

# Nonlinear currents and magnetic field in a metal irradiated with $p$ -polarized visible radiation

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The electric field structure and electron distribution function in a metal irradiated by  $p$ -polarized visible-frequency radiation have been studied. The resulting nonlinear currents and the magnetic field generated by them have been determined.

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## Introduction

The study of the interaction of electromagnetic radiation with metals has a long history [1–4]. However, some aspects of the theory are of considerable interest today, from both fundamental and applied points of view.

When studying the interaction of radiation with conducting media, the task was often set to calculate characteristics such as impedance or reflection coefficient (see, for example, [5–10]), which is explained by the simplicity of their experimental measurement. The structure of the electromagnetic field, being difficult to access for direct measurements, attracted less attention. However, modern problems of photonics and nonlinear optics require knowledge of the spatial distribution of electromagnetic fields and the electron distribution function inside a metal.

An example of a problem where knowledge of the field distribution in space is required is the problem of THz pulse generation via the interaction of femtosecond laser pulses with a metal. In the development of Ref. [11,12], where the effect of  $s$ -polarized radiation on metal was studied, this paper provides a detailed description of the structure of the electromagnetic field and the electron distribution function in metal when it is irradiated with  $p$ -polarized radiation. The motivation of the proposed study is that the dependence of the power of the generated low-frequency field on the polarization of pulses that affected the metal was observed in experimental studies in Ref. [13–15]. Using the kinetic equation and Maxwell's equations, the structure of fields at the fundamental frequency of radiation is studied below. The found fields and the correction to the distribution function made it possible to calculate nonlinear currents at zero frequency. The magnetic fields generated by these currents are determined.

## 1. Distribution function and current density

Let us consider the oblique incidence from a vacuum of a monochromatic  $p$ -polarized wave, whose electric field vector lies in the plane of incidence, onto a flat metal surface located in the half-space  $z > 0$ . The field in vacuum at  $z < 0$  has the form

$$\mathbf{B}_{vac} = \frac{1}{2} \left[ e^{i(\omega/c)z \cos \theta} + R \cdot e^{-i(\omega/c)z \cos \theta} \right] \times B_{inc} \mathbf{e}_y e^{-i\omega t + i(\omega/c)x \sin \theta} + c.c., \quad (1)$$

$$\mathbf{E}_{vac} = \frac{1}{2} \left[ e^{i(\omega/c)z \cos \theta} - R \cdot e^{-i(\omega/c)z \cos \theta} \right] \times \cos \theta B_{inc} \mathbf{e}_x e^{-i\omega t + i(\omega/c)x \sin \theta} - \left[ e^{i(\omega/c)z \cos \theta} + R \cdot e^{-i(\omega/c)z \cos \theta} \right] \times \sin \theta B_{inc} \mathbf{e}_z e^{-i\omega t + i(\omega/c)x \sin \theta} + c.c., \quad (2)$$

where  $B_{inc}$  is the amplitude of the magnetic field strength,  $\omega$  is the frequency,  $\theta$  is the angle of incidence,  $R$  is the reflection coefficient,  $c$  is the speed of light,  $\mathbf{e}$  are unit vectors,  $c.c.$  is the complex conjugation. Penetrating into the metal, the field affects the electrons. The description of the electrons will be carried out using the kinetic equation for their distribution function  $f$  with the collision integral in the relaxation time approximation:

$$\frac{\partial f}{\partial t} + \mathbf{v} \frac{\partial f}{\partial \mathbf{r}} + \frac{e}{m} \left( \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right) \frac{\partial f}{\partial \mathbf{v}} = -\nu(f - f_F), \quad (3)$$

where  $e$  and  $m$  is the electron charge and mass,  $\mathbf{E}$ ,  $\mathbf{B}$  are fields in metal,  $\mathbf{v}$  is the electron velocity,  $\nu$  is the effective frequency of electron collisions. In the case of a weak field, we represent the distribution function as the sum

of the Fermi distribution function  $f_F = 3n\eta(v_F - v)/4\pi v_F^3$  and small corrections — proportional to the field correction  $f_1$  and the stationary correction quadratic in the field  $f_0$ , where  $n$  is the electron density,  $\eta(x)$  is the Heaviside function,  $v_F$  is the Fermi velocity. Due to the stationarity in time and uniformity along the direction  $x$ , the field in the metal and the linear field perturbation of the distribution function depend on  $t$  and  $x$  as  $\sim \exp(-i\omega t + iq x)$ , where  $q = (\omega/c) \sin \theta$ :

