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Suppression of Fiske Steps in Long Josephson junction

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In this paper, we test a method for suppressing Fiske steps on the current-voltage characteristic of a long Josephson junction using matched load at its non-radiating end. The matched load is a four-stage Chebyshev transformer made of four sections of microstrip lines. The upper electrode of matched load is made of normal metal. The material chosen is molybdenum with a thickness of about 100 nm, which is a normal metal at the experiment temperature. Suppression of the Fiske steps will partially enable continuous tuning of the oscillator frequency at arbitrary bias current, including operating points in the resonant mode. Smoothing of the steps will lead to the expansion of the frequency range of the oscillator operation. We demonstrate that with the help of the proposed modification it was possible to smooth the Fiske steps on the current-voltage characteristics of the long Josephson junction and increase the range of voltages, in which it is possible to generate a signal with the power sufficient for use in heterodyne receivers.

Keywords: superconducting electronics, flux-flow oscillator, submillimeter-range oscillator, Fiske steps suppression.

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1. Introduction

Superconducting devices based on superconductor-insulator-superconductor (SIS) tunnel junctions are the main element of superconducting should be low noise receivers that are used in many fields of physics, terrestrial and space radio astronomy [1–5], biology, spectroscopy and metrology. The tunnel SIS junction is used in a superconducting heterodyne receiver as a mixing element. The noise level in superconducting heterodyne receivers is approaching the quantum limit, which is a fundamental limitation for receivers of this type [6–10]. If a receiving antenna and a local oscillator are placed on the same chip along with the mixer, then a superconducting integrated receiver (SIR) [11] will be obtained, the functionality of which will be much higher than that of a receiver with an external local oscillator [12]. The superconducting integrated receiver has been repeatedly tested in the above-mentioned tasks.

The long Josephson junction (LJJ) is traditionally used as a terahertz and subterahertz signal generator, including as a local oscillator in SIR. The magnetic field directed in the junction plane penetrates into it in the form of vortices (Josephson vortices), each of which carries a quantum of magnetic flux. When the displacement current is passed through the LJJ, the Josephson vortices begin to move. When the vortex reaches the edge of the LJJ, a single-quantum voltage pulse occurs and an electromagnetic wave is emitted [13]. In this way, a signal is generated, the frequency of which is related to the voltage at the junction through the Josephson relation [14]: $hf = 2\,\text{eV}$.

The LJJ has two operating ranges, which differ in the nature of the movement of the vortices. At voltages $V < V_g/3$, where V_g is the gap voltage of the tunnel junction, the so-called resonant generation mode is observed. In this mode, part of the electromagnetic radiation generated by the LJJ is reflected from the radiating end of the junction and reaches its opposite side. This leads to the appearance of standing electromagnetic waves inside the junction, which manifest themselves on the current-voltage characteristic (IVC) as quasi-constant voltage segments following a certain voltage period; this structure is called Fiske steps [15]. Stable signal generation in this mode is possible only at these steps. This leads to a limitation of the frequency range in which it is possible to select the necessary operating point at a given frequency for stable signal generation. As a result, a smooth frequency tuning in this mode of operation of the LJJ is possible only when selecting the position of the stage with a certain combination of the applied magnetic field and current through the LJJ.

At voltages $V > V_g/3$, the so-called „self-pumping effect“ occurs [11,16] due to which the attenuation coefficient of the waves in the Josephson junction sharply increases. In this mode, the IVC of the LJJ are quite smooth, and a smooth adjustment of the operating frequency is possible.

The purpose of this paper is to study the possibility of suppressing Fiske steps in a LJJ by adding a matched load to its non-emitting end to implement continuous frequency tuning at voltages $V < V_g/3$. Experimental studies were carried out on samples that include the studied LJJ, a microwave signal detector based on a lumped SIS contact,

as well as an integrated matching structure for transmitting a signal from the generator to the detector; all elements except the matched load are superconducting and are located on the same substrate [17].

2. Resonant stages on the LJJ IVC, selection of the operating point

A schematic representation of the LJJ section without load elements is shown in Figure 1. This type of structure, the so-called „overlap“ [11], is a resonator for waves propagating through it.

The resonance is observed when an integer number of half-waves n fits over the length of the LJJ: $\lambda_n = 2L/n$, where L is the length of the LJJ. The resonant frequencies $f_n = nc_{sw}/2L$, where c_{sw} is the propagation velocity of electromagnetic waves in the junction, or the Swihart velocity. These modes interact with the Josephson current, and when the frequency of the Josephson current coincides with the frequency of the resonant modes, steps of the Josephson current appear on the IVC of the junction at voltages $V_n = n\hbar\pi c_{sw}/4eL$, called Fiske steps. On the steps, the differential resistance is low, and therefore the line width will be quite narrow [18]. At the same time, the differential resistance in the space between the steps is very large, and often the choice of operating points is almost impossible. Thus, a pronounced resonant structure is observed on the high-voltage characteristics of the LJJ in this voltage range, which makes the implementation of smooth frequency tuning impossible with a constant bias current [19].

