04

Portable properties of low-temperature thermochemically nonequilibrium plasma

© M.M. Stepanov

Baltic State Technical University „VOENMEKH“ named after Marshal D.F. Ustinov,
St. Petersburg, Russia
e-mail: lecros@mail.ru

Received July 14, 2025
Revised September 22, 2025
Accepted October 23, 2025

The effect of two temperatures (the difference between the temperature of electrons and the temperature of „heavy“ particles) on the transport properties of a partially ionized thermochemically nonequilibrium plasma is investigated. Diffusion and heat fluxes are calculated in the highest approximations of the Chapman–Ensky theory. At the same time, a technique is used when „forces“ are calculated through „flows“, which allows you to obtain the values of flows, bypassing the direct calculation of transfer coefficients. This is combined with neglecting the terms of the order $\varepsilon = (m_e/m_h)0.5 \ll 1$, where me, mh are the masses of electrons and heavy particles, significantly reduces the number of calculations and makes this method really applicable for gas-dynamic calculations even while keeping thermo and barodiffusion in mind.

Keywords: low-temperature thermochemically nonequilibrium plasma, diffusion and heat flows, thermo and barodiffusion.

DOI: 10.61011/TP.2026.03.63156.180-25

Introduction

Many applied problems require fairly accurate data on the momentum, heat, and mass fluxes for low-temperature plasma flows, especially those characterized by low Reynolds numbers. As was noted in a number of studies [1–10], these fluxes for weakly ionized plasma need to be calculated using accurate methods based on the molecular kinetic theory of gases. This is especially important for calculation of such quantities as thermal conductivity, diffusion, thermal diffusion, and barodiffusion coefficients. When calculating these parameters (using, e.g., the Chapman–Enskog method), one needs to take into account higher approximations in finding these coefficients in the form of series in Sonin polynomials. If one fails to do so, the error of flow parameter calculation may become unacceptably large. Therefore, the present study relies on the use of higher approximations of the Chapman–Enskog theory and is focused on examining the accuracy of calculations of the transport properties of partially ionized air plasma under the conditions of possible deviation of electron temperature T_e from temperature T of „heavy“ components. One particular field where this is important is the study of parameters of the atmospheric region (both in the immediate vicinity of flying bodies and in their wakes) disturbed during hypersonic flight. In the general case, this region is a thermochemically non-equilibrium flow of weakly ionized plasma. However, the authors of most published studies focused on the determination of parameters of such flows relied on the assumption of their thermal equilibrium. The simplest formulae are used to calculate the transport coefficients, which is attributable to the fact that more accurate methods of the molecular kinetic theory of gases require a large amount of computation. The formulae used most often for determining the coefficients of thermal conductivity, diffusion, and viscosity in modeling of gas-dynamic fields around the aforementioned high-speed aircraft are the Wilke and Mason–Saxena ones, which correspond approximately to the first approximation of the Chapman–Enskog method with the presence of two temperatures, thermal diffusion, and barodiffusion neglected. However, one may note a number of studies where a method for determining transport properties was proposed in a more precise formulation that requires significantly fewer calculations even when the possible difference between the electron temperature and the temperature of „heavy“ particles and the indicated effects are taken into account [1–9]. Therefore, this method is actually applicable to gas-dynamic calculations of high-speed aircraft flight.

We follow the authors of [3,4,6,7] in calculation of diffusion and heat fluxes. The methodology of these studies with „forces“ being calculated through „fluxes“ allows one to obtain the values of fluxes without calculating the transport coefficients directly. Combined with suppression of terms on the order of $\varepsilon = \left(\frac{m_e}{m_h}\right)^{0.5} \ll 1$, where m_e and m_h are the masses of electrons and „heavy“ particles, this reduces significantly the amount of computation and makes the method truly applicable in gas-dynamic calculations performed with account for the above-noted possible difference between T_e and T and the effects of thermal diffusion and barodiffusion.

1. Calculation procedure

Following [3–7], we limit ourselves to the third approximation for the electron component and the second approximation for „heavy“ particles. The corresponding system of equations then takes the following form:

1. For „heavy“ particles,

$$\frac{\nabla x_i}{x_i} = \sum_{j=1}^{NH} \frac{x_j}{D_{ij}} \left(\frac{I_j}{\rho_j} - \frac{I_i}{\rho_i} \right) - \frac{x_e}{D_{ei}} \times \frac{I_e}{\rho_e}$$

$$- \sum_{j=1}^{NH} \frac{x_j}{D_{ij}} \frac{m_{ij}}{m} a_{ij}^{01} (g_j - g_i) - \frac{x_e}{D_{ei}} a_{ei}^{01} g_e$$

$$+ \frac{x_e}{D_{ei}} a_{ei}^{02} f_e - \left(1 - \frac{m_i}{m}\right) \frac{P}{nkT} \nabla \ln P$$

$$- x_e \left(1 - \frac{T_e}{T}\right) \nabla \ln P_e - x_e \nabla \ln \frac{T}{T_e} + z_i e \frac{E}{T}; \quad (1)$$

$$\sum_{j=1}^{NH} \left[\frac{C_i^2}{D_{ii}} a_{ii}^{11} (1 - C_i) \delta_{ij} - \frac{C_j^2}{D_{jj}} C_j a_{jj}^{11} (1 - \delta_{ij}) \right.$$

$$\left. + \sum_{j=1}^{NH} \frac{x_j - x_k}{D_{jk}} \left(\frac{m_{kj}}{m} \right)^2 A_{jk}^i \right] g_{j=.}$$

$$= \sum_{j=1}^{NH} \frac{x_i x_{.jk}}{D_{ij}} \frac{m_{ij}}{m} a_{ij}^{10} \left(\frac{I_j}{\rho_j} - \frac{I_i}{\rho_i} \right). \quad (2)$$

