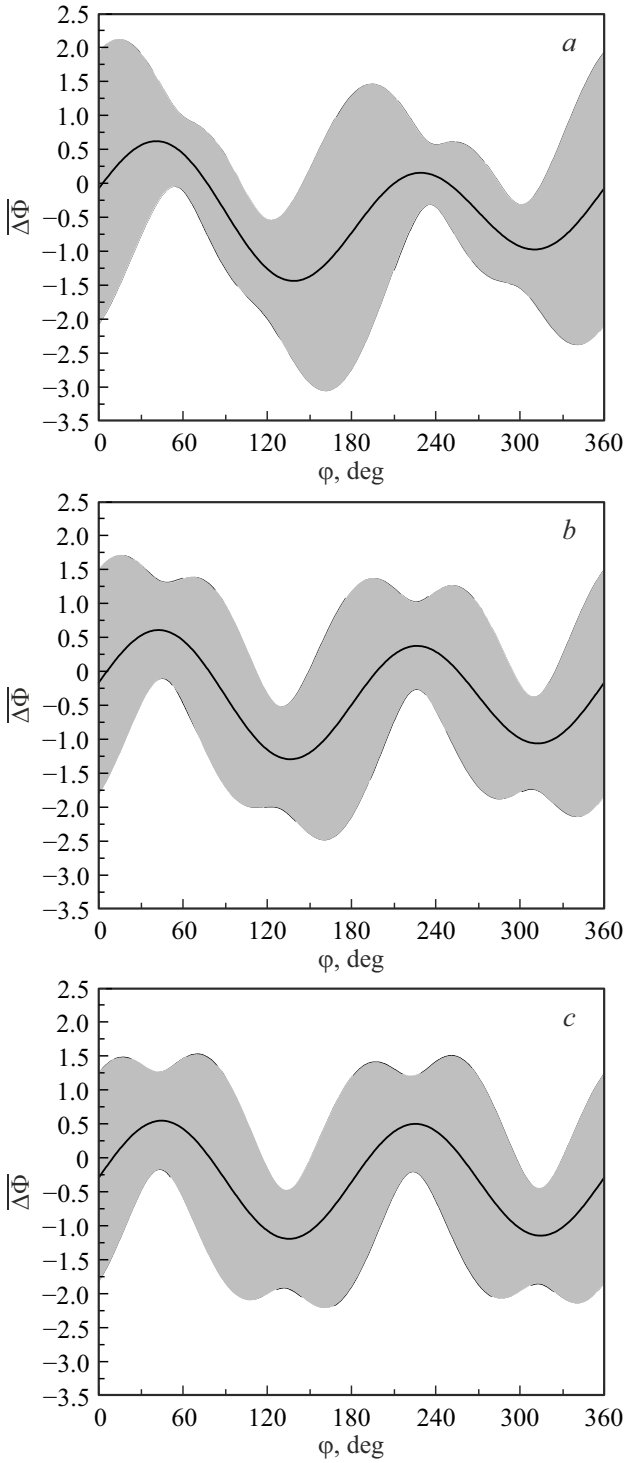


Figure 4. Dependence of the average value and amplitude (shaded area) (in units of deg·10^{-3}) on the angle of position of the main plane φ: $\gamma_i = 10'$ (*a*), $5'$ (*b*), $1'$ (*c*).

