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Low-frequency transition radiation of particle bunch on grid screen

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We analyze electromagnetic radiation of a charged particle bunch passing through a planar grid structure composed of thin conductors. The conductors cross each other, have galvanic contact in the cross points and form square cells whose size is small compared to the lengths of the electromagnetic waves under consideration. It is also assumed that the size of the cell is significantly greater than the transverse size of the conductor. In the considered approximation, the grid structure is described by the averaged boundary conditions of M.I. Kontorovich. With the help of these conditions we obtain a general analytical solution which is then investigated asymptotically by methods of the complex variable function theory. It is shown that the radiation consists only of volume waves. The Fourier-transforms of the electromagnetic field components are obtained, the dependences of the energy characteristics on the bunch properties and geometrical parameters of the structure are analyzed, and a comparison with the case of a solid perfectly conductive plane is given.

Keywords: Radiation of charged particles, transition radiation, grid structure, average boundary conditions, charged particle bunch.

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Introduction

Transition radiation of charged particles has been discovered in the middle of the 20th century [1–3] and is still being actively studied today. It has found wide application in accelerator physics and particle beam research. Depending on the nature of inhomogeneity, one may obtain widely different radiation properties for various uses, such as particle detection [4–6] or particle bunch diagnostics [7–9].

One research direction is associated with the analysis of radiation from planar periodic conducting structures within the range of wavelengths that exceed significantly the period of the structure. In this case, the structure may be substituted with a certain surface characterized by a particular „averaged" boundary condition. Corrugated conductive structures are of note in this context. In the low-frequency part of the spectrum, their effect is characterized using the Weinstein–Sivov equivalent boundary conditions [10]. Transition radiation with such structures was examined in [11,12].

Planar systems of thin parallel conductors represent another common type of periodic structures. Transition radiation produced by a charge crossing the plane of such a structure was analyzed in [13–17]. Various approximations ranging from the simplest one with ideal conductivity along the wires [13,14] to the use of averaged Kontorovich boundary conditions [18,19], which provide an opportunity to factor in the geometric and electrical properties of conductors [15–17], were used in these studies for characterizing the structure.

It should be noted that the studied structures were markedly anisotropic. Therefore, transition radiation at wavelengths exceeding significantly the structure period included both the volume component and surface waves. This may be fairly useful in certain contexts. For example, the nature of surface waves may help estimate the size and shape of a particle bunch that generated them, which is of interest for bunch diagnostics problems [11,12,16,17].

At the same time, if there is no need to excite surface waves, they will be a parasitic effect. An isotropic structure close in its properties to a solid highly conductive surface may be used in this case. Naturally, this may be a thin layer of metal (foil) with a small hole for the particle bunch to pass through, but precise bunch positioning in the plane of the screen is required in this scenario. Note that radiation of a charge propagating through a round hole in an infinitely thin and perfectly conducting screen was analyzed in [20,21].

A grid structure of conductors may serve as an alternative to a solid conductive screen. The fundamental mechanism behind the emergence of radiation then remains unchanged: the particle bunch field affects electrons in conductors, exciting time-dependent currents, and their field contains transition radiation as a wave component. Grid cells should be smaller than the wavelengths under study if the influence of the grid on the field is to be made close to that a perfectly conducting plane. At the same time, for the bunch to pass relatively freely through the grid, the thickness of wires must remain small compared to the size of cells. In this scenario, it is sufficient to find the field averaged over the structure periods instead of solving rigorously the electrodynamic problem with a complex system of conductors (this cannot be performed analytically). The structure is then substituted

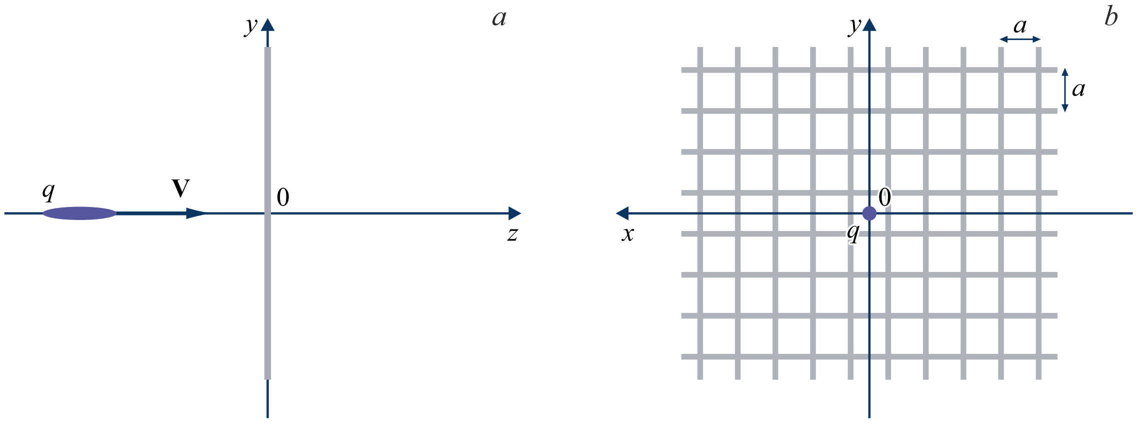

Figure 1. Beam of charged particles with total charge q moving with velocity $\mathbf{V}$ through a structure of thin conductors with square cells. a — View in the (y, z) plane; b — view in the (x, y) plane.

with a solid surface on which averaged boundary conditions (ABCs) are set [19]. As is known [19], if the distance from the grid to the observation point is greater than the cell size, the field is close to its average value, which is obtained using the ABC method.