The following relation may be used instead of one of the above equations:

$$\nabla \ln T = - \sum_{j=1}^{NH} \left[\frac{C_j^2}{D_{jj}} a_{jj}^{11} + \sum_{k=1}^{Nh} \frac{x_j x_k}{D_{jk}} \left(\frac{m_{kj}}{m} \right)^2 A_{jk}^0 \right] g_j; \quad (3)$$

2. For electrons,

$$\frac{\nabla x_e}{x_e} = - \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} \frac{I_e}{\rho_e} + \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{01} g_e$$

$$- \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{02} f_e \frac{P}{nkT} \nabla \ln P$$

$$+ (1 - x_e) \left(1 - \frac{T_e}{T}\right) \nabla \ln P_e - e \frac{E}{kT}, \quad (4)$$

$$\nabla \ln T_e = \frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{10} \frac{I_e}{\rho_e}$$

$$- \left(\frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{11} + \frac{x_e}{D_{ee}} a_{ee}^{11} \right) g_e$$

$$+ \left(\frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{12} + \frac{x_e}{D_{ee}} a_{ee}^{12} \right) f_e, \quad (5)$$

$$\left(\frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{22} + \frac{x_e}{D_{ee}} a_{ee}^{22} \right) f_e =$$

$$- \frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{20} \frac{I_e}{\rho_e}$$

$$+ \left(\frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{21} + \frac{x_e}{D_{ee}} a_{ee}^{21} \right) g_e; \quad (6)$$

The following notation is introduced in these formulae:

$$g_i = \frac{m}{kT} \frac{J_i}{\rho_i}; \; g_e = \frac{m_e}{kT} \frac{J_e}{\rho_e}; \; f_e = \frac{35}{8} g_{e2}; \; m = \sum_{j=1}^{NH} x_j m_j;$$

$$m_{ij} = \frac{m_i m_j}{m_i + m_j}; \; D_{ij} = \frac{3}{16 n m_{ij}} \frac{kT}{\Omega_{ij}^{11}}; \; D_{ee} = \frac{3}{8 n m_e} \frac{kT_e}{\Omega_{ee}^{11}};$$

$$C_i = \frac{\rho_i}{\rho}; \; x_i = \frac{n_i}{n}; \; n = \sum_{j=1}^{N} n_j.$$

Here, I_i and J_i are the diffusion and reduced heat fluxes of the ith component; T_e and T are the temperatures of electrons and „heavy“ components; P, P_e, and P_h are the mixture, electron, and „heavy“ particle pressures; n and n_i are the numbers of particles in unit volume of the mixture and the ith component; ρ and ρ_i are the densities of the mixture and the ith component; D_{ij} and E are the binary diffusion coefficients in the first approximation and the internal electric field strength; e and Z_{ie} are the charges of an electron and the ith component; N and $NH = N - 1$ are the numbers of components and „heavy“ components; $V_i = I_i/\rho_i$ is the diffusion velocity of the ith component; and Ω_{ij}^{mn} are the collision integrals.

In the above equations, a_{ij}^{mn}, A_{ij}^0, and A_{ij}^1 are linear functions of collision integrals $a_{ij}^{mn} = F_1(\omega_{ij}^{ls})$:

$$A_{jk}^0 = F_2\left(\frac{m_j}{m_k}, a_{jk}^{22}\right); \; A_{jk}^1 = F_3\left(\frac{m_j}{m_k}, a_{jk}^{11}, a_{jk}^{22}, C_i\right),$$

where $\omega_{ij}^{ls} = \Omega_{ij}^{ls}/\Omega_{ij}^{11}$.

The expressions for them were given in [4,5].

This system of equations is closed with a relation arising from the quasi-neutrality condition, which allows one to eliminate the strength of internal induced electric field E:

$$\sum_{i=1}^{N} \frac{Z_i I_i}{m_i} = 0. \quad (7)$$

The given transport equations are a system of linear algebraic equations (SLAE) in diffusion and heat fluxes I_i, J_i and auxiliary flux f_e. Notably, the transport equations for electrons are separated from the transport equations for „heavy“ particles, and the right-hand sides have identical matrices with respect to the sought-for projections of fluxes onto the axes of Cartesian coordinate system (X, Y, Z). Thus, to find the fluxes, one needs to solve this SLAE, which

reduces significantly the amount of computation compared to the standard method of calculating them by determining the corresponding transport coefficients.