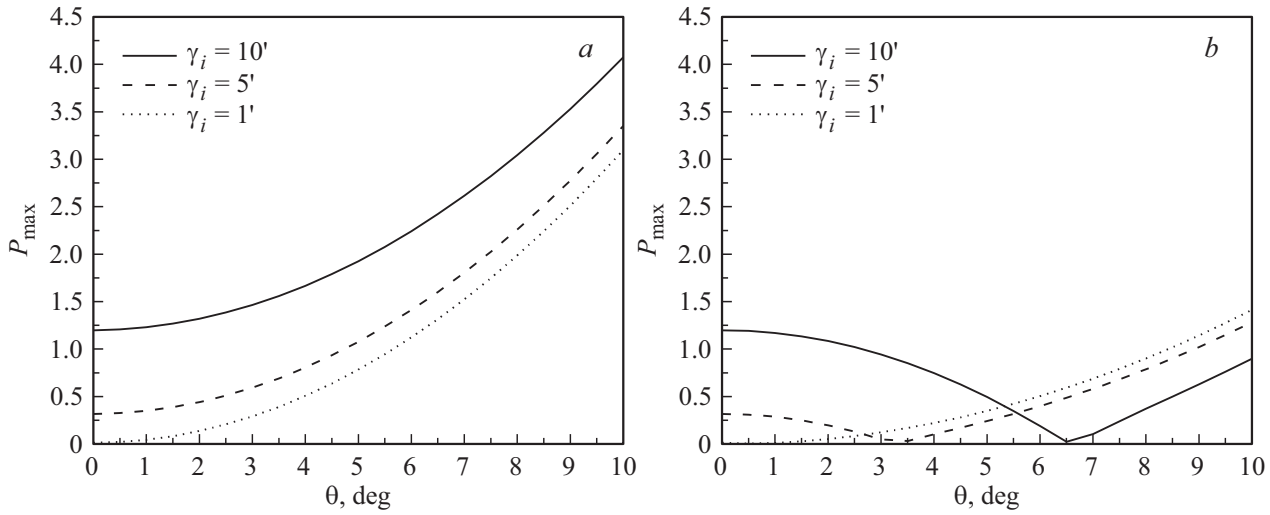


Figure 5. The dependence of the maximum and minimum variation of $\Delta\Phi(\xi)$ values (in units of deg·10^{-3}) on the deviation of the optical axis from the perpendicular to the working face θ: (*a*) $P_{\max}(\theta)$, (*b*) $P_{\min}(\theta)$.

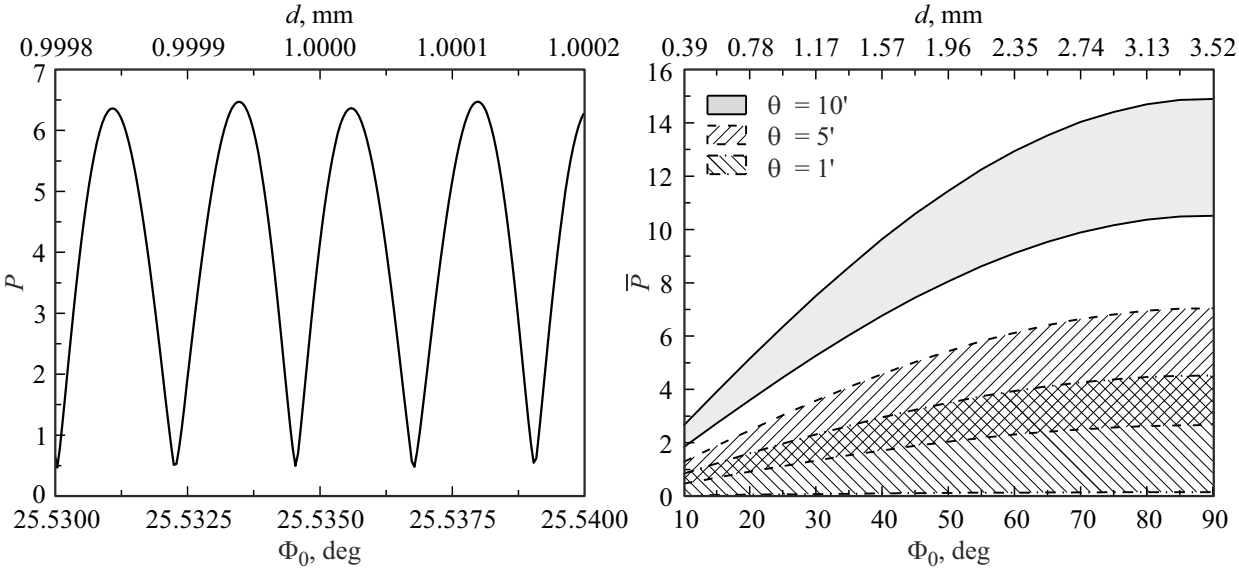


Figure 6. Dependence of the maximum spread of changes in values of $\Delta\Phi(\xi)$ (in units of deg·10^{-3}) on the nominal value of the optical rotation angle: (*a*) $P_{\max}(\Phi_0)$ in the vicinity of the point $\Phi_0 = 25.535°$ ($d = 1$ mm), $\gamma_i = 10'$, $\theta = 10'$; (*b*) — dependency envelopes $\tilde{P}_{\max}(\Phi_0)$ calculated for γ_i values in the range from $1'$ to $10'$ (shaded areas) with optical axis deviations θ, equal to $1'$, $5'$ and $10'$.