In the present study, we consider a grid with square cells and galvanic contacts between conductors at the intersections. The advantage of such a structure is that its ABCs are isotropic [19] (e.g., the diagonal direction is „on average“ electrodynamically indistinguishable from the direction along the conductors). Since the conductor thickness is small compared to the size of cells, the particle bunch passes through the grid relatively easily, and the issue of bunch positioning in the plane of the screen is inessential. At the same time, it will be demonstrated below that the radiation characteristics may be very close to those of a solid screen.

It must be emphasized that only relatively long-wave radiation (i.e., radiation with wavelengths exceeding significantly the period of the structure) is considered here. Naturally, radiation at wavelengths comparable to or shorter than the period may also be generated. This radiation is beyond the scope of the present study; its examination requires a different approach. It should also be noted that virtually no radiation is excited at wavelengths smaller than the bunch size. Therefore, if a bunch is larger than the grid period, the only radiation present is the one that is studied here. If the bunch is not as long, radiation at shorter wavelengths will be added, but it is left unaddressed below. Naturally, all the results obtained for the long-wave part of the radiation field remain valid regardless of the bunch size.

1. ABCs

The problem of radiation of a bunch of charged particles propagating through an infinite planar structure of thin cylindrical conductors is considered. Figure 1 illustrates the geometry of the problem. The structure is positioned in the $z = 0$ plane in vacuum and has square cells (with side length a) and perfect contacts at the intersections. It is assumed that the cell size is significantly larger than wire diameter d_0:

$$a \gg d_0. \tag{1}$$

Another assumption is that the cell size is small compared to the considered electromagnetic wavelengths λ:

$$\lambda \gg a. \tag{2}$$

Thus, relatively long-wave radiation of a charge is analyzed.

If conditions (1) and (2) are met, the grid structure may be substituted with another surface on which the following ABCs for the Fourier transform of tangential component $\mathbf{E}_{\tau\,\omega}$ of the electric field are satisfied [19]:

$$\mathbf{E}_{\tau\,\omega} = -\frac{2ik_0a}{c}\ln\left(\frac{a}{\pi d_0}\right) \times \left\{\left[1 + \frac{\mu_i f\,(p)}{4\ln\left(a/\pi d_0\right)}\right]\mathbf{I}_\omega + \frac{1}{2k_0^2}\nabla_\tau\,\mathrm{div}\,\mathbf{I}_\omega\right\}. \tag{3}$$

Here, $\nabla_\tau = \partial/\partial x + \partial/\partial y$, i is the imaginary unit, $\mathbf{I}_\omega$ is the Fourier transform of surface current density, $k_0 = \omega/c = 2\pi/\lambda$, ω is the angular frequency, c is the speed of light in vacuum, μ_i is the permeability of wires, and function $f\,(p)$ takes a simple form in two limit cases:

$$f\,(p) = \begin{cases} -i + 1/p^2, & p \ll 1 \\ (1-i)/p, & p \gg 1 \end{cases}, \tag{4}$$

where $p = d_0/4d_s$ and d_s is the skin depth. Note that the skin depth of metals does not exceed several micrometers at frequencies from a gigahertz to a terahertz; therefore, condition $p \gg 1$ is almost always satisfied. For example, the skin depth for copper at 1 GHz is $d_s = 2.1\,\mu$m, and this value decreases with increasing frequency. Thus, the difference between the real structure and the perfectly conducting one at the considered frequencies is insignificant even for very thin conductors. The latter is true not only for copper, but also for several other metals.

Boundary conditions (3) relate electric field $\mathbf{E}_{\tau\,\omega}$ averaged over a grid cell to averaged surface current density $\mathbf{I}_\omega$. Subscripts ω and τ denote a Fourier harmonic and a vector component tangential to the boundary, respectively. The current is related to the magnetic field strength in the following way:

$$\mathbf{I}_\omega = \frac{c}{4\pi}\left[\mathbf{e}_z, \{\mathbf{H}_\omega\}\right], \tag{5}$$

where square brackets denote a vector product, $\mathbf{e}_z$ is a unit vector along z, and

$$\{\mathbf{H}_\omega\} = \mathbf{H}_\omega\Big|_{z=+0} - \mathbf{H}_\omega\Big|_{z=-0}$$

is the jump of the Fourier transform of the magnetic field on the grid.

Note that the cross-sectional shape of conductors is nonessential. If wires are not circular cylinders, diameter d_0 is replaced by a certain effective diameter. In the case of thin strips and wires with a square cross section, it is equal to half the width of a strip and the side length of a square multiplied by 1.18, respectively [19].