As an example, we consider the flow of weakly ionized air plasma, which is a mixture of the following components: O_2, O, N_2, N, NO, Ar, NO^+, e^-. The possible lack of both chemical and thermodynamic equilibrium with electron temperature T_e differing from temperature T of „heavy" particles (two-temperature gas mixture) is assumed. A further reduction in the amount of computation is achieved by replacing the original system of equations (1)–(7) with a certain modification of it where all fluxes associated with the electronic component (I_e, J_e, f_e) and strength (E) are excluded using Eqs. (1)–(7). Equation (1) then takes the form

$$\frac{\nabla x_i}{x_i} - K2_i = \sum_{j=1}^{NH} \frac{x_j}{D_{ij}} \left(\frac{I_j}{\rho_j} - \frac{I_i}{\rho_i} \right) + K1_i \frac{I_7}{\rho_7} - \sum_{j=1}^{NH} \frac{x_j}{D_{ij}} \frac{m_{ij}}{m} a_{ij}^{01} (g_j - g_i), \tag{8}$$

where $i = 1$–6 for neutral components O, O_2, N, N, NO, and Ar, respectively; $i = 7$ for NO^+; and $i = 8 = e^-$ for electrons.

$$K1_i = \frac{x_e}{D_{ei}} \left[1 + a_{ei}^{01} \frac{P_{11}}{P_{12}} + \frac{S_3}{S_1} \left(a_{ei}^{01} \frac{P_{13}}{P_{12}} + a_{ei}^{02} \right) \right] + \frac{S}{S_1} \delta_{i7},$$

$$K2_i = \frac{x_e}{D_{ei}} P_2 \frac{P_{12}}{S_1} \left(a_{ei}^{01} \frac{P_{13}}{P_{12}} + a_{ei}^{02} \right) + \left(P_2 \frac{S_2}{S_1} - P_3 \right) \delta_{i7},$$

$$P_2 = \nabla \ln T_e;$$

$$P_3 = \frac{\nabla x_e}{x_e} + \frac{P}{nkT} \nabla \ln P - (1 - x_e) \left(1 - \frac{T_e}{T} \right) \nabla \ln P_e - (1 - x_e) \nabla \ln \frac{T}{T_e};$$

$$S_1 = P_{12} P_{23} - P_{22} P_{13};\ S_2 = P_{12} P_{33} - P_{32} P_{13};$$

$$S_3 = P_{11} P_{22} - P_{21} P_{12};$$

$$S = P_{11} (P_{22} P_{33} - P_{32} P_{23}) - P_{21} S_2 + P_{31} S_1;$$

$$P_{11} = -\frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{20};$$

$$P_{12} = \frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{21} + \frac{x_e}{D_{ee}} a_{ee}^{21};$$

$$P_{21} = \frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{10};$$

$$P_{13} = -\frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{22} - \frac{x_e}{D_{ee}} a_{ee}^{22};$$

$$P_{22} = -\frac{T}{T_e} \sum_{j=1}^{NH} \frac{x_j}{D_{ej}} a_{ej}^{11} - \frac{x_e}{D_{ee}} a_{ee}^{11};$$

$$P_{23} = P_{12};\ P_{32} = \frac{T_e}{T} P_{21};\ P_{33} = \frac{T_e}{T} P_{11}.$$

Equations (2), (3), (8) represent the final closed system of $2^*(N-1) = 14$ linear algebraic equations in fluxes I_i, J_i ($i = 1 \ldots (N - 1 = 7)$).

The transport properties of the electron component and the electric field strength are defined as

$$I_e = \frac{\rho_e}{\rho_7} I_7;\ g_e = -\frac{P_{13}}{S_1} P_2 - \left(\frac{p_{11}}{p_{12}} + \frac{P_{13}}{P_{12}} \frac{S_3}{S_1} \right) \frac{I_e}{\rho_e};$$

$$f_e = \frac{P_{12}}{S_1} P_2 + \frac{S_3}{S_1} \frac{I_e}{\rho_e};$$

$$E = \frac{kT}{eS_1} \left(S \frac{I_e}{\rho_e} + S_2 P_2 - S_1 P_3 \right).$$

Total heat flux of „heavy" particles Q is determined with account for the fluxes induced by the transport of energy of the internal degrees of freedom of polyatomic molecules and the energy flux due to mass transport [11]:

$$Q = J_i + \sum_{j=1}^{NH} h_i I_i;\ J_h = \sum_{j=1}^{NH} (J_i - \lambda_i^{\mathrm{b}} \nabla T_{\mathrm{b}}^i - \lambda_i^k \nabla T_k^i).$$

Here, h_i, λ_i^{b}, λ_i^k, T_{b}^i, and T_k^i are the enthalpy of molecules type i, the thermal conductivity coefficients, and the temperatures of rotational and vibrational degrees of freedom, respectively.

To solve the given system of equations, one needs to know coefficients a_{ij}^{mn}, which may be calculated by specifying the potentials of interaction between particles. Data on these potentials were provided in [8–13] and other studies. The indicated coefficients for the $\varphi = \frac{\mathrm{const}}{r^n}$ potential and the rigid-sphere potential are constant pre-calculated values (see [3,4,10,11] and other studies), which also contributes significantly to a reduction in the amount of computation needed to determine the transport properties. Thus, the values of $\omega_{ij}^{ls} = \mathrm{const}$ for these potentials, which implies that $a_{ij}^{ls} = \mathrm{const}$, since they are their linear combinations. Table 1 lists the parameters of $\varphi = \frac{\mathrm{const}}{r^n}$ potentials used in calculation of collision integrals.

Following [13,14], we chose rigid-sphere potentials for the interaction of NO^+–N, NO^+–O, and NO^+–Ar particles, since they have little effect on the final values of flows of weakly ionized plasma. The truncated Coulomb potential is used for charged particles.

Coefficients a_{ij}^{mn} are calculated with the above interaction potentials. The approximation dependencies obtained in [5] are used for charged particles. Table 2 lists the values of a_{ij}^{mn} for „heavy" particles. Here, $A_e = \frac{kT_e}{e^2} Rd$ and $A_7 = \frac{kT}{e^2} Rd$, where $Rd = \left[\frac{kTT_e}{4\pi e^2 (T_e n_7 + T n_e)} \right]^{0.5}$ is the Debye screening radius.