$$f = f_F(v) + f_0(\mathbf{v}, z) + \frac{1}{2} \{f_1(\mathbf{v}, z) \exp(-i\omega t + iq x) + c.c.\}, \quad (4)$$

$$\mathbf{E} = \frac{1}{2} [E_x(z)\mathbf{e}_x + E_z(z)\mathbf{e}_z] \exp(-i\omega t + iq x) + c.c., \quad (5)$$

$$\mathbf{B} = \frac{1}{2} B_y(z)\mathbf{e}_y \exp(-i\omega t + iq x) + c.c.. \quad (6)$$

Let's write down the equation for the high-frequency correction linear in the field  $f_1(\mathbf{v}, z)$  and solve it in the case of specular reflection of electrons from the metal boundary  $z = 0$ . In this case, it is possible to extend the equation (3) to the area  $z < 0$ , extending  $f_1(\mathbf{v}, z)$  and  $E_x(z)$  in the even manner and extending  $E_z(z)$  in the odd manner. After that, using the Fourier transform and relations (4), (5) from (3), we find

$$-i(\omega - qv_x - kv_z)f_1(\mathbf{v}, k) + \frac{e}{m}\mathbf{E}(k)\frac{\partial f_F}{\partial \mathbf{v}} = -\nu f_1(\mathbf{v}, k), \quad (7)$$

where is the Fourier image of the field in the metal  $\mathbf{E}(k) = E_x(k)\mathbf{e}_x + E_z(k)\mathbf{e}_z$ . Due to the isotropy of the Fermi velocity distribution function, the term proportional to  $[\mathbf{v} \times \mathbf{B}] \cdot \partial f_F / \partial \mathbf{v} \equiv 0$ . Considering the smallness of  $v_F/c \ll 1$  from (7) we have

$$f_1(\mathbf{v}, k) = -\frac{ie}{m} \frac{1}{\omega + i\nu - kv_z} \left[ 1 + \frac{qv_x}{\omega + i\nu - kv_z} \right] \times \left( E_x(k) \frac{v_x}{v} + E_z(k) \frac{v_z}{v} \right) \frac{\partial f_F}{\partial v}. \quad (8)$$

The function  $f_1(\mathbf{v}, k)$  defines the Fourier image of the current density at the frequency  $\omega$ :

$$\mathbf{j} = \frac{1}{2} j_\alpha(k)\mathbf{e}_\alpha \exp(-i\omega t + iq x) + c.c., \quad (9)$$

$$j_\alpha(k) = \int e v_\alpha f_1(\mathbf{v}, k) d\mathbf{v} + c.c. = \sigma_{\alpha\beta}(k) E_\beta(k). \quad (10)$$

The components of the conductivity tensor included in this expression have the form

$$\sigma_{xx}(k) = \frac{i\omega_p^2}{4\pi(\omega + i\nu)} \cdot \frac{3}{2} \left\{ \text{Ln} - \frac{(\omega + i\nu)^2}{k^2 v_F^2} (\text{Ln} - 1) \right\}, \quad (11)$$

$$\sigma_{zz}(k) = \frac{i\omega_p^2}{4\pi(\omega + i\nu)} \cdot 3(\text{Ln} - 1) \frac{(\omega + i\nu)^2}{k^2 v_F^2}, \quad (12)$$

$$\sigma_{xz} = \sigma_{zx} = \frac{i\omega_p^2}{4\pi(\omega + i\nu)} \cdot \frac{3\omega \sin \theta}{2ck} \times \left[ 3 \frac{(\omega + i\nu)^2}{k^2 v_F^2} (\text{Ln} - 1) - \text{Ln} \right]. \quad (13)$$

where  $\omega_p = (4\pi n e^2 / m)^{1/2}$  is the plasma frequency,  $\text{Ln} \equiv \text{Ln}(k v_F / (\omega + i\nu))$ ,

$$\text{Ln}(x) = \frac{1}{2x} \ln \left( \frac{1+x}{1-x} \right) = \frac{\arctan h(x)}{x}. \quad (14)$$