Boundary conditions (3) may be written for Fourier transforms in frequency and tangential wave vector component. Using relation (5), we obtain two boundary conditions in Cartesian coordinates:

$$E_{x\,\omega,k_x,k_y} = ik_0 aA\left[\left(2B - \frac{k_x^2}{k_0^2}\right)\{H_{y\,\omega,k_x,k_y}\} + \frac{k_x k_y}{k_0^2}\{H_{x\,\omega,k_x,k_y}\}\right], \tag{6}$$

$$E_{y\,\omega,k_x,k_y} = -\,ik_0 aA\left[\left(2B - \frac{k_y^2}{k_0^2}\right)\{H_{x\,\omega,k_x,k_y}\} + \frac{k_x k_y}{k_0^2}\{H_{y\,\omega,k_x,k_y}\}\right], \tag{7}$$

where

$$A = \frac{1}{4\pi}\ln\left(\frac{a}{\pi d_0}\right), \quad B = 1 + \frac{\mu_i f(p)}{4\ln(a/\pi d_0)}. \tag{8}$$

2. General solution of the problem

We assume that a bunch of particles with total charge q moves with constant velocity $\mathbf{V} = V\mathbf{e}_z$ and has infinitely small transverse dimensions and an arbitrary charge distribution along the trajectory of motion. In this case, the volume charge and current density may be written as

$$\rho = q\delta(x)\,\delta(y)\,\varkappa_\parallel(\xi), \quad j_z \equiv j = \rho V,$$

where $\delta(\,)$ is the Dirac delta function, $\xi = z - Vt$, and function $\varkappa_\parallel(\xi)$ characterizes the charge distribution along the bunch length. Since the bunch charge is q, $\int_{-\infty}^{+\infty}\varkappa_\parallel(\xi)\,d\xi = 1$.

It should be emphasized that the model of an infinitely thin bunch considered here is justified by the fact that

We use the Hertz vector to solve the problem. The total bunch field is the sum of an „forced“ field of a charge moving in infinite vacuum and a „free“ field, which arises due to the influence of the periodic structure:

$$\mathbf{\Pi}_\omega = \mathbf{\Pi}_\omega^{(i)} + \mathbf{\Pi}_\omega^{(r)}.$$

The „forced“ bunch field is well known [2] and is characterized by a one-component Hertz vector, the Fourier transform of which (in frequency and two components of the wave vector) takes the form

$$\Pi^{(i)}_{z\,\omega,k_x,k_y} = \frac{iq\tilde{\varkappa}_\parallel}{\pi c k_0}\,\frac{\exp(ik_0 z/\beta)}{k_x^2 + k_y^2 + k_0^2(1-\beta^2)/\beta}.$$

Here, $\beta = V/c$ and $\tilde{\varkappa}_\parallel$ is the Fourier transform of the bunch profile defined as

$$\tilde{\varkappa}_\parallel = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\varkappa_\parallel(\xi)\exp(-ik_0\xi/\beta)\,d\xi. \tag{9}$$

The Fourier transform of the Hertz vector of the „free“ field may be written as

$$\Pi^{(r)}_{z\,\omega,k_x,k_y} = -\operatorname{sgn}(z)\,\frac{iq\tilde{\varkappa}_\parallel}{\pi c k_0^2}\,\frac{\exp(ik_{z0}|z|)}{k_{z0}}\,T,$$

$$\Pi^{(r)}_{x\,\omega,k_x,k_y} = \Pi^{(r)}_{y\,\omega,k_x,k_y} = 0, \tag{10}$$

where T is an arbitrary coefficient, which may be determined using the boundary conditions, and $k_{z0} = \sqrt{k_0^2 - k_x^2 - k_y^2}$ is the projection of the wave vector onto axis Oz. Note that if the expression under the radical sign is positive, then k_{z0} is a purely real and positive quantity. If the expression under the radical sign is negative, k_{z0} is a purely imaginary quantity with a positive imaginary part. This ensures propagation of an electromagnetic wave in the direction away from the boundary when $k_0^2 > k_x^2 + k_y^2$ and an exponential field decay with increasing $|z|$ when $k_0^2 < k_x^2 + k_y^2$.

The Fourier transforms of electric and magnetic fields are related to the Fourier transform of the Hertz vector in the following manner:

$$\mathbf{E}_\omega = \nabla\operatorname{div}\mathbf{\Pi}_\omega + k_0^2\mathbf{\Pi}_\omega, \quad \mathbf{H}_\omega = -ik_0\operatorname{rot}\mathbf{\Pi}_\omega. \tag{11}$$

Expression (11) makes it easy to obtain the Fourier transform of the total electromagnetic field of the bunch. Inserting this field into boundary condition (6) or (7), one can find an expression for the unknown coefficient in (10):

$$T = \frac{k_0^2\beta k_{z0}}{(k_0^2 - \beta^2 k_{z0}^2)\{k_{z0} - 2iaA[(2B-1)k_0^2 + k_{z0}^2]\}}. \tag{12}$$

3. Field in the far-field region

Let us examine the „free“ field. According to (10) and (12), the Fourier transform of the Hertz vector of this field is given by

$$\Pi^{(r)}_{z\,\omega} = -\,\mathrm{sgn}\,(z)\,\frac{iq\tilde{\varkappa}_{\|}\beta}{\pi c}\int_{-\infty}^{+\infty}\int dk_x dk_y$$
$$\times\frac{\exp\left(ik_x x + ik_y y + ik_{z0}|z|\right)}{(k_0^2-\beta^2k_{z0}^2)\{k_{z0}-2iaA[(2B-1)k_0^2+k_{z0}^2]\}}.$$

Given the symmetry of the problem, it makes sense to switch to cylindrical coordinates, both in the usual space (r, φ, z) and in the space of wave vectors (k_r, φ_k, k_z). The integral over φ_k is easy to solve in these new coordinates (it is tabulated, see [25]), while the integral over k_r is reduced to the integral over the entire axis. We thus obtain

$$\Pi^{(r)}_{z\,\omega} = -\,\mathrm{sgn}(z)\,\frac{iq\tilde{\varkappa}_{\|}\beta}{ck_0}$$
$$\times\int_{\infty e^{i\pi}}^{\infty}\frac{\tilde{k}_r H_0^{(1)}\left(k_0\tilde{k}_r r\right)\exp\left(ik_0\tilde{k}_{z0}|z|\right)d\tilde{k}_r}{\left(1-\beta^2\tilde{k}_{z0}^2\right)\left[\tilde{k}_{z0}-2ik_0aA\left(2B-1+\tilde{k}_{z0}^2\right)\right]}, \tag{13}$$

where $\tilde{k}_r = k_r/k_0$, $\tilde{k}_{z0} = k_{z0}/k_0 = \sqrt{1-\tilde{k}_r^2}$, and $H_0^{(1)}()$ is the Hankel function.