The data from [4,5,8–14] (Table 3) are used for the interaction of electrons with other particles.

In addition, $a_{ee}^{12} = a_{ee}^{21} = 0.132$, $a_{ee}^{22} = 0.328$, and $a_{ee}^{11} = 0.383 - 0.24 \times A_e$.

Coefficients $a_{ei}^{02} = a_{ei}^{20}$, $a_{ei}^{01} = a_{ei}^{10}$, $a_{ei}^{21} = a_{ei}^{12}$, a_{ei}^{11}, and a_{ei}^{22} with $i =$ O, N_2, which depend strongly on temperature,

Table 1. Parameters of potentials of the $\varphi = \frac{\text{const}}{r^n}$ type

Interaction type	const (8 V)	n	Interaction type	const (8 V)	n
N_2–N_2	550	7.4	N–N	13.8	4.28
N_2–O_2	330	6.8	N–O	19.1	5.13
N_2–NO	880	8.48	O–O	2.64	5.98
O_2–O_2	240	6.3	Ar–Ar	944	8.46
O_2–NO	1007	8.72	N_2–NO^+	12.67	4
NO–NO	403	7.5	O_2–NO^+	11.52	4
N_2–N	76.6	6.31	NO–NO^+	12.67	4
N_2–O	22.5	5	Ar–O	158	7.22
O_2–N	362	8.3	Ar–N	114	6.37
O_2–O	13.25	4.4	Ar–O_2	1270	8.38
NO–N	379	8.12	Ar–N_2	755	7.78
NO–O	801	9	Ar–NO	567	7.06

are calculated directly using the dependences and data from [4,5,10].

Linear system of equations (2), (3), (8) may be solved by any existing method (e.g., the Gauss method). However, it is ill-conditioned in a number of applied problems. The ill-conditioning of these equations may be associated, e.g., with a strongly heterogeneous composition of the gas mixture under consideration: widely (by orders of magnitude) differing component concentrations, particles with widely differing masses, etc. All of this is characteristic of weakly ionized plasma. Therefore, it normally requires the use of more complex calculation methods. In the present study, we used an iterative method for solving the specified $AX = B$ system: an approximate solution $X^{(p)}$ is calculated first using the Gauss method with pivoting and is then refined in iterative process $X^{(p+1)} = X^{(p)} + d^{(p)}$, where $d^{(p)}$ is the solution of system $A^* d^{(p)} = B - A^* X^{(p)}$.

If the system is not too ill-conditioned, this process is convergent; the number of iterations is chosen based on the specified relative accuracy $\delta \leq 1\,\%$ of the obtained result.

The flow of low-temperature plasma of heated air in a thermochemically non-equilibrium state was studied as an example. Transport properties were considered only in projections onto the Y axis. The initial profiles of all parameters are presented in Fig. 1. These are the parameters corresponding approximately to the flow in a hypersonic axisymmetric far wake behind a „thin“ body flying with velocity $V_\infty = 7.4\,\text{km/s}$ in air with undisturbed flow parameters $\rho_\infty = 0.21 \cdot 10^{-4}\,\text{kg/m}^3$, $T_\infty = 185\,\text{K}$ at distance $x \approx 20d$ from the body ($d = 2R$ is its characteristic dimension) [15–17].

The electron temperature was set as $\frac{T_e}{T} = K \cdot \exp(-A^* y_a)$, where $y_a = y/R$ and A, K, and R are constants.

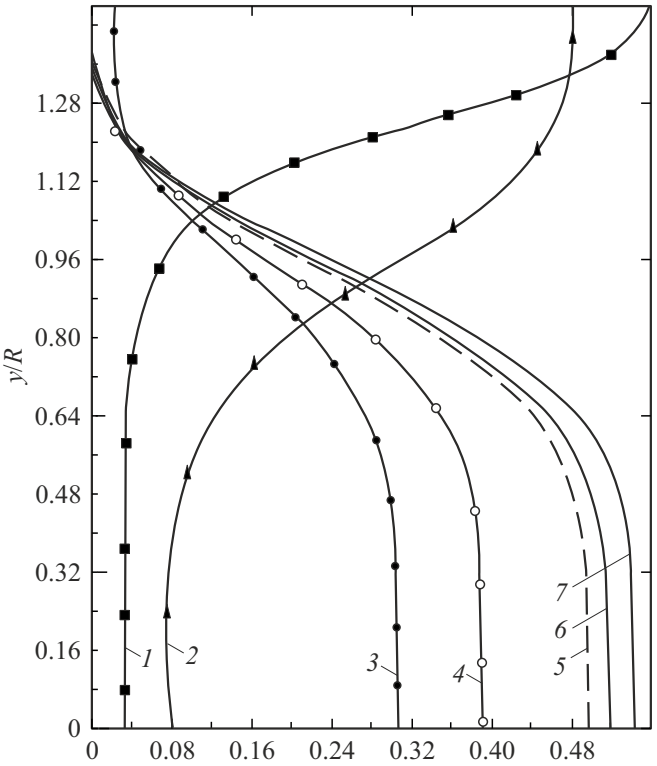

Figure 1. Initial profiles of relative mass concentrations C_i^*, temperature $T^* = T/T_\infty$, and density $\rho^* = \rho/\rho_\infty$: *1* — $C_{O_2} = 0.136 + 2C_{O_2}^*$; *2* — $\rho^* = \rho/\rho_\infty$; *3* — $T^* = (T/T_\infty)/55$; *4* — $C_N = 0.5 \cdot 10^{-3} C_N^*$; *5* — $C_e = 2.0 \cdot 10^{-10} C_e^*$; *6* — $C_O = 10^{-1} C_O^*$; *7* — $C_{NO} = 10^{-1} C_{NO}^*$.

