Model assessment of accuracy of the Tunka-Grande array data reconstruction

© A.L. Ivanova,[1,2] I.I. Astapov,[3] P.A. Bezyazeekov,[5] A.V. Bulan,[4] A. Blinov,[5] A.N. Borodin,[5] N.M. Budnev,[2] P.V. Busygin,[2] N.V. Volkov,[6] P.A. Volchugov,[4,2] D.M. Voronin,[7] A.R. Gafarov,[2] A.Yu. Garmash,[1,8] V.M. Grebenyuk,[5,9] O.A. Gress,[2] T.I. Gress,[2] E.O. Gress,[2] A.A. Grinyuk,[5] O.G. Grishin,[2] A.N. Dyachok,[2] V.A. Erofeeva,[2] D.P. Zhurov,[2] A.V. Zagorodnikov,[2] V. Zirakashvili,[10] A.D. Ivanova,[2,11] M.A. Iliushin,[2] I.A. Kabannik,[1] N.N. Kalmukov,[4] V.V. Kindin,[3] S.N. Kiryuhin,[2] V.A. Kozhin,[4] K.G. Kompaniets,[3] E.E. Korosteleva,[4] E.A. Kravchenko,[1,8] A.P. Kryukov,[4] L.A. Kuzmichev,[4] A. Chiavassa,[12] A.A. Lagutin,[6] Yu.E. Lemeshev,[2] B.K. Lubsandorzhiev,[7] N.B. Lubsandorzhiev,[4,2] A. Lukanov,[7] S.D. Malakhov,[2] R.R. Mirgazov,[2] R.D. Monkhoev,[1,2] E.A. Okuneva,[4,2] E.A. Osipova,[4] A. Pan,[5] A.D. Panov,[4] L.V. Pankov,[2] A.L. Pakhorukov,[2] A.A. Petrukhin,[3] I. Poddubny,[2] V.A. Poleschuk,[2] E.G. Popova,[4] E.B. Postnikov,[4] V.V. Prosin,[4] A.A. Pushnin,[2] R.I. Raikin,[6] A.Yu. Razumov,[4,2] G.I. Rubtsov,[7] E.V. Ryabov,[2] I. Satyshev,[5] V.S. Samoliga,[2] L.G. Sveshnikova,[4] A.Yu. Sidorenkov,[7] A.A. Silaev,[7] A.A. Silaev (junior),[4] A.V. Skurikhin,[4] A.V. Sokolov,[1,8] V.A. Tabolenko,[2] A.B. Tanaev,[2] M.Yu. Ternovoy,[2] N.A. Ushakov,[7] A. Shaikovsky,[5] M.V. Shulga,[2] I.I. Yashin[3]

[1]Novosibirsk State University,
630090 Novosibirsk, Russia
[2]Irkutsk State University,
664003 Irkutsk, Russia
[3]National Research Nuclear University „MEPhI“,
115409 Moscow, Russia
[4]Skobeltsyn Institute of Nuclear Physics, Moscow State University,
119991 Moscow, Russia
[5]Joint Institute for Nuclear Research,
141980 Dubna, Moscow oblast, Russia
[6]Altai State University,
656049 Barnaul, Russia
[7]Institute for Nuclear Research, Russian Academy of Sciences,
117312 Moscow, Russia
[8]Budker Institute of Nuclear Physics, Siberian Branch, Russian Academy of Sciences,
630090 Novosibirsk, Russia
[9]Dubna State University,
141980 Dubna, Moscow oblast, Russia
[10]Pushkov Institute of Terrestrial Magnetism, Ionosphere and Radio Wave Propagation of the Russian Academy of Sciences,
142191 Moscow, Russia
[11]Moscow Institute of Physics and Technology (National Research University),
117303 Dolgoprudny, Moscow Region, Russia
[12]Dipartimento di Fisica Generale Universiteta di Torino and INFN,
Turin, Italy
e-mail: annaiv.86@mail.ru

Received May 4, 2025
Revised July 30, 2025
Accepted August 7, 2025

The work is devoted to a model assessment of the accuracy of reconstructing the EAS and CR parameters according to the data of the Tunka-Grande array. The technique for reconstructing events and results of processing artificial showers are presented. The comparison of the obtained accuracy of EAS and CR parameters reconstruction with the results of the experimental assessment is made.