The field in the far-field region (i.e., at $k_0R \gg 1$, where $R=\sqrt{r^2+z^2}$) may be determined by the saddle-point method. Calculations reveal that integral (13) is determined in this case by the contribution of the saddle point, while the contributions of other singularities are insignificant in the relatively long-wave approximation under consideration. In other words, the radiation field within the given frequency range is determined solely by the contribution of the saddle point.

Let us take into account the Hankel function asymptotics:

$$H_0^{(1)}\left(k_0\tilde{k}_r r\right) \simeq \sqrt{\frac{2}{\pi k_0\tilde{k}_r r}}\exp\left(ik_0\tilde{k}_r r - i\frac{\pi}{4}\right).$$

Owing to the parity of (13) in z, it is sufficient to examine the $z>0$ region only. The saddle-point contribution in spherical coordinates (R, θ, φ) may be written as [26]

$$\begin{Bmatrix}\Pi^{(r)}_{R\,\omega}\\ \Pi^{(r)}_{\theta\,\omega}\end{Bmatrix} = \begin{Bmatrix}-\cos\theta\\ \sin\theta\end{Bmatrix}\frac{2q\tilde{\varkappa}_{\|}\beta}{ck_0^2}$$
$$\times\frac{\cos\theta}{(1-\beta^2\cos^2\theta)\left[\cos\theta-2ik_0aA(2B-\sin^2\theta)\right]}$$
$$\times\frac{\exp(ik_0R)}{R}. \tag{14}$$

Using (14) and formulae (11), we find the Fourier transforms of components of the electromagnetic field:

$$E^{(r)}_{R\,\omega} = H^{(r)}_{R\,\omega} = E^{(r)}_{\varphi\,\omega} = H^{(r)}_{\theta\,\omega} = 0,$$
$$E^{(r)}_{\theta\,\omega} = H^{(r)}_{\varphi\,\omega} = \frac{2q\tilde{\varkappa}_{\|}\beta}{c}$$
$$\times\frac{\cos\theta\sin\theta}{(1-\beta^2\cos^2\theta)[\cos\theta-2ik_0aA(2B-\sin^2\theta)]}$$
$$\times\frac{\exp(ik_0R)}{R}. \tag{15}$$

Expressions (15) characterize the Fourier transform of the electromagnetic field of a volume wave generated by a bunch of charged particles propagating through a fine-cellular structure orthogonally to its plane. These formulae were derived for $z>0$ and positive frequencies $(k_0=\omega/c>0)$. The corresponding expressions for negative frequencies may be obtained from (15) according to the $F_{-\omega}=F^*_{\omega}$ rule, which follows from real-valuedness of the original field components.

Let us now examine certain energy characteristics. As was noted above, real structures almost always have $p \gg 1$; therefore, we assume that $B=1$ (see (4) and (8)). Let us analyze the total radiation energy of a moving charge. It may be determined as the time integral of the energy flux through hemisphere Σ_+ in the $z>0$ region multiplied by 2 (owing to the symmetry of fields in regions $z>0$ and $z<0$):

$$W = 2\int_{-\infty}^{+\infty}dt\int_{\Sigma_+}\mathbf{S}d\mathbf{\Sigma}, \tag{16}$$

where $\mathbf{S}=c(4\pi)^{-1}[\mathbf{E}^{(r)}, \mathbf{H}^{(r)}]$ is the Umov–Poynting vector and $d\mathbf{\Sigma}$ is an element of the hemisphere surface with the normal directed outward.

According to (15), the Umov–Poynting vector has a single component along the radius vector

$$S_R = \frac{c}{4\pi}E^{(r)}_{\theta}H^{(r)}_{\varphi} = \frac{c}{4\pi}\left(E^{(r)}_{\theta}\right)^2. \tag{17}$$

Inserting (17) into (16), we obtain

$$W = \frac{cR_0^2}{2\pi}\int_{-\infty}^{+\infty}dt\int_0^{2\pi}d\varphi\int_0^{\pi/2}\sin\theta\left(E^{(r)}_{\theta}\right)^2d\theta. \tag{18}$$

The time integral may be reduced to a frequency one [27]:

$$\int_{-\infty}^{+\infty}\left(E^{(r)}_{\theta}\right)^2dt = 4\pi\int_0^{+\infty}\left|E^{(r)}_{\theta\,\omega}\right|^2d\omega. \tag{19}$$

With (19) and the cylindrical symmetry of the problem taken into account, expression (18) for the total energy takes the form

$$W = 4\pi\int_0^{+\infty}d\omega\int_0^{\pi/2}\sin\theta\,\frac{d^2W}{d\omega d\theta}\,d\theta, \tag{20}$$