Let us consider the dependence of $\Delta\Phi$ on the rotation angle of the plate ξ and the angle of incidence γ_i. The simulation results are shown in Table 3. The figure on the left shows the values of $\Delta\Phi(\xi, \gamma_i)$ in polar coordinates, where the angle ξ is calculated along a circle from a vertical line, and γ_i is represented by a radius shifted from the center. The corresponding values of $\Delta\Phi(\xi)$ are shown on the right for some fixed values of the deviation angle γ_i, i.e. they represent circular sections of the graphs on the left. The graphs in Fig. 3, *a, b* are calculated for $\theta = 0$, the rest are calculated for $\theta = 10'$.

It can be seen from the graphs presented that when the quartz control plate is rotated, the observed dependence has a sinusoidal character. The amplitude and average value depend on the angle of incidence and the orientation of the main plane relative to the plane of incidence. The experimental results published in Ref. [24] confirm the shape of the resulting dependencies. However, the difference between the maximum and minimum values obtained experimentally may in some cases differ from the simulation results by one–two orders of magnitude.

Fig. 4 shows how the average value $\overline{\Delta\Phi}(\varphi)$ changes for some fixed values of the angle γ_i, depending on the angle between the plane of incidence and the main plane φ. It can be seen that the average value can be either less or more than the nominal value Φ_0. The shaded area is determined

by the amplitude $A_\Phi(\varphi)$, deferred from the average and calculated as

$$A_\Phi(\varphi) = \frac{1}{2}\left(\max_\xi [\Delta\Phi(\xi, \varphi)] - \min_\xi [\Delta\Phi(\xi, \varphi)]\right).$$

To consider how the optical rotation angle changes when the optical axis deviates from the perpendicular to the working surface of the plate, we calculate the maximum and minimum spread of values $\Delta\Phi(\xi)$:

$$P_{\max}(\theta) = \max_\varphi \left(\max_\xi [\Delta\Phi(\xi, \varphi, \theta)] - \min_\xi [\Delta\Phi(\xi, \varepsilon, \theta)]\right),$$

$$P_{\min}(\theta) = \min_\varphi \left(\max_\xi [\Delta\Phi(\xi, \varphi, \theta)] - \min_\xi [\Delta\Phi(\xi, \varepsilon, \theta)]\right).$$

It can be seen from the obtained graphs (Fig. 5) that the spread increases significantly with increasing deviation of the optical axis from the normal. At the same time, the difference between the maximum and minimum spreads becomes larger, which can lead to greater uncertainty in the measurement results.

Finally, we can evaluate how the maximum changes $\Delta\Phi(\nu)$ depend on the nominal value of optical rotation angle of the plate Φ_0, which is determined by the plate thickness d:

$$P_{\max}(\Phi_0) = P_{\max}(d) = \max_\varphi\Big(\max_\xi[\Delta\Phi(\xi, \varphi, d)] - \min_\xi[\Delta\Phi(\xi, \varphi, d)]\Big).$$

During the modeling process, it was found that the $P_{\max}(\Phi_0)$ dependence is periodic with a high frequency (Fig. 6, *a*), which persists over the entire range of nominal values under consideration. To evaluate the change in the spread of $\tilde{P}_{\max}(\Phi_0)$ values of this dependency (Fig. 6, *b*). The envelope is considered in the range of γ_i from $1'$ to $10'$ (shaded areas) with optical axis deviations of θ equal to $1'$, $5'$ and $10'$. The obtained zones can be considered as the upper limit of the measurement uncertainty prescribed by the tolerances for the manufacture of plates. It can be seen from from the graphs obtained that with an increase in the nominal value, the uncertainty increases smoothly.

Conclusion

It can be seen from the above graphs that the angle of rotation of the plane of polarization depends in a complex way on the deviatio of the parameters of the polarimetric plates from the ideal ones. In the worst case scenario, the uncertainty will exceed the margin of error for working standards of the first category $\delta = 0.003°$. It is recommended to manufacture them with higher requirements than specified in GOST 22409-77 to reduce measurement uncertainty and create plates corresponding to the level of the first-class working standard in the range from minus 90° to plus 90°. Reducing the maximum deviation of the optical axis from the perpendicular to the working surface to $5'$ will reduce by about 2 times the uncertainty of measuring the optical rotation angle of plates in the range of nominal optical rotation angle values from minus 90° to plus 90°. It is also recommended to reduce the tolerance for deviation from the perpendicular of the working surface of the plate to the axis of incident radiation to $5'$, which can be achieved by revising the recommended cylindrical tube design. The presented model is of great practical importance, since it describes the process of radiation passing through a quartz control plate from the point of view of a strict theory. The contribution of various parameters to the overall uncertainty can be numerically estimated using this model.