Keywords: EAS, Tunka-Grande array, model assessment of reconstruction accuracy.

DOI: 10.61011/TP.2025.12.62385.230-25

Introduction

The Tunka-Grande scintillation array [1] is located in the Tunka Valley, 50 km from the lake Baikal. It consists of a network of 19 observation stations deployed over an area of about $0.5\,km^2$. The array is part of the TAIGA astrophysical complex [2] and is designed for studying cosmic rays (CR) and searching for diffuse gamma radiation in the energy range from 10 PeV to 1 EeV by detecting electron-photon and muon components of extensive atmospheric showers (EAS).

The analysis of the accuracy of the reconstruction of the EAS and CR parameters is an important stage in the processing of experimental data. Two main approaches are used to assess the quality of parameter recovery: experimental and model-based.

The first method is based on a comparison of the data processing results of the studied and reference arrays and is suitable for experiments located on the same site and recording the same wide atmospheric showers. Within the framework of this approach, the accuracy of reconstruction of the parameters of EAS and CR according to the Tunka-Grande array was estimated using the analysis of EAS registered simultaneously with the Cherenkov arrays Tunka-133 [2,3] and TAIGA-HiSCORE [2,4], assuming that the accuracy of reconstruction of the characteristics of EAS according to the Cherenkov arrays is higher, than according to the data of the scintillation array. All other things being equal, this is achieved due to the nature of the observed phenomena and the density of the detectors. The key advantage of the Cherenkov arrays is the registration of the light flux integrated over all the depths of the EAS development in the atmosphere, which provides significant smoothing of fluctuations inherent in the charged component of the EAS. Based on this, the values of the EAS and CR parameters, restored according to the Cherenkov arrays, were accepted as reference values. But, strictly speaking, this approach allowed us to obtain only upper limits on the errors in reconstructing the EAS parameters according to the Tunka-Grande array [3,4], since Cherenkov arrays have their own errors in reconstruction.

The second alternative method for estimating the accuracy of the recovery of EAS parameters is based on modeling the processes of EAS development in the atmosphere and the interaction of secondary particles with the detectors of the array. In the simulation process, a bank of artificial showers is formed, containing the results of the interaction of the EAS with the detectors of the array. The simulated events are processed using a software package for processing experimental data. The accuracy of reconstruction of the EAS and CR parameters is determined by comparing the reconstructed and initial values.

The model estimation of the accuracy of the reconstruction of the parameters of EAS and CR according to Tunka-Grande data was performed by a comparative analysis of the reconstructed and initial parameters of artificial showers generated using CORSIKA [5] and Geant4 [6].

1. Simulation of the Tunka-Gande array

The simulation of the Tunka-Grande array's response to secondary EAS particles was performed in two stages. In the first step using the CORSIKA software package (model of electromagnetic interactions — EGS4, models of hadron interactions — QGSJET-II-04, Geisha) [5] 1,600,000 artificial showers were modeled (50,000 for each variation of zenith angle and energy). Simulation parameters: primary particle — proton, energy range $15 \leq \lg(E/1\,\text{eV}) \leq 16.75$ in increments of 0.25, zenith angles are 0°, 15°, 30°, 45°, the range of azimuthal angles $0^\circ-360^\circ$, the position of the EAS axis is uniformly played out inside a circle with a radius of 800 m with the coordinate center coinciding with the coordinates of the Tunka-Grande central station. At the second stage, the responses of scintillation stations to EAS particles were modeled in the digital model of the Tunka-Grande array [7], implemented using the Geant4 toolkit [6].

2. Reconstruction and analysis of simulated data

The Tunka-Grande experimental data reconstruction program was used to process the simulated EAS. The algorithm for reconstruction of EAS parameters based on Tunka-Grande data is described in detail in Ref. [3,8]. Section 2 provides only a brief overview specifying those stages of reconstruction that are associated with the restoration of the spatial distribution of charged particles of EAS and energy.