Table 2. Values of coefficients a_{ij}^{mn} for „heavy“ particles

	O	O_2	N	N_2	NO	Ar	NO^+
$a_{iJ}^{01} = a_{iJ}^{10}$							
O	−0.0662						
O_2	−0.0182	−0.0730					
N	−0.0441	−0.1036	−0.0131				
N_2	−0.0400	−0.0824	−0.0732	−0.0919			
NO	−0.1111	−0.1083	−0.1015	−0.1057	−0.0933		
Ar	−0.0892	−0.1045	−0.0744	−0.0972	−0.0867	−0.1054	
NO^+	−0.2000	0	−0.2	0	0	−0.2	$0.44 + 0.108 \cdot 10^{-2} A_7$
a_{iJ}^{11}							
O	0.3944						
O_2	0.4076	0.3930					
N	0.4196	0.4522	0.4107				
N_2	0.4176	0.4397	0.4346	0.3856			
NO	0.4568	0.4550	0.4509	0.4534	0.3885		
Ar	0.4436	0.4527	0.4353	0.4483	0.4422	0.3780	
NO^+	0.52	0.4	0.52	0.4	0.4	0.52	$0.383 - 0.24 \cdot 10^{-3} A_7$
a_{iJ}^{22}							
O	0.7889						
O_2	0.8200	0.7859					
N	0.8047	0.7622	0.8313				
N_2	0.8070	0.7771	0.7861	0.7712			
NO	0.7530	0.7557	0.7631	0.7589	0.7671		
Ar	0.7718	0.7578	0.7843	0.7649	0.7724	0.7561	
NO^+	0.6400	0.8268	0.6400	0.8268	0.8268	0.6400	$0.766 - 0.48 \cdot 10^{-3} A_7$

Table 3. Values of coefficients a_{ij}^{mn} for electrons

i	O_2	N	NO	Ar	NO^+
$a_{ei}^{01} = a_{ei}^{10}$	−0.284	$-0.381 + 0.77 \cdot 10^{-5} \cdot T_e$	−0.284	−0.500	$0.486 + 0.32 \cdot 10^{-3} \cdot A_e$
a_{ei}^{11}	0.630	$0.603 - 0.4 \cdot 10^{-5} \cdot T_e$	0.630	0.880	$0.435 + 0.16 \cdot 10^{-3} \cdot A_e$
$a_{ei}^{02} = a_{ei}^{20}$	0.002	$-0.073 + 0.12 \cdot 10^{-5} \cdot T_e$	0.002	0.057	$a_{e7}^{01} - 0.188$
$a_{ei}^{21} = a_{ei}^{12}$	−0.279	$-0.224 + 0.36 \cdot 10^{-5} \cdot T_e$	−0.279	−0.620	$a_{e7}^{01} - 0.204$
a_{ei}^{22}	0.520	$0.472 - 0.58 \cdot 10^{-5} \cdot T_e$	0.520	0.760	$0.263 + 0.16 \cdot 10^{-3} \cdot A_e$

2. Calculation results

Figure 2 shows the diffusion fluxes of the electron component calculated with the following parameters:

1. $A = -\frac{\partial}{\partial_y}\left(\ln\frac{T_e}{T}\right) = 0.3$; *a*: $K = \left(\frac{T_e}{T}\right)_{y=0} = 1.9$; *b*: $K = 1.7$; *c*: $K = 1.5$;

2. $A = 0$; *a*: $K = 0.1$; *b*: $K = 0.2$; *c*: $K = 0.5$, i.e., at different ratios between electron temperature T_e and

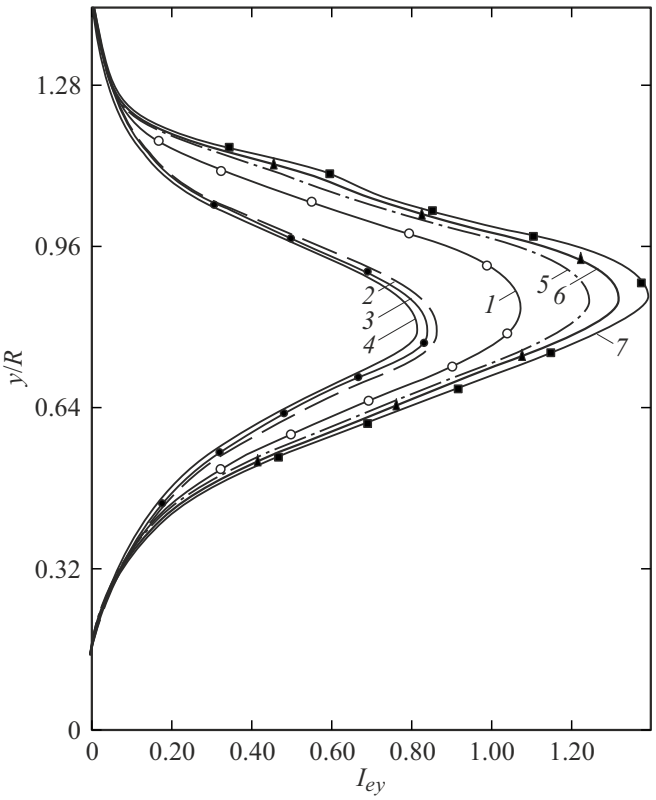

Figure 2. Profiles of diffusion electron fluxes $I_{ey} \cdot 10^{13}$, [kg/(m^2·s)]: *1* — $A = 0$, $K = 1$;*2* — $A = 0$, $K = 0.5$; *3* — $A = 0$, $K = 0.2$; *4* — $A = 0$, $K = 0.1$; *5* — $A = 0.3$, $K = 1.5$; *6* — $A = 0.3$, $K = 1.7$; *7* — $A = 0.3$, $K = 1.9$.