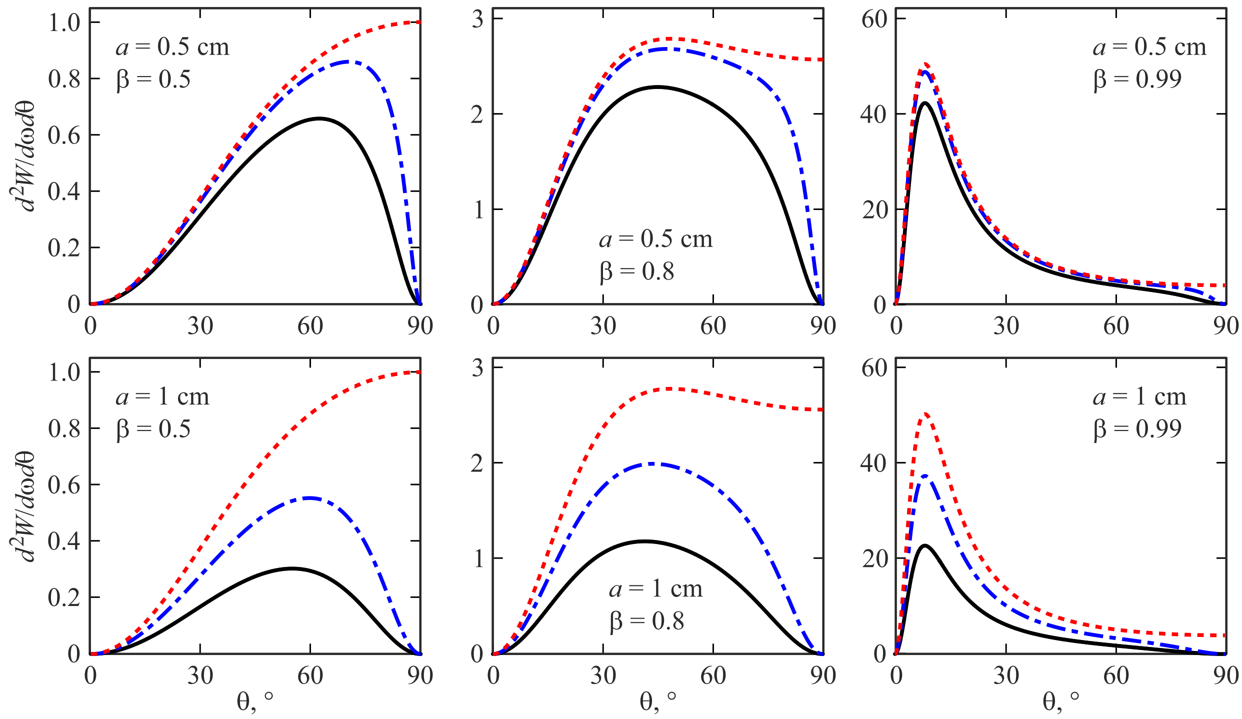

Figure 2. Dependence of the spectral-angular energy density of volume radiation $d^2W/d\omega d\theta$ on angle θ in the case of a fine-cellular structure (black solid and blue dash-dotted lines) and an ideal screen (red dotted lines). Beam velocity $\beta = 0.5$ (left column), $\beta = 0.8$ (middle column), and $\beta = 0.99$ (right column). Structure period $a = 0.5$ cm (top) and $a = 1$ cm (bottom). Wire diameter $d_0 = 0.1$ mm (black solid lines) and $d_0 = 0.5$ mm (blue dash-dotted lines). Wavenumber $k_0 = 1\,\text{cm}^{-1}$ (wavelength $\lambda \simeq 6.28$ cm). The spectral-angular energy density is normalized to $q^2 \left|\tilde{\varkappa}_{\|}\right|^2/c$.

where

$$\frac{d^2W}{d\omega d\theta} = cR_0^2 \left|E_{\theta\omega}^{(r)}\right|^2. \tag{21}$$

Inserting (15) into (21), we find

$$\frac{d^2W}{d\omega d\theta} = \frac{4q^2|\tilde{\varkappa}_{\|}|^2\beta^2}{c} \times \frac{\cos^2\theta \sin^2\theta}{(1-\beta^2\cos^2\theta)^2 \left[\cos^2\theta + 4k_0^2a^2A^2(2-\sin^2\theta)^2\right]}. \tag{22}$$

Thus, we obtained the spectral-angular energy density of radiation generated by a bunch of charged particles propagating through a fine-cellular structure of thin ideal conductors. The bunch has an arbitrary charge distribution along the trajectory of motion. It bears reminding that the longitudinal profile of the bunch is function $\varkappa_{\|}(\xi)$, where $\xi = z - c\beta t$, and its Fourier transform $\tilde{\varkappa}_{\|}$ is given by expression (9).

It should be emphasized that the obtained formulae are valid when conditions (1) and (2) are satisfied. Letting small parameter a/λ of the problem to zero, we obtain a formula for a solid and perfectly conducting plane:

$$\frac{d^2W}{d\omega d\theta} = \frac{4q^2|\tilde{\varkappa}_{\|}|^2\beta^2}{c}\,\frac{\sin^2\theta}{(1-\beta^2\cos^2\theta)^2}. \tag{23}$$

If the charge is a point one, $\tilde{\varkappa}_{\|}^{point} = (2\pi)^{-1}$, and (23) transforms into the well known expression for the spectral-angular energy density of volume radiation generated by a point charge moving through an ideal screen [2].