The direction of arrival and the position of the shower axis, the number of charged particles in the EAS, the age parameter of the showers, and the energy are recovered from the energy release and response times of ground-based detectors using an iterative procedure involving 3 stages. In the first step, the number and corresponding density of particles trapped in the detectors are calculated. The direction of arrival of the downpour is restored using the triangle method based on the response times of the ground detectors. This procedure uses up to four detectors with the maximum density of detected particles. The initial coordinates of the position of the shower axis at the array site x, y are calculated using the center of mass method, the number of charged particles in the shower N_e is estimated in the zero approximation as the average weighted by particle density and the value of the spatial distribution function of particles at a fixed age of the shower $s = 1$ in triggered ground detectors. At the second stage, the direction of arrival of the shower is adjusted by taking into account the curvature of the front of the EAS [8]. The coordinates of the axis x, y and N_e are specified by the maximum likelihood method by minimizing the function of two variables x and y by the modified simplex method [9.10] with a fixed shower age parameter $s = 1$. At the last stage, the maximum likelihood method is also used, but with three free parameters — x, y, and s. The results of the previous stage are used as input values. Finally, the density of charged particles is calculated at a distance of 200 m from the EAS axis ρ_{200} and the primary energy is restored [8].

The key point of the reconstruction procedure is to restore the spatial distribution of secondary particles in the EAS.

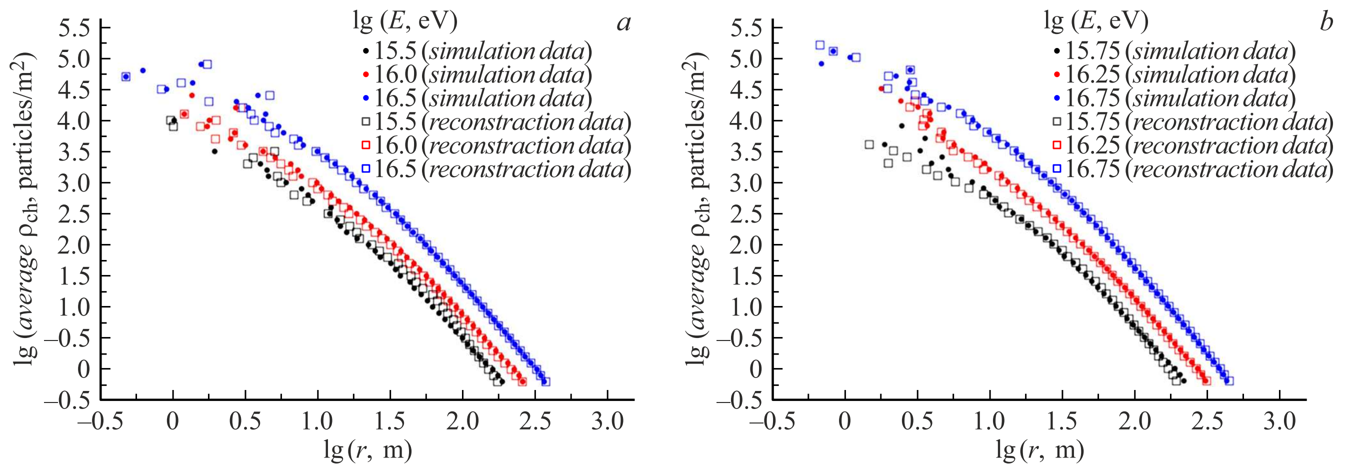

Figure 1. The density of particles in the ground-based detectors of the array as a function of the orthogonal distance to the EAS axis for energies $\lg(E/1\,\text{eV})$ 15.5, 16, 16.5 (it a) and 15.75, 16.25, 16.75 (*b*).

The spatial distribution of charged particles in the EAS is restored using a variation of the Nishimura–Kamata–Greisen function, obtained empirically at the EAS-MSU array [11]. The Greisen function is used as the spatial distribution function (SDF) of muons [12]. Otherwise, the restoration of the number of muons in the EAS N_μ according to the muon detector data is performed using the same algorithm, in parallel with the analysis of ground-based detector data.