## 2. High-frequency field in metal

Now, knowing the conductivity tensor, we can find the fields. Taking into account the extension of fields to the region  $z < 0$ , Maxwell's equations inside the metal have the form

$$ikE_x - \frac{i\omega}{c} \sin \theta E_z = \frac{i\omega}{c} B_y, \quad (15)$$

$$-ikB_y + 2B_0 = -\frac{i\omega}{c} (\varepsilon_{xx} E_x + \varepsilon_{xz} E_z), \quad (16)$$

$$i\frac{\omega}{c} \sin \theta B_y = -\frac{i\omega}{c} (\varepsilon_{zx} E_x + \varepsilon_{zz} E_z), \quad (17)$$

where  $B_0 = B_y(+0)$  is an oddly extended magnetic field at the boundary,  $\varepsilon_{\alpha\beta}(k) = \varepsilon_0 \delta_{\alpha\beta} + 4i\pi \sigma_{\alpha\beta}(k) / \omega$  is the dielectric permittivity tensor,  $\varepsilon_0$  is the contribution to the dielectric tensor from bound electrons and the lattice. From (15)–(17) we find the Fourier images of fields in metal:

$$B_y(k) = \frac{-2ik}{D} \left( \varepsilon_{zz} - \frac{\omega \sin \theta}{ck} \varepsilon_{zx} \right) B_0, \quad (18)$$

$$E_x(k) = \frac{-2i(\omega/c)}{D} \left( \varepsilon_{zz} - \sin^2 \theta \right) B_0, \quad (19)$$

$$E_z(k) = \frac{2ik}{D} \left( \sin \theta - \frac{\omega}{ck} \varepsilon_{zx} \right) B_0, \quad (20)$$

where

$$D = \left( k^2 - \frac{\omega^2 \varepsilon_{xx}}{c^2} \right) \varepsilon_{zz} + \frac{\omega^2}{c^2} \left( \varepsilon_{xx} \sin^2 \theta + \varepsilon_{zx} \varepsilon_{xz} \right) + \frac{k\omega \sin \theta}{c} (\varepsilon_{xz} + \varepsilon_{zx}). \quad (21)$$

In metals  $v_F \ll c$ , which makes it possible to omit the off-diagonal components of the dielectric tensor in formulas (18)–(20). In the visible frequency range for typical metals at room temperature the following inequalities are fulfilled

$$\omega_p, \omega_p / \sqrt{|\varepsilon_0|} \gg \omega \gg \nu, \omega_p \frac{v_F}{c}. \quad (22)$$

Taking into account the relations (22) in formulas (18)–(21), the terms containing  $\sin^2 \theta$  can be omitted. In this case,  $\varepsilon_{zz}$  falls out of the expressions (18).

Under these assumptions, the poles of the Fourier images of the fields are determined by the zeros of the

expressions  $k^2 - \omega^2 \epsilon_{xx}/c^2$  and  $\epsilon_{zz}$ . Zero of the expression  $k^2 - \omega^2 \epsilon_{xx}/c^2$  is achieved with a wave number

$$k^2 = \kappa^2 \simeq \frac{\omega_p^2}{c^2} \frac{\omega}{(\omega + i\nu)}, \quad (23)$$

which determines the depth of the skin layer in the case of a high-frequency skin effect. The zero of the  $\epsilon_{zz}$  occurs when

$$k^2 = \xi^2 = \frac{\omega_p^2}{v_F^2} \frac{3}{\epsilon_0}. \quad (24)$$

This wave number corresponds to the Debye radius of the degenerate electron gas. For typical metals  $|\xi| \gg |\kappa|$ . Near these zeros, the Fourier images (18), (19) have the form

$$B_y(k) \simeq \frac{-2ik}{k^2 + \kappa^2} B_0, \quad (25)$$

$$E_x(k) \simeq \frac{-2i\omega/c}{k^2 + \kappa^2} B_0. \quad (26)$$

The Fourier image  $E_z(k)$  contains two poles (Fig. 1, 2). Near them, it can be written as

$$E_z(k) \simeq 2ik \left[ \frac{-\omega(\omega + i\nu)}{(k^2 + \kappa^2)\omega_p^2} + \frac{1}{(k^2 + \xi^2)\epsilon_0} \right] \sin \theta B_0. \quad (27)$$

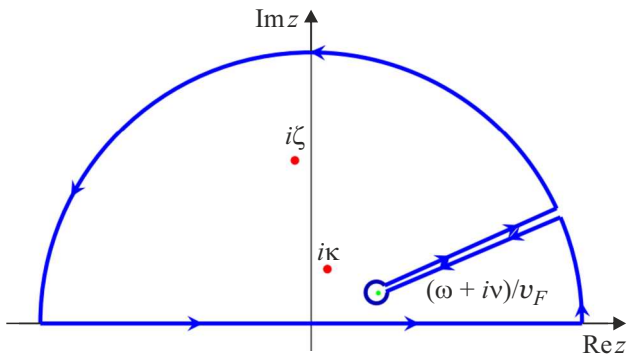


Figure 1. Integration contour in the calculation of  $E_z(k)$ , (20).