4. Calculation results and discussion

Figure 2 shows the dependences of the spectral-angular density of radiation energy (in units of $q^2\left|\tilde{\varkappa}_{\|}\right|^2/c$) on angle θ obtained at different values of bunch velocity β, structure period a, and wire diameter d_0 for a fine-cellular grid (black solid and blue dash-dotted lines, formula (22)) and an ideal screen (red dotted lines, formula (23)). It follows from these dependences that radiation maximum shifts toward lower values of θ as the bunch velocity increases. The maximum magnitude of spectral-angular energy density also increases, and the rate of its rise is proportional to the bunch velocity. In the case of an ultrarelativistic bunch, radiation is maximized in the direction close to the line of motion of the charge. These patterns also hold for a solid conducting plane.

Note that the most significant difference between the examined radiation and radiation in the case of an ideal screen is found in the direction close to the structure plane ($\theta \simeq 90^\circ$). The spectral-angular density is very low in this scenario and tends to zero at $\theta \to 90°$. Significant differences may also arise at other angles θ: the difference intensifies with increasing grid period a and decreasing wire diameter d_0. It follows from Fig. 2 that the maximum value of spectral-angular energy density decreases approximately by a factor of 2 if one increases the period from 0.5 cm (top row) to 1 cm (bottom row) with the wire diameter kept constant at 0.1 mm (black solid lines). If the transverse size of the wire is larger (e.g., $d_0 = 0.5$ mm, which corresponds to blue dash-dotted lines), the spectral-angular density decreases more slowly with an increasing period. It should be emphasized that the radiation distribution is affected more profoundly by the structure period variation than by the change in wire diameter. This follows directly from formula (22) and is confirmed by the results of numerical calculations presented in Fig. 2.

Let us now consider the total spectral density of transition radiation. To do this, we rewrite (20) as

$$W = \int_0^{+\infty} \frac{dW}{d\omega} d\omega.$$

Quantity

$$\frac{dW}{d\omega} = \frac{16\pi q^2 |\tilde{\varkappa}_{\|}|^2 \beta^2}{c} \times \int_0^{\pi/2} \frac{\cos^2\theta \sin^3\theta d\theta}{(1-\beta^2\cos^2\theta)^2 \left[\cos^2\theta + 4k_0^2 a^2 A^2 (2-\sin^2\theta)^2\right]}$$

is the spectral energy density of volume radiation. Note that one needs to specify bunch profile $\varkappa_{\|}$ and, consequently, its Fourier transform $\tilde{\varkappa}_{\|}$, which is determined by expression (9) and is generally dependent on frequency, before analyzing the spectrum. We consider bunches with Gaussian and rectangular charge distributions:

$$\varkappa_{\|}^{gaus}(\xi) = \frac{\exp(-\xi^2/2\sigma^2)}{\sqrt{2\pi}\sigma},$$

$$\varkappa_{\|}^{rect}(\xi) = \frac{\Theta(\sigma - |\xi|)}{2\sigma},$$

where $\Theta()$ is the Heaviside step function and σ is one half of the bunch length. According to (9), we then obtain

$$\tilde{\varkappa}_{\|}^{gaus} = \frac{\exp(-k_0^2\sigma^2/2\beta^2)}{2\pi}, \quad \tilde{\varkappa}_{\|}^{rect} = \frac{1}{2\pi}\frac{\sin(k_0\sigma/\beta)}{k_0\sigma/\beta}. \quad (24)$$

Figure 3 presents the dependences of $dW/d\omega$, $[q^2/c]$, on frequency $f = ck_0/(2\pi)$ for Gaussian (top) and rectangular (bottom) bunches at different bunch velocities β and wire diameters d_0. As before, black solid and blue dash-dotted lines correspond to a fine-cellular grid, while red dotted lines correspond to an ideal screen. It is evident that the width of the spectrum depends both on the bunch properties and on the geometric parameters of the surface. Specifically, the radiation spectrum for a rectangular bunch is somewhat wider than the one for a Gaussian bunch. Note that the dependences in Fig. 3 for the grid structure were obtained with bunch length 2σ being equal to period a; in this scenario, the radiation spectrum may only contain wavelengths that are significantly larger than the period (i.e., those belonging to the wave range considered here). It should be emphasized that the high-frequency part of the spectrum is suppressed not only when the bunch length increases, but also (see Fig. 3) when the bunch velocity decreases. The spectrum width also depends on the geometric parameters of the structure. Specifically, the spectrum becomes narrower with an increase in grid period and with a reduction in wire diameter (black solid and blue dash-dotted lines in Fig. 3 correspond to $d_0 = 0.1$ and 0.5 mm, respectively).

Note that the obtained results are also approximately true for bunches of a finite thickness (if it is significantly smaller than the bunch length). The finite thickness of a bunch introduces a new factor (transverse form factor $\tilde{\varkappa}_{\perp}$, which is equal to unity for an infinitely thin bunch) into the expressions for the Fourier transforms of field components. If the bunch is, e.g., a uniform cylinder with radius r_0 and its axis parallel to the velocity, $\tilde{\varkappa}_{\perp}^{cyl} = 2J_1(k_r r_0)/k_r r_0$, where k_r is the transverse wavenumber (see [2]). In the present case, $k_r = ik_0/\beta\gamma$, and we get

$$\tilde{\varkappa}_{\perp}^{cyl} = -i\frac{2\beta\gamma}{k_0 r_0} I_1\left(\frac{k_0 r_0}{\beta\gamma}\right), \quad (25)$$