The recovery procedure was applied to all EAS events with three or more triggered ground-based detectors. Fig. 1, *a, b* shows a comparison of the average modeled and calculated densities of charged particles for showers with fixed energy in the range of $15.5 \leq \lg(E/1\,\text{eV}) \leq 16.75$. The circles correspond to the distribution of the simulated particle densities in ground-based detectors as a function of the distance to the simulated EAS axis, and the squares correspond to the distribution of calculated particle densities as a function of the distance to the reconstructed shower axis. The reconstructed densities of charged particles were obtained by solving the inverse problem using the SDF values calculated for each detector and the reconstructed N_e in EAS.

The simulation results confirmed the experimental estimate of the threshold energy for registering EAS on the array area — the efficiency of registering artificial showers with $E = 10$ PeV in a circle with a radius of 400 m was $\approx 95\%$. The radius of the effective registration circle increases with increase of the EAS energy, and for showers with $E = 30$ PeV reaches 650 m, which significantly exceeds the size of the array itself. However, a comparison of the restored and initial parameters of the external EAS showed that in the current configuration of the Tunka-Grande array, the analysis of showers that fell outside it requires separate consideration. First of all, it concerns vertical EAS. In such events, the position of the shower axis is restored inside the array, being attracted to the local maximum of particle density, which can be observed both in the station of the outer and in the station of the inner circle.

EAS parameter reconstruction errors

lg (E/eV)	15.	15.25	15.5	15.75	16.	16.25	16.5	16.75
ψ_{68}, °	3.4	2.8	2.2	1.7	1.4	1.2	1.1	1
R_{68}, m	81	68	50	31	21	15	12	10
σ_E, %	86	73	55	43	36	34	30	26

Additional selection conditions have been introduced to exclude such events: 1) if less than 6 stations operated in the event, the number of internal triggered stations exceeds or is equal to the number of external triggered stations; 2) if the station with the maximum density of charged particles belongs to the outer circle, the number of particles registered in it is not less than 50; 3) the distance between the restored position of the EAS axis and the station with the maximum number of charged particles ≤ 150 m.

The accuracy of reconstruction of the position of the axis and the direction of arrival of EAS and primary energy was estimated from the remaining showers after additional thinning.

3. Model assessment

The angular resolution of the Tunka-Grande array was estimated by 68th percentile in the event distribution depending on the flat angle ψ between the reconstructed and the original direction of arrival of the EAS, the accuracy of reconstruction of the position of the EAS axis was estimated by 68th percentile in the event distribution depending on the distance R between the reconstructed and the original position of the shower axis. The energy restoration error σ_E was calculated from the standard deviation σ in the event

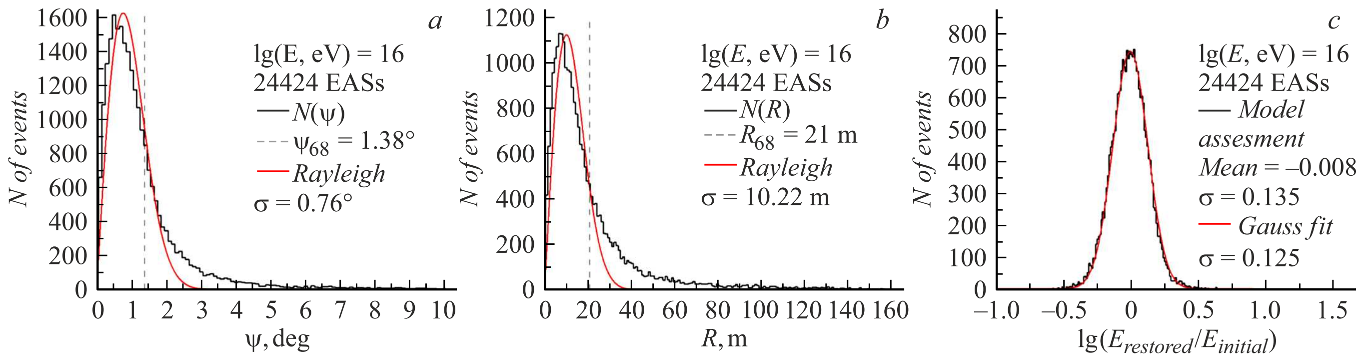

Figure 2. Event distributions ($E = 10$ PeV, $0°$ — $45°$, the position of the axis in a circle with a radius of 400 m) according to the parameters — ψ (a), R (b) and $\lg(E_{restored}/E_{initial})$ (c).