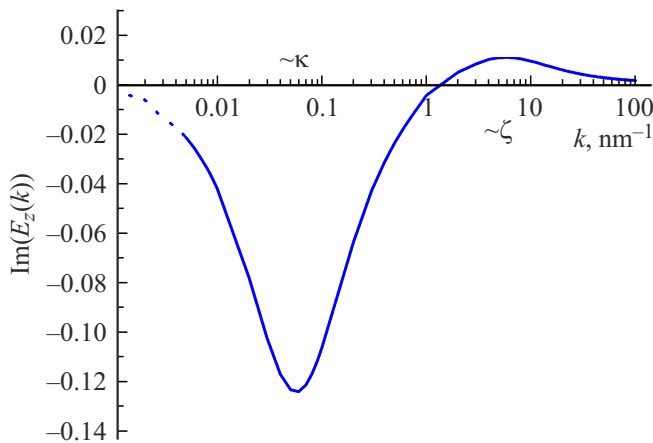


Figure 2. The imaginary part of the Fourier image of the electric field component normal to the surface,  $\text{Im}(E_z(k))$ .

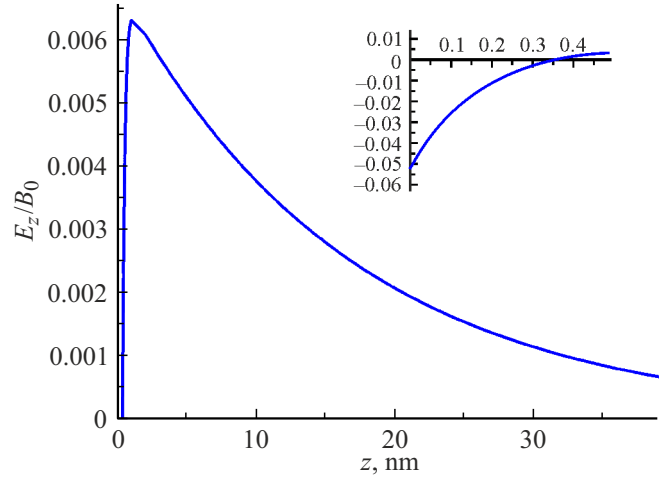


Figure 3. The electric field component perpendicular to the surface,  $E_z(z)$ . Calculations were performed for aluminum [19,20] at  $\omega = 1.7 \cdot 10^{15} \text{ s}^{-1}$ ,  $\omega_p = 1.9 \cdot 10^{16} \text{ s}^{-1}$ ,  $\nu = 8.5 \cdot 10^{13} \text{ s}^{-1}$ ,  $v_F = 1.6 \cdot 10^8 \text{ cm/s}$ ,  $\epsilon_0(\omega) = 12$  and  $\theta = \pi/4$ .

Under conditions (22), the poles make the main contribution to the field components (cf. [16]). Thus, from relations (25)–(27) we find

$$B_y(z) = B_0 e^{-\kappa z}, \quad (28)$$

$$E_x(z) = -\frac{i\omega}{c\kappa} B_0 e^{-\kappa z}, \quad (29)$$

$$E_z(z) = \left[ \frac{\omega(\omega + i\nu)}{\omega_p^2} e^{-\kappa z} - \frac{1}{\epsilon_0} e^{-\xi z} \right] \sin \theta B_0. \quad (30)$$

Under typical metal conditions, the field (30) is shown in Fig. 3. The value  $B_0$  included in (28), (29) is found from the condition of continuity of the tangential components of the field in metal and vacuum (1), (2). In the conditions under consideration

$$B_0 = \frac{2c\kappa \cos \theta}{c\kappa \cos \theta - i\omega} B_{inc}. \quad (31)$$