References

[1] ICUMSA Specification and Standard SPS-1 (International Commission for Uniform Methods of Sugar Analysis, 2017) https://www.icumsa.org/methods/icumsa-specification-and-standard-sps-1-2017

[2] R14 (1995): Polarimetric saccharimeters graduated in accordance with the ICUMSA International Sugar Scale. (International Organization of Legal Metrology, Paris, France) URL: https://www.oiml.org/en/files/pdf_r/r014-e95.pdf

[3] E.A. Volkova. Trudy metrologicheskih institutov SSSR. Vsesoyuz. nauch.-issled. in-t metrologii im. D.I. Mendeleeva, Issledovaniya v oblasti opticheskih izmerenij i izmerenij luchistoj energii, **56** (116), 27 (1961) (in Russian).

[4] M. Schulz, A. Fricke, K. Stock, A.D. Alvarenga, H. Belaidi. In: *XVIII IMEKO World Congress. Metrology for a Sustainable Development.* (Rio de Janeiro, Brazil, 2006)

[5] A.P.D. Alvarenga, N.C.E. Pereira, L.V.G. Tarelho, R.S. França, H. Belaidi. Produto & Produco, **11** (1), 1 (2010). DOI: 10.22456/1983-8026.11818

[6] H. Horinaka, K. Tomii, H. Sonomura, T. Miyauchi. Jap. J. Appl. Phys., **24**, 755 (1985). DOI: 10.1143/JJAP.24.755

[7] H. Zhong, Z.H. Levine, D.C. Allan, J.W. Wilkins. Phys. Rev. B, **48**(3), 1384 (1993). DOI: 10.1103/PhysRevB.48.1384

[8] J. Kobayashi, T. Asahi, S. Takahashi, A.M. Glazer. J. Appl. Crystall., **21**, 479 (1988). DOI: 10.1107/S0021889888005503

[9] O. Arteaga, A. Canillas, G.E. Jellison. Jr. Appl. Opt., **48**(28), 5307 (2009). DOI: 10.1364/AO.48.005307

[10] F.I. Fedorov. *Teoriya girotropii* (Nauka i tekhnika, Minsk, 1976) (in Russian)

[11] A.F. Konstantinova, B.N. Grechushnikov, B.V. Bokut', E.G. Valyashnikov. *Opticheskie svojstva kristallov* (Nauka i tekhnika, Minsk, 1995) (in Russian).

[12] A.F. Konstantinova, K.K. Konstantinov, B.V. Nabatov, E.A. Evdishchenko. Kristallografiya, **47** (4), 702 (2002) (in Russian).

[13] A.F. Konstantinova, B.V. Nabatov, E.A. Evdishchenko, K.K. Konstantinov. Kristallografiya, **47** (5), 879 (2002) (in Russian).

[14] A.F. Konstantinova, K.A. Rudoj, B.V. Nabatov, E.A. Evdishchenko, V.I. Stroganov, O.YU. Pikul'. Kristallografiya, **48** (5), 884 (2003) (in Russian).

[15] A.F. Konstaninova. PFMT, **2** (7), 75 (2011) (in Russian).

[16] A.F. Konstantinova, T.G. Golovina, K.K. Konstantinov. Crystallography Reports, **9** (4), 447 (2014). DOI: 10.1134/S1063774514040105.

[17] K.P. Loveckij, A.A. Hohlov. Vestnik RUDN. Seriya Matematika. Informatika. Fizika, **1**, 93 (2010) (in Russian).
[18] D.W. Berreman. JOSA, **62** (4), 502 (1972). DOI: 10.1364/JOSA.62.000502
[19] V.M. Potapov. *Stereohimiya* (Himiya, Moskva, 1988) (in Russian).
[20] A.V. SHubnikov. *Kvarc i ego primenenie* (Moskva, 1958) (in Russian).
[21] S.P. Palto. JETP, **119** (4), 638 (2001) [S.P. Palto. JETP, **92** (4), 552 (2001)] (in Russian).
[22] G. Ghosh. Opt. Commun., **163**, 95 (1999). DOI: 10.1016/S0030-4018(99)00091-7
[23] J. Keitel. In: *ICUMSA Proceedings, 22nd Session, Subject 4. Polarimetry and Quartz Plates*, (1998).
[24] A.A. Golopolosov. *III Mezhdunarodnaya nauchno-prakticheskaya konferenciya molodyh uchenyh i specialistov „ZA NAMI BUDUSHCHEE" i pilotnyj Mezhdunarodnyj konkurs „Luchshij molodoj metrolog MGS SNG" 2024 goda* (OOO „Tipografiya Litas+", 2024), p. 63 (in Russian).

Translated by A.Akhtyamov