distribution according to the logarithm of the ratio of the restored energy to the set one ($\sigma_E = (10^\sigma - 1) \cdot 100\,\%$). Fig. 2 demonstrates the accuracy of reconstruction of the direction of arrival (Fig. 2, a), the position of the axis (Fig. 2, b) and energy (Fig. 2, c) of simulated showers with an energy of 10 PeV, arriving at an angle of $0°–45°$ on the array area bounded by a circle with a radius of 400 m. The vertical dashed lines in Fig. 2, a, b correspond to the values of the parameters ψ and R, which contain 68 % of the values of the corresponding desired distributions. A model estimate of the accuracy of reconstruction of the parameters of the EAS with energies in the range of $15. \leq \lg(E/1\,\text{eV}) \leq 16.75$ using the Tunka-Grande array data is presented in the table.

4. Comparison with experimental evaluation results

An analysis of the joint Tunka-Grande events with the Cherenkov Tunka-133 and TAIGA-HISCORE arrays showed that when reconstruction of events with energy of $E \geq 10$ PeV, limited by the zenith angle of arrival 45°, the angular resolution of the scintillation array is 2.3°, the accuracy of reconstruction of the axis position is not worse than 26 m, the energy resolution is 36 % [8]. The results of the model evaluation (see table) are slightly better than the results obtained experimentally. This is because the experimental evaluation allows us to obtain only the total error, including the errors of each of the arrays involved in the analysis.

Conclusion

According to the results of the model assessment, when recording showers with energy $E > 10$ PeV, which entered the Tunka-Grande array area at an angle up to 45°, the current technique allows reconstructing the position of the EAS axis with an error of no more than 26 m, the direction of arrival with an accuracy of no worse than 1.4° and primary energy with an error of no more than 36 %. The obtained values demonstrate good agreement with the results of the experimental evaluation and prove the effectiveness of using the Tunka-Grande array as a tool for studying primary cells and searching for diffuse gamma quanta in the energy range of 10 PeV–1 EeV.

Funding

The work was performed on the basis of the UNU „Astrophysical Complex of MSU-IGU“. The study was supported by the Russian Science Foundation (project 23-72-00016, section 3), 23-72-00054 (section 4)) and Ministry of Science and Higher Education of the Russian Federation (projects FZZE-2024-0005, FZZE-2026-0006, FSUS-2025-0008).

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] A.L. Ivanova, I.I. Astapov, P.A. Bezyazykov, E.A. Bonvech, A.N. Borodin, N.M. Budnev, A.V. Bulan, A. Vaidyanathan, N.V. Volkov, P.A. Volchugov, D.M. Voronin, A.R. Gafarov, A.Yu. Garmash, V.M. Grebenyuk, O.A. Gres, T.I. Gres, E.O. Gres, A.A. Grinyuk, O.G. Grishin, A.N. Dyachok, D.P. Zhurov, A.V. Zagorodnikov, A.D. Ivanova, M.A. Ilyushin, N.N. Kalmykov, V.V. Kindin, S.N. Kiryukhin, V.A. Kozhin, R.P. Kokoulin, N.I. Kolosov, K.G. Companietz, E.E. Korosteleva, E.A. Kravchenko, A.P. Kryukov, L.A. Kuzmichev, A. Chiavassa, M.V. Lavrova, A.A. Lagutin, Yu.E. Lemeshev, B.K. Lubsandorzhiev, N.B. Lubsandorzhiev, S.D. Malakhov, R.R. Mirgazov, R.D. Monkhoev E.A. Okuneva, E.A. Osipova, A. Pan, A.D. Panov, L.V. Pankov, A.L. Pakhorukov, A.A. Petrukhin, D.A. Podgrudkov, E.G. Popova, E.B. Postnikov, V.V. Prosin, V.S. Ptuskin, A.A. Pushnin, R.I. Raikin, A.V. Razumov, G.I. Rubtsov, E.V. Ryabov, I. Satyshev, V.S. Samoliga, L.G. Sveshnikova, A.Yu. Sidorenkov, A.A. Silaev, A.A. Silaev (Jr.), A.V. Skurikhin, A.V. Sokolov, V.A. Tabolenko, A.B. Tanaev, M.Y. Ternovoy, L.G. Tkachev, N.A. Ushakov, D.V. Chernov, I.I. Yashin. ZhTF, **93** (12), 1799 (2023) (in Russian).
DOI: 10.61011/JTF.2023.12.56825.f235-23