### 3. Low frequency distribution function and drag current

Expressions (28)–(30) can be used to calculate the nonlinear current along the metal surface  $j_x = \int e v_x f_0 d\mathbf{v}$ . Let's break this current into two parts, different in origin:  $j_{x,L}$ , which occurs due to the action of the Lorentz force on electrons;  $j_{x,C}$ , which appears due to movement along the surface of the charge squeezed from it. To obtain an equation for the nonlinear correction  $f_0$  to the electron distribution function  $f_F$ , in addition to explicit formulas for fields (28)–(30), an expression for the high-frequency correction to the distribution function in real space is

needed. Let's divide the field (30) into one that varies on the electromagnetic  $\kappa^{-1}$  and Debye  $\xi^{-1}$  scales,

$$\begin{aligned} E_z(z) &= E_z^{(\kappa)} + E_z^{(\xi)}, \\ E_z^{(\kappa)} &= \frac{\omega(\omega + i\nu)}{\omega_p^2} e^{-\kappa z} \sin \theta B_0, \\ E_z^{(\xi)} &= -\frac{1}{\varepsilon_0} e^{-\xi z} \sin \theta B_0. \end{aligned} \quad (32)$$

Writing the correction  $f_1$ , we take into account the inequalities (22). Then at the fundamental frequency for  $E_x$  and  $E_z^{(\kappa)}$ , the spatial derivative is a small correction. On the contrary, for  $E_z^{(\xi)}$  it makes the main contribution. In this case, for  $f_1(\mathbf{v}, z)$  we have

$$\begin{aligned} f_1(\mathbf{v}, z) &= \frac{ie}{m} \left\{ \left[ 1 + \frac{\omega \sin \theta}{\omega + i\nu} \frac{v_x}{c} + i \frac{\kappa v_z}{\omega + i\nu} \right] \right. \\ &\quad \left. \times \frac{E_x v_x + E_z^{(\kappa)} v_z}{\omega + i\nu} - \frac{i E_z^{(\xi)}}{\xi} \right\} \frac{1}{v} \frac{\partial f_F}{\partial v}. \end{aligned} \quad (33)$$

Let's substitute this correction to the distribution function into the equation for  $f_0$ . Leaving the terms odd in  $v_x$  in the the side of the equation (3), since only they contribute to the current along the surface, we obtain

$$\begin{aligned} \left( v_z \frac{\partial}{\partial z} + \nu \right) f_0(\mathbf{v}, z) + \frac{e}{m} \varepsilon \frac{\partial f_F}{\partial \mathbf{v}} &= \frac{ie^2}{4m^2} \left\{ \left( E_z^{(\kappa)} + E_z^{(\xi)} \right) \right. \\ \times \frac{\partial}{\partial v_z} \left[ \left( 1 - i \frac{\kappa v_z}{\omega - i\nu} \right) \frac{E_x^* v_x}{\omega - i\nu} + \frac{\omega \sin \theta}{\omega - i\nu} \frac{v_x}{c} \frac{E_z^{*(\kappa)} v_z}{\omega - i\nu} \right] \\ + \left[ E_x \frac{\partial}{\partial v_x} + \frac{B_y}{c} \left( v_x \frac{\partial}{\partial v_z} - v_z \frac{\partial}{\partial v_x} \right) \right] \\ \times \left[ \left( 1 - i \frac{\kappa v_z}{\omega - i\nu} \right) \frac{E_z^{*(\kappa)} v_z}{\omega - i\nu} + \frac{i E_z^{*(\xi)}}{\xi} \right] \\ + \left[ E_x \frac{\partial}{\partial v_x} + \frac{B_y}{c} \left( v_x \frac{\partial}{\partial v_z} - v_z \frac{\partial}{\partial v_x} \right) \right] \\ \times \left. \left[ \frac{\omega \sin \theta}{\omega - i\nu} \frac{v_x}{c} \frac{E_x^* v_x}{\omega - i\nu} \right] \right\} \frac{1}{v} \frac{\partial f_F}{\partial v} + c.c., \end{aligned} \quad (34)$$

where  $\varepsilon$  is a static electric field, quadratic in high-frequency field strength.<sup>1</sup> For nonlinear terms varying on the electromagnetic scale  $\sim e^{-\kappa z}$ , the main contribution to the equation is given by the term with  $\nu$  in the left part, since  $v_z \partial / \partial z \sim v_z \kappa \sim v_F \omega_p / c$ , while according to the last of the conditions (22),  $\nu \gg \omega_p v_F / c$ . For terms varying on the Debye scale  $\sim e^{-\xi z}$ , on the contrary, the derivative is  $v_z \partial / \partial z \sim v_z \xi \sim v_z \omega_p / v_F \sim \omega_p$ , and according to (22), this contribution is greater than the collision frequency. Given this, we substitute (29), (30), and (32) into (34)