constant temperature T of „heavy" particles. As was already noted, $V_e = V_{\mathrm{NO+}}$ and $I_e \cdot \rho_{\mathrm{NO+}} = I_{\mathrm{NO+}} \cdot \rho_e$, since electrons e^- and NO^+ ions diffuse together. Therefore, these same plots illustrate the NO^+ ion diffusion fluxes with scaling factor $\frac{m_e}{m_{\mathrm{NO^+}}} = 1.8 \cdot 10^{-5}$.

A strong dependence of fluxes on the electron temperature is evident; the smaller T_e is, the less pronounced is the diffusion of electrons e^- and NO^+ ions to the periphery of the wake. This difference is largely attributable to phenomena that may be denoted tentatively as „two-temperature barodiffusion" and „two-temperature thermal diffusion" and are characterized by the following terms of Eqs. (1), (4):

$$S_p \sim \left(1 - \frac{T_e}{T}\right) \nabla \ln P_e;\ S_T \sim \ln \frac{T}{T_e}.$$

Under typical far wake conditions, where $\frac{\partial x_e}{\partial y} < 0$ and $\frac{\partial P_e}{\partial y} < 0$, electron diffusion flux I_{ey} to the wake boundary grows due to S_P at $T_e > T$ and, in addition, due to S_T at $\frac{\partial}{\partial y}\left(\frac{T_e}{T}\right) < 0$. The far wake is also characterized by small values of electron concentration $x_e \ll 1$ and constant pressure over the cross section $\frac{\partial P}{\partial y} = 0$. Taking this into account and neglecting the influence of the Soret and Dufour effects and auxiliary flux f_e, which plays an insignificant part here, we may rewrite Eq. (4) as

$$\frac{T_e}{T} \frac{\partial \ln x_e}{\partial y} + \frac{\partial}{\partial y}\left(\frac{T_e}{T}\right) + e \frac{E_y}{kT} = -\frac{I_{ey}}{\rho_e} \sum_{i=1}^{NH} \frac{x_i}{D_{ei}}. \qquad (9)$$

Analyzing Eq. (9), we find that I_{ey} increases due to concentration diffusion (Fick's law) by a factor of $m = \frac{T_e}{T}$ and due to „two-temperature thermal diffusion" at $\frac{\partial}{\partial y}\left(\frac{T_e}{T}\right) < 0$.

The final values of diffusion fluxes of charged particles (e^-, NO^+) and internal electric field strength E are formed as a result of complex interaction of all types of diffusion (concentration, two-temperature, Soret effect, barrodiffusion) in combination with ambipolar diffusion.

With the above conditions of the far wake taken into account, Eq. (1) for „heavy" particles may be written as

$$\frac{\partial \ln x_i}{\partial y} = -x_e(A_1 + B + C) - e\frac{E_y}{kT} = \sum_{j=1}^{NH} \frac{x_j}{D_{ij}} \left(\frac{I_{jy}}{\rho_j} - \frac{I_{iy}}{\rho_i}\right), \qquad (10)$$

where

$$A_i = \frac{1}{D_{ei}} \frac{I_{ey}}{\rho_e};\ B = \left(\frac{T_e}{T} - 1\right) \frac{\partial \ln x_e}{\partial y};\ C = \frac{\partial}{\partial y}\left(\frac{T_e}{T}\right).$$

It follows from Eq. (10) that the overall influence of terms A_1, B, and C, which mediate the effect of the difference between T_e and T on „heavy" particle fluxes I_{iy}, may lead both to reduction and enhancement of these fluxes. The above-mentioned conditions for enhancement of electron diffusion flux I_{ey} are also the conditions under which term A_1 of Eq. (10) yields component $I_{iy}^{(1)}$ that is directed toward the periphery of the wake and, consequently, increases I_{iy} if $\frac{\partial x_i}{\partial y} < 0$ (for O, N, NO, NO^+) and reduces I_{iy} if $\frac{\partial x_i}{\partial y} > 0$ (for O_2, N_2). However, these same conditions serve as conditions for the emergence of component $I_{iy}^{(2)}$, which is directed opposite to $I_{iy}^{(1)}$, due to terms B and C. It follows from (10) that the contribution of two temperatures to the fluxes of „heavy" particles is proportional to x_e and may become significant only at a sufficiently high electron concentration. In the far wake, $x_e \ll x_i$, and the difference between T_e and T exerts a negligible influence.