[2] I.I. Astapov, P.A. Bezyazykov, M. Blank, E.A. Bonvech, A.N. Borodin, M. Bruckner, N.M. Budnev, A.V. Bulan, A. Vaidyanatan, R. Vishnevsky, N.V. Volkov, P.A. Volchugov, D.M. Voronin, A.R. Gafarov, O.A. Gress, T.I. Gress, O.G. Grishin, A.Y. Garmash, V.M. Grebenyuk, A.A. Grinyuk, A.N. Dyachok, D.P. Zhurov, A.V. Zagorodnikov, A.L. Ivanova, N.N. Kalmykov, V.V. Kindin, S.N. Kiryukhin, R.P. Kokoulin, K.G. Kompaniets, E.E. Korosteleva, V.A. Kozhin, E.A. Kravchenko, A.P. Kryukov, L.A. Kuzmichev, A.A. Kompaniets, E.E. Korosteleva, V.A. Kozhin, E.A. Kravchenko, A.P. Kryukov, L.A. Kuzmichev, A. Kyavassa, A.A. Lagutin, M.V. Lavrova, Y.E. Lemeshev, B.K. Lubsandorzhiev, N.B. Lubsandorzhiev, R.R. Mirgazov, R. Mirzoyan, R.D. Monkhoev, E.A. Osipova, A.L. Pakhorukov, A. Pan, M.I. Panasyuk, L.V. Pankov, A.A. Petrukhin, D.A. Podgrudkov, V.A. Poleshchuk, E.G. Popova, A. Porelli, E.B. Postnikov, V.V. Prosin, V.S. Ptuskin, A.A. Pushnin, A.V. Razumov, R.I. Raikin, G.V. Raimov I. Rubtsov, E.V. Ryabov, Y.I. Sagan, V.S. Samoliga, I. Satyshev, A.A. Silaev, A.A. Silaev (Jr.), A.Yu. Sidorenkov, A.V. Skurikhin, A.V. Sokolov, L.G. Sveshnikova, Y.V. Suvorkin, V.A. Tabolenko, A.B. Tanaev, B.A. Tarashchansky, M.Y. Ternovoy, L.G. Tkachev, M. Tluzhikont, N.A. Ushakov, D. Horns, D.V. Chernov, I.I. Yashin. ZhETF, **161** (4), 548 (2022) (in Russian).
DOI: 10.31857/S0044451022040095

[3] R.D. Monkhoev, I.I. Astapov, P.A. Bezyazykov, E.A. Bonvech, A.N. Borodin, N.M. Budnev, A.V. Bulan, A. Vaidyanatan, N.V. Volkov, P.A. Volchugov, D.M. Voronin, A.R. Gafarov, E.O. Gres, O.A. Gres, T.I. Gres, O.G. Grishin, A.Yu. Garmash, V.M. Grebenyuk, A.A. Grinyuk, A.N. Dyachok, D.P. Zhurov, A.V. Zagorodnikov, A.D. Ivanova, A.L. Ivanova, M.A. Ilyushin, N.N. Kalmykov, V.V. Kindin, S.N. Kiryukhin, R.P. Kokoulin, N.I. Kolosov, K.G. Companietz, E.E. Korosteleva, V.A. Kozhin, E.A. Kravchenko, A.P. Kryukov, L.A. Kuzmichev, A. Chiavassa, A.A. Lagutin, M.V. Lavrova, Yu.E. Lemeshev, B.K. Lubsandorzhiev, N.B. Lubsandorzhiev, S.D. Malakhov, R.R. Mirgazov, E.A. Okuneva, E.A. Osipova, A.D. Panov, A.L. Pakhorukov, A. Pan, L.V. Pankov, A.A. Petrukhin, D.A. Podgrudkov, E.G. Popova, E.B. Postnikov, V.V. Prosin, V.S. Ptuskin, A.A. Pushnin, A.V. Razumov, R.I. Raikin, G.I. Rubtsov, E.V. Ryabov, V.S. Samoliga, I. Satyshev, L.G. Sveshnikova, A.A. Silaev, A.A. Silaev, A.Yu. Sidorenkov, A.V. Skurikhin, A.V. Sokolov, V.A. Tabolenko, A.B. Tanaev, B.A. Tarashchansky, M.Yu. Ternovoy, L.G. Tkachev, N.A. Ushakov, D.V. Chernov, I.I. Yashin. Izv. RAN. Ser. fiz., **87** (7), 954 (2023) (in Russian). DOI: 10.31857/S0367676523701673