<sup>1</sup> The component of this field along the surface, due to uniformity along the  $x$  axis, is zero, and the component perpendicular to the surface does not affect the calculation of nonlinear currents along  $x$ . Therefore, no explicit expression for  $\varepsilon$  is presented.

and find  $f_0$ . Knowing  $f_0$ , one can find the densities of nonlinear currents along the surface:

$$j_{x,C}(z) \simeq \frac{e I_0 \sin \theta}{mc \omega_p} \cdot \xi e^{-\xi z}, \quad (35)$$

$$j_{x,L}(z) \simeq -\frac{e I_0 \sin \theta}{mc \omega_p} \cdot 2\kappa e^{-2\kappa z}, \quad (36)$$

where  $I_0 = c B_0^2 / 8\pi$  is the energy flux density in the metal. The current density (35) corresponds to that obtained in Ref. [17] (see formula (24)), but takes into account the finite depth of field penetration, which is determined by the Debye radius  $\xi^{-1}$ . The expression (36) coincides, up to a numerical factor, with the formula (36) from Ref. [11] obtained in the same approximation for  $s$ -polarized radiation and the formula (25) from Ref. [18] obtained for the pulse of the finite duration. The amplitudes of the current densities (35), (36) are referred to as  $\kappa/\xi$ , and the integrals of them over  $z$ , i.e. the surface currents, coincide in order of magnitude. It should be noted that the fulfillment of inequality  $\nu \gg \kappa v_F$ , which was used to write the current density  $j_{x,L}$  is rather difficult. This should be kept in mind when comparing with the experiment. It can be seen from the above that taking into account the Lorentz force leads to another significant contribution to the surface current.

Let's compare the magnetic fields created by these currents. It follows from Maxwell's equations that

$$B_{y,C}(z=0) \simeq \frac{\pi e \sin \theta}{mc^2 \omega_p} \left| \frac{2c\kappa \cos \theta}{c\kappa \cos \theta - i\omega} \right|^2 I_{inc}, \quad (37)$$

$$B_{y,L}(z=0) \simeq \frac{\pi e \sin \theta}{2mc^2 \omega_p} \left| \frac{2c\kappa \cos \theta}{c\kappa \cos \theta - i\omega} \right|^2 I_{inc}, \quad (38)$$

$$B_{y,L}^{(s)}(z=0) \simeq \frac{2\pi e \sin \theta \omega_p^2}{mc^2 \omega_p \omega^2} \left| \frac{2\omega \cos \theta}{\omega \cos \theta + i c \kappa} \right|^2 I_{inc}, \quad (39)$$

where  $I_{inc}$  is the energy flux density of the field in a vacuum. The first two expressions correspond to currents (35), (36), and the third expression corresponds to the current generated by  $s$ -polarized radiation (see formula (37) from [11]).

The dependence of the amplitude of THz radiation pulses generated by nonlinear currents was measured in Ref. [13,15] in the case of femtosecond pulses acting on gold at different angles  $\theta$ , different polarizations and intensities of optical radiation. The quadratic nonlinearity observed in them and the comparable value of fields in low-frequency pulses (proportional to the corresponding quasi-stationary currents) for various exciting polarizations are consistent with the following magnetic field ratio from (37) to (39).

## Conclusion

Above, using the kinetic equation together with Maxwell's equations, the structure of the perturbation of the distribution function of electrons and the electromagnetic field in a metal is studied. When solving the equations

for the field and the distribution function, non-locality is taken into account both at the frequency of radiation acting on the metal and at the zero frequency. The correction to the distribution function quadratic in field and the corresponding nonlinear current along the metal surface are calculated. A quasi-static magnetic field generated by the current has been found. It is shown that two mechanisms of current generation along the surface are important for a  $p$ -polarized wave. One of them is caused by the action of the magnetic component of the Lorentz force simultaneously with the action of the electric field. It is similar to that occurring in the case of exposure to  $s$ -polarized radiation. The second is caused by the simultaneous action of two components of the electric field: the component normal to the surface generates a density disturbance at the metal surface, and the tangent to the surface shifts this disturbance.

The results obtained expand the understanding of optical rectification mechanisms and may be useful in the subsequent study of terahertz radiation generation.

## Conflict of interest

The authors declare that they have no conflict of interest.

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