Figures 3 and 4 illustrate the above reasoning with I_{iy}/I_{iy}^* profiles characterizing the ratio of fluxes in projection onto axis Y at $T_e \neq T$ (versions 1b, 1c, 2b) to a similar flux determined in the case of complete thermodynamic equilibrium $T_e = T$ for atomic oxygen O (in Fig. 3) and for argon Ar (in Fig. 4), the concentration of which was set constant over the cross section and equal to $x_{\mathrm{Ar}} = 0.7 \cdot 10^{-2}$. It should be noted that $I_{\mathrm{Ary}} = 0$ if traditional transport equations are used (e.g., Wilke formulae for diffusion and viscosity coefficients and/or the Mason–Saxena formula for thermal conductivity coefficients). The obtained diffusion fluxes of argon arose exclusively as a result of ordinary and two-temperature thermal diffusion and barodiffusion. It is clear

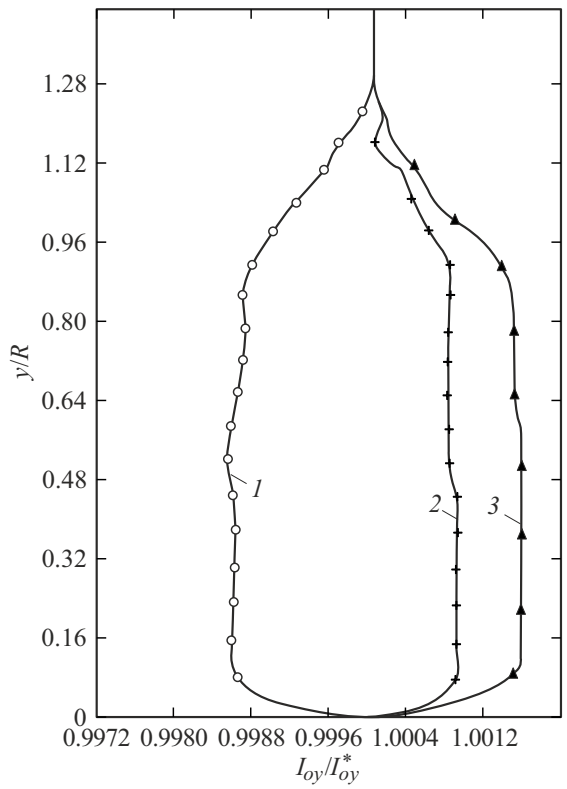

Figure 3. Profiles of diffusion fluxes of atomic oxygen I_{oy} normalized to similar flux I^*_{oy} determined at $A = 0$, $K = 1$: *1* — $A = 0$, $K = 0.2$; *2* — $A = 0.3$, $K = 1.5$; *3* — $A = 0.3$, $K = 1.7$.

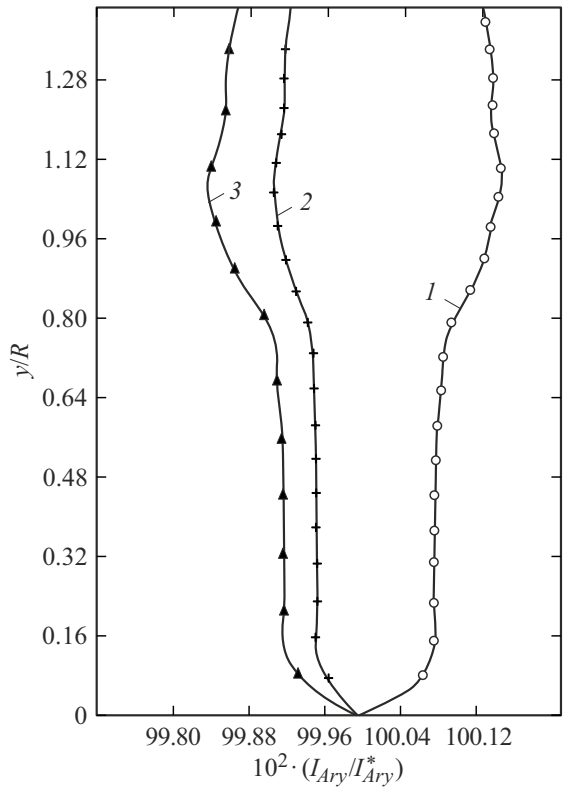

Figure 4. Profiles of diffusion fluxes of argon I_{Ary} normalized to similar flux I^*_{Ary} determined at $A = 0$, $K = 1$. See Fig. 3 for notation.

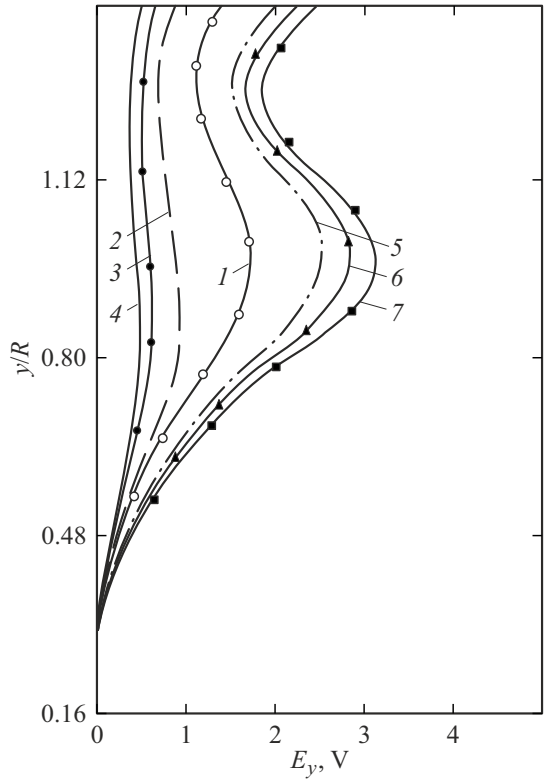

Figure 5. Profiles of internal electric field strength E_y (V). See Fig. 2 for notation.

that the influence of two temperatures is insignificant and amounts to fractions of a percent. However, such influences for O and Ar differ, which is attributable to the fact that the terms of Eq. (10) produce dissimilar contributions to fluxes I_{oy}, I_{Ary}. Since $D_{e-o} < D_{e-\mathrm{Ar}}$, $A_o > A_{\mathrm{Ar}}$. In the case of oxygen, condition $A_1 > B + C$ is satisfied and, as a consequence, flux I_{oy} increases as T_e grows relative to T. In the case of argon, the diffusion flux of which is driven exclusively by the Soret effect (the thermal diffusion flux of Ar molecules, which are the „heaviest“ of all particles, is directed against the temperature gradient toward the periphery of the wake), condition $A_1 < B + C$ is satisfied. Therefore, I_{Ary} decreases with increasing $m = \frac{T_e}{T}$.