[4] A.L. Ivanova, R. Monkhoev, I. Astapov, P. Bezyazeekov, M. Blank, E. Bonvech, A. Borodin, M. Brückner, N. Budnev, A. Bulan, D. Chernov, A. Chiavassa, A. Dyachok, A. Gafarov, A. Garmash, V. Grebenyuk, E. Gress, O. Gress, T. Gress, A. Grinyuk, O. Grishin, D. Horns, A.D. Ivanova, N. Kalmykov, V. Kindin, S. Kiryuhin, R. Kokoulin, K. Kompaniets, E. Korosteleva, V. Kozhin, E. Kravchenko, A. Kryukov, L. Kuzmichev, A. Lagutin, M. Lavrova, Y. Lemeshev, B. Lubsandorzhiev, N. Lubsandorzhiev, A. Lukanov, D. Lukyantsev, S. Malakhov, R. Mirgazov, R. Mirzoyan, E. Osipova, A. Pakhorukov, L. Pankov, A. Pan, A. Panov, A. Petrukhin, I. Poddubnyi, D. Podgrudkov, V. Poleschuk, V. Ponomareva, E. Popova, A. Porelli, E. Postnikov, V. Prosin, V. Ptuskin, A. Pushnin, R. Raikin, A. Razumov, G. Rubtsov, E. Ryabov, Y. Sagan, V. Samoliga, A. Satyshev, A. Silaev, A. Silaev (junior), A. Sidorenkov, S. Sinegovsky, A. Skurikhin, A. Sokolov, V. Sulakov, L. Sveshnikova, V. Tabolenko, B. Tarashchansky, L. Tkachev, M. Tluczykont, A. Tanaev, M. Ternovoy, R. Togoo, N. Ushakov, A. Vaidyanathan, P. Volchugov, N. Volkov, D. Voronin, R. Wischnewski, A. Zagorodnikov, A. Zhaglova, D. Zhurov, I. Yashin. J. Phys. Conf. Ser., **2156**, 012196 (2021). DOI: 10.1088/1742-6596/2156/1/012196

[5] R. Engel, D. Heck, T. Huege, T. Pierog, M. Reininghaus, F. Riehn, R. Ulrich, M. Unger, D. Veberič. Comput. Softw. Big Sci., **3** (1), 2 (2019). DOI: 10.1007/s41781-018-0013-0