where $\gamma = (1-\beta^2)^{-1/2}$ is the Lorentz factor and $I_1()$ is the modified Bessel function. Since the bunch thickness is much smaller than the bunch length (and certainly smaller than the wavelength), $k_0 r_0 \ll 1$. Then, $I_1(k_0 r_0/\beta\gamma) \simeq k_0 r_0/2\beta\gamma$, and the modulus of the transverse form factor is close to unity. Specifically, if a bunch with thickness $2r_0 = 200\,\mu$m [22–24] propagates with velocity $\beta = 0.9$, $\left|\tilde{\varkappa}_{\perp}^{cyl}\right| \simeq 1.00001$ at frequency $f = 10$ GHz ($\lambda = 3$ cm). The value of $\left|\tilde{\varkappa}_{\perp}^{cyl}\right|$ decreases and approaches unity as the bunch velocity increases.

Note that the influence of the bunch size on transition radiation with a solid screen was investigated in [28]. According to the results reported there, if the wavelength is one and a half times greater than the radius of a spherical bunch, radiation differs little from that of a point charge, and the difference becomes even less noticeable as the wavelength/bunch radius ratio increases. This example confirms that the bunch size is of minor importance at relatively long wavelengths. Our calculations for a grid screen verify this conclusion.

In the case of a bunch in the form of a uniform cylinder with radius r_0, we obtain the following expression for the spectral-angular density of radiation energy with account for

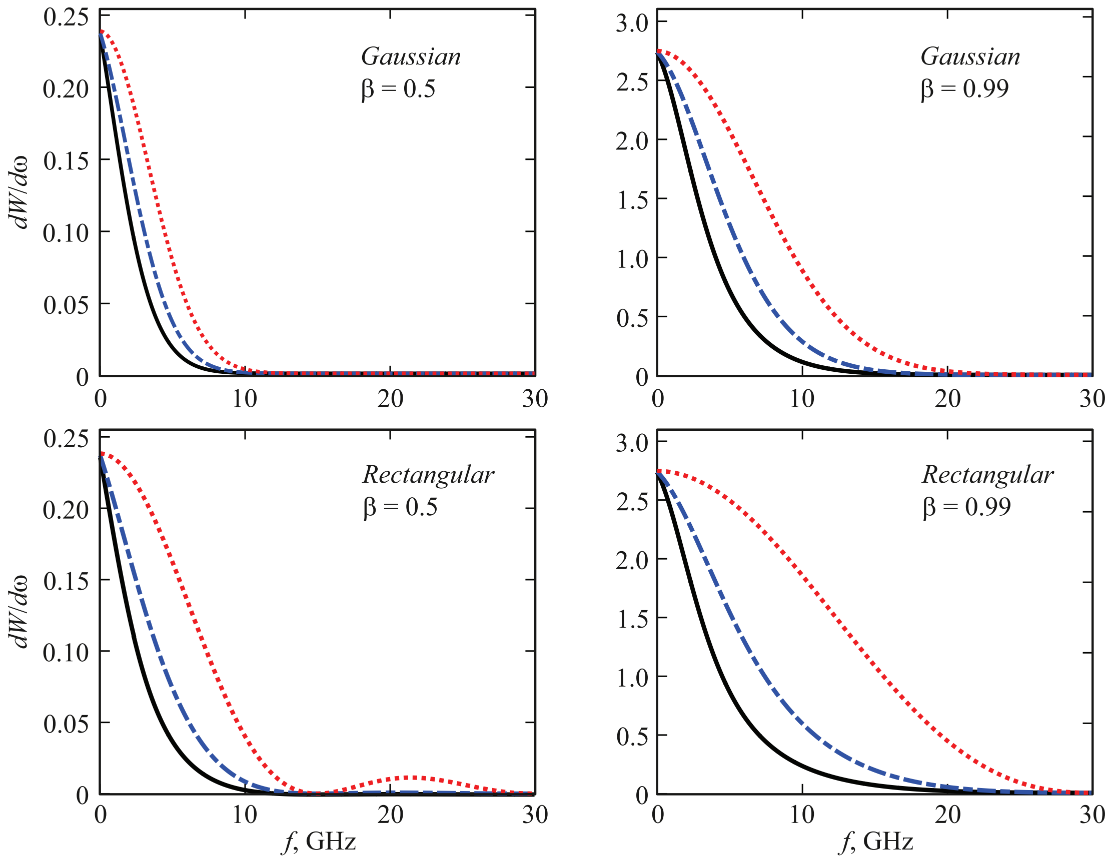

Figure 3. Dependence of the spectral energy density of volume radiation $dW/d\omega$ on frequency f in the case of a fine-cellular structure (black solid and blue dash-dotted lines) and an ideal screen (red dotted lines). Beam profile: Gaussian (top); rectangular (bottom). Length of the bunch $2\sigma = 1$ cm; its velocity $\beta = 0.5$ (left) and $\beta = 0.99$ (right). Structure period $a = 1$ cm; wire diameter $d_0 = 0.1$ mm (black solid lines) and $d_0 = 0.5$ mm (blue dash-dotted lines). The energy density is normalized to q^2/c, and the frequency is indicated in GHz.

transverse (25) and longitudinal (24) form factors:

$$\frac{d^2W}{d\omega d\theta} = \frac{4q^2\beta^6\gamma^2 I_1^2(k_0r_0/\beta\gamma)}{c\pi^2 k_0^4\sigma^2 r_0^2} \times \frac{\cos^2\theta \sin^2\theta \sin^2(k_0\sigma/\beta)}{(1-\beta^2\cos^2\theta)^2\left[\cos^2\theta + 4k_0^2a^2A^2(2-\sin^2\theta)^2\right]}.$$