Figure 5 shows the profiles of internal electric field strength in projection onto the Y axis determined for the parameter combinations from Fig. 2. It can be seen that the value of E_y formed under the influence of all types of diffusion of charged particles and ambipolar diffusion depends strongly on the ratio between temperatures T_e and T.

Conclusions

The obtained results provide an opportunity to evaluate the influence of thermodynamic non-equilibrium on the transport properties of partially ionized air plasma and may be used to choose a proper physical and mathematical

model of these properties for flows characterized by low Reynolds numbers. With all the considered methods for reducing the amount of necessary computation implemented, the used transport model becomes actually applicable in modeling of low-temperature plasma even when higher approximations of the Chapman–Enskog theory, chemical and thermal non-equilibrium, and normal and two-temperature diffusion are taken into account (e.g., for flows around and behind high-velocity vehicles flying in the upper layers of the atmosphere). Under such flight conditions and fairly complex trajectories and shapes of the said vehicles, the discussed transport model will allow for significant refinement of the calculated parameters.

Conflict of interest

The author declares that he has no conflict of interest.

References

[1] I.A. Sokolova, S.A. Vasil'evskii, A.V. Andriatis. Fiz.-Khim. Kinet. Gazov. Din., **3**, 1 (2005) (in Russian). http://chemphys.edu.ru/issues/2005-3/articles/80/
[2] I.A. Sokolova, S.A. Vasil'evskii, A.V. Andriatis. Fiz.-Khim. Kinet. Gazov. Din., **3**, 1 (2005) (in Russian). http://chemphys.edu.ru/issues/2005-3/articles/81/
[3] A.F. Kolesnikov, G.A. Tirskii. *Uravneniya gidrodinamiki dlya chastichno ionizovannykh mnogokomponentnykh smesei gazov s koeffitsientami perenosa v vysshikh priblizheniyakh*. In: *Molekulyarnaya gazodinamika* (M., 1982), pp. 24–44 (in Russian).
[4] A.F. Kolesnikov. *Uravneniya dvizheniya mnogokomponentnoi chastichno ionizovannoi dvukhtemperaturnoi smesi gazov s koeffitsientami perenosa v vysshikh priblizheniyakh* (Inst. Mekh. Mosk. Gos. Univ. M., 1974) (in Russian)
[5] A.F. Kolesnikov. *Uravneniya perenosa dlya vysokotemperaturnykh ionizirovannykh smesei gazov v elektromagnitnykh polyakh*. In: *Nauchnye trudy Instituta mekhaniki MGU* (M., 1975) (in Russian)
[6] A.F. Kolesnikov. Fluid Dyn., **50**, 153 (2015).
[7] A.F. Kolesnikov. Fluid Dyn., **53**, 315 (2018). DOI: 10.7868/S0568528118020147/
[8] V. Rat, J. André, J. Obregon, M.F. Elchinger, J. Fauchais, D. Vacher. J. Phys. D: Appl. Phys., **35** (10), 981 (2002).
[9] V. Colombo, E. Ghedini, P. Sanibondi. J. Phys. D. Appl. Phys., **42** (24), 055213 (2009). DOI: 10.1088/0022-3727/42/5/055213
[10] I.A. Sokolova. In: *Fizicheskaya kinetika. Aerofizicheskie issledovaniya* (Inst. Teor. Prikl. Mekh. Sib. Otd. Akad. Nauk SSSR, Novosibirsk, 1974), p. 39 (in Russian)
[11] J.H. Ferziger, H.G. Kaper. *Mathematical Theory of Transport Processes in Gases* (North-Holland, Amsterdam, 1972)
[12] J.E. Jordan, S.O. Colagate, J. Amdur, E.A. Mason. Chem. Phys, **52** (3), 1143 (1970). DOI: 10.1063/1.1673109
[13] R.S. Devoto. Phys. Fluid, **10** (2), 2105 (1967). DOI: 10.1063/1.1762005
[14] R.S. Devoto. Phys. Fluid, **19** (1), 22 (1976).
[15] M.M. Stepanov. Fluid Dyn., **37**, 138 (2002).
[16] M.M. Stepanov, A.V. Moldavanov, V.I. Sementsov. *Issledovanie termokhimicheski neravnovesnogo techeniya okolo tel, letyashchikh v atmosfere*. In: *Materialy mezhdunarodnoi nauchno-tekhnicheskoi konferentsii „Chetvertye Utkinskie chteniya"* (Balt. Gos. Tekh. Univ., SPb., 2009), pp. 209–215 (in Russian).
[17] Yu.P. Savel'ev, M.M. Stepanov. Zh. Tekh. Fiz., **57** (11), 2178 (1987) (in Russian).

Translated by D.Safin