[6] S. Agostinelli, J. Allison, K. Amako, J. Apostolakis, H. Araujo, P. Arce, M. Asai, D. Axen, S. Banerjee, G. Barrand, F. Behner, L. Bellagamba, J. Boudreau, L. Broglia, A. Brunengo, H. Bur khardt, S. Chauvie, J. Chuma, R. Chytracek, G. Cooperman, G. Cosmo, P. Degtyarenko, A. Dell'Acqua, G. Depaola, D. Dietrich, R. Enami, A. Feliciello, C. Ferguson, H. Fesefeldt, G. Folger, F. Foppiano, A. Forti, S. Garelli, S. Giani, R. Giannitrapani, D. Gibin, J.J. GómezCadenas, I. González, G. Gracia Abril, G. Greniaus, W. Greiner, V. Grichine, A. Grossheim, S. Guatelli, P. Gumplinger, R. Hamatsu, K. Hashimoto, H. Hasui, A. Heikkinen, A. Howard, V. Ivanchenko, A. Johnson, F.W. Jones, J. Kallenbach, N. Kanaya, M. Kawabata, Y. Kawabata, M. Kawaguti, S. Kelner, P. Kent, A. Ki mura, T. Kodama, R. Kokoulin, M. Kossov, H. Kurashige, E. Lamanna, T. Lampén, V. Lara, V. Lefebure, F. Lei, M. Liendl, W. Lockman, F. Longo, S. Magni, M. Maire, E. Medernach, K. Minamimoto, P. MoradeFreitas, Y. Morita, K. Murakami, M. Nagamatu, R. Nartallo, P. Nieminen, T. Nishimura, K. Ohtsubo, M. Okamura, S. O'Neale, Y. Oohata, K. Paech, J. Perl, A. Pfeiffer, M.G. Pia, F. Ranjard, A. Rybin, S. Sadilov, E. DiSalvo, G. Santin, T. Sasaki, N. Savvas, Y. Sawada, S. Scherer, S. Sei, V. Sirotenko, D. Smith, N. Starkov, H. Stoecker, J. Sulkimo, M. Takahata, S. Tanaka, E. Tcherniaev, E. SafaiTehrani, M. Tropeano, P. Truscott, H. Uno, L. Urban, P. Urban, M. Verderi, A. Walkden, W. Wander, H. Weber, J.P. Wellisch, T. Wenaus, D.C. Williams, D. Wrigh, T. Yamada, H. Yoshida, D. Zschiesche. Nucl. Instrum. Meth. A, **506**, 250 (2003). DOI: 10.1016/S0168-9002(03)01368-8

[7] R. Monkhoev M. Ternovoy, I. Astapov, P. Bezyazeekov, A. Borodin, M. Brueckner, N. Budnev, A. Chiavassa, A. Dyachok, A. Gafarov, A. Garmash, V. Grebenyuk, O. Gress, T. Gress, A. Grinyuk, O. Grishin, D. Horns, A. Igoshin, A. Ivanova, Al. Ivanova, N. Kalmykov, V. Kindin, S. Kiryuhin, R. Kokoulin, K. Kompaniets, E. Korosteleva, V. Kozhin, E Kravchenko, A. Kryukov, L. Kuzmichev, A. Lagutin, Y. Lemeshev, B. Lubsandorzhiev, N. Lubsandorzhiev, D. Lukyantsev, S. Malakhov, R. Mirgazov, R. Mirzoyan, E. Osipova, A. Pakhorukov, L. Pankov, A. Petrukhin, I. Poddubnyi, V. Poleschuk, V. Ponomareva, M. Popesku, E. Popova, A. Porelli, E. Postnikov, V. Prosin, V. Ptuskin, A. Pushnin, R. Raikin, G. Rubtsov, E. Ryabov, Y. Sagan, V. Samoliga, B. Sabirov, A. Silaev, A. Sidorenkov, S. Sinegovsky, A. Skurikhin, V. Slunecka, A. Sokolov, Y. Suvorkin, L. Sveshnikova, V. Tabolenko, B. Tarashchansky, L. Tkachev, M. Tluczykont, A. Tanaev, R. Togoo, N. Ushakov, A. Vaidyanathan, P. Volchugov, D. Voronin, R. Wischnewski, A. Zagorodnikov, A. Zhaglova, D. Zhurov, I. Yashin. J. Phys: Conf. Ser., **2103**, 012001 (2021).
DOI: 10.1088/1742-6596/2103/1/012001

[8] R.D. Monkhoev. Pisma v zhurnal Physika elementarnykh chastits i atomnogo yadra, **20** (5 (250)), 1117 (2023) (in Russian). DOI: 10.1134/S1547477123050552

[9] J.A. Nelder, R. Mead. Comp. J., **7**, 308 (1964).

[10] D. Himmel'blau. *Prikladnoe nelinejnoe programmirovanie* (Mir, M., 1975), p. 534 (in Russian).
[11] Y.A. Fomin, N.N. Kalmykov, I.S. Karpikov, G.V. Kulikov, M.Yu. Kuznetsov, G.I. Rubtsov, V.P. Sulakov, S.V. Troitsky. JINST, **11** (08), T08005 (2016). DOI: 10.1088/1748-0221/11/08/T08005
[12] K. Greisen. Ann. Rev. Nucl. Part. Sci., **10** (1), 63 (1960).

Translated by A.Akhtyamov