Let us use this formula to analyze the total radiation energy that is determined by expression (20). Figure 4 presents the results of a numerical calculation of the total energy as a function of bunch velocity in the case of a fine-cellular grid and an ideal screen. It is evident that the energy of transition radiation increases with conductor thickness and bunch velocity (notably, the rate of energy rise also increases with bunch velocity). A bunch 10 cm in length and 200 μm in thickness with a charge of 50 nC (approximately $3 \cdot 10^{11}$ particles) generates radiation with an energy on the order of mJ (Fig. 4, *a*). The energy of radiation excited by a shorter bunch with the same thickness and a charge of 1 nC (approximately $6 \cdot 10^9$ particles) is on the order of μJ (Fig. 4, *b*). The bunch size and charge were chosen so as to be consistent with the values used in actual experiments. Specifically, bunches with similar characteristics are used as drivers in wake acceleration of particle bunches [22–24].

Note that the indicated energy is concentrated in a pulse with its duration comparable to the one of the particle bunch (i.e., fractions of a nanosecond). Therefore, the power of a pulse generated by bunches with $q = 50$ nC, $2\sigma = 10$ cm (Fig. 4, *a*) and $q = 1$ nC, $2\sigma = 5$ cm (Fig. 4, *b*) is on the order of MW and kW, respectively. Such pulses (and even shorter, picosecond ones [29–32]) are easy to record in an experiment using well known electromagnetic radiation detectors, such as bolometers [29] or pyroelectric sensors [30]. Note that a broadband antenna with an approximate sensitivity of 0.3 mV/W [31,32] may also serve as a detector.

Conclusion

Electromagnetic radiation of a thin bunch of charged particles moving at a constant velocity through a planar grid structure with square cells was investigated under the assumption that the cell size is small compared to the wavelengths under consideration and the wire thickness is small compared to the cell size. It was assumed that wires

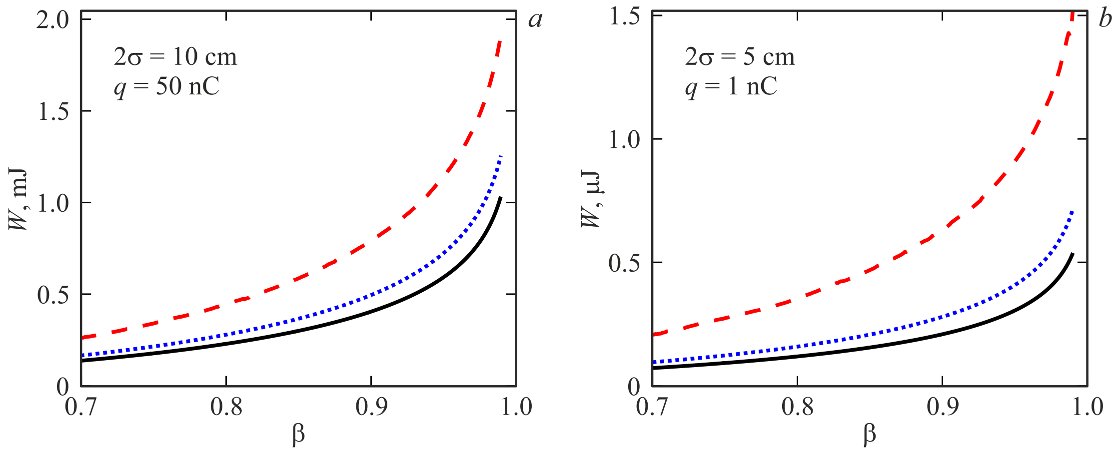

Figure 4. Dependence of the total energy of volume radiation W on bunch velocity β in the case of a fine-cellular structure (black solid and blue dash-dotted lines) and an ideal screen (red dotted lines). The bunch is a uniform cylinder with radius $r_0 = 100\,\mu$m. The bunch charge and length are $q = 50$ nC, $2\sigma = 10$ cm (a) and $q = 1$ nC, $2\sigma = 5$ cm (b). Structure period $a = 3$ cm; wire diameter $d_0 = 0.5$ mm (blue dash-dotted lines) and $d_0 = 0.1$ mm (black solid lines). The energy units are mJ (a) and μJ (b).

are in contact at the intersections, which ensures that the averaged boundary conditions are isotropic.

It was demonstrated as a result that the charge radiation field at the studied frequencies consists of volume waves only. The directional patterns of transition radiation and its spectrum were analyzed, and it was found that the properties of generated radiation depend to a significant extent on the geometric parameters of the structure. Specifically, a smaller grid period and a larger wire diameter contribute to enhancement of the radiation field, with the dependence on the period being stronger than on the diameter. Following a certain adjustment of grid parameters, one may generate radiation with such characteristics that differ little from those obtained with an ideal screen (with the exception of radiation at angles close to tangential). In contrast, if the generated field needs to be attenuated, one may reduce the thickness of wires or increase the period.

The obtained results may help develop new methods for detection and diagnostics of particle bunches. It is important to note that the use of a grid structure of thin conductors does not require precise bunch positioning, and the mentioned diagnostic methods may be non-invasive (i.e., non-destructive to the bunch).

Conflict of interest

The authors declare that they have no conflict of interest

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