The Fiske steps are not vertical, but slightly inclined in this experiment. When the magnetic field changes, „jumps“ occur between the steps. The motion of the vortices in the junction is controlled by the bias current I_B, and their density is set by the magnetic field, which depends on the current in the control line I_{CL}. Thus, the positions of the operating points depend on both the displacement current I_B and the magnetic field generated by the current in the control line I_{CL}. To obtain information about the presence of operating points at certain frequencies at which stable signal generation of the required power is possible, instead of directly searching for a generation line, it is more convenient to measure the dependence of differential resistance on voltage, which is related to frequency according to the Josephson formula. A series of dependences of the voltage at the LJJ on the value of the bias current I_B is removed at fixed values of the current in the control line I_{CL}. When differentiating this curve, the dependence of the differential resistance on the voltage $R_D(\mathrm{V_{LJJ}})$ is obtained. At the Fiske steps, the differential resistance $R_D = dV/dI$ is small, which affects signal generation, since the width of the radiation line is proportional to R_D^2. However, choosing the operating point between the steps turns out to be almost impossible (Figure 2); even if there is an operating point between the steps, the generation line will be very wide, and neither phase nor frequency stabilization will be possible. This means that in order to generate a signal at the appropriate frequency, it is necessary to set a different value I_{CL}, at which an operating point exists at this frequency. If such a value of I_{CL} is found in the range of small I_B, the power of the generated signal will be insufficient for the LJJ to operate as a local oscillator as part of an integrated receiver. It should be noted that for LJJ with a high tunnel current density, the radiation at the edges of the Fiske steps is practically impossible due to the transition to the chaotic generation mode caused by the compression of the vortices when moving at a speed close to the Swihart velocity. As a result, in the resonant radiation mode, the LJJ does not have the ability to continuously adjust the frequency.

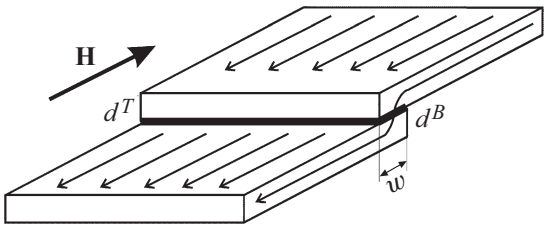

Figure 1. LJJ device used as a generator.

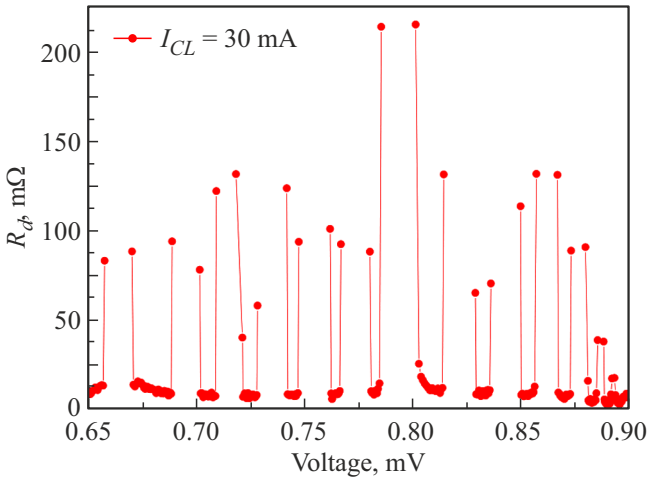

Figure 2. Dependence of differential resistance on voltage for a generator without load cells at current $I_{CL} = 30$ mA. There are clearly areas where there are no operating points. Generation is not possible at these voltages for a given current value I_{CL}.

Previously, the co-authors of the work implemented a procedure for searching for operating points at a fixed frequency for a LJJ as part of a superconducting integrated receiver [11]. However, even with this procedure, it is not possible to find operating points with sufficient generation power at all frequencies. In addition, the position of the working points varies greatly from one thermal cycling to another, and because of this, after each thermal cycling,

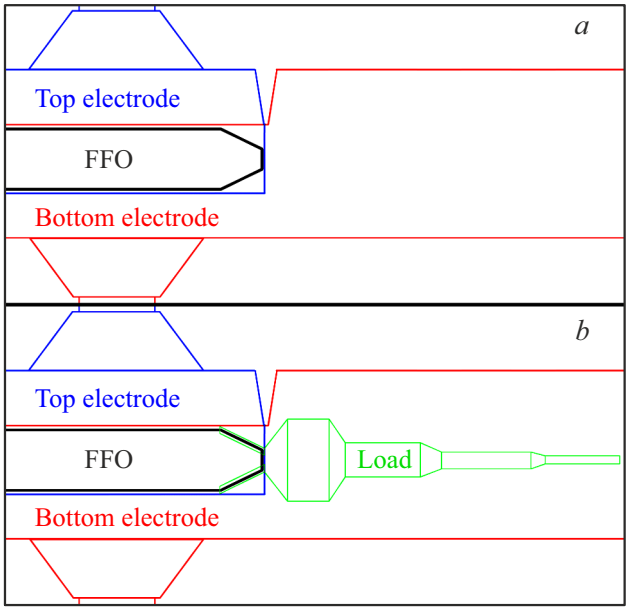

Figure 3. Schematic representation of the non-emitting end of the LJJ, top view. *a* — LJJ in traditional geometry; *b* — LJJ with the addition of a matched load. Top electrode, bottom electrode — upper and lower electrodes, respectively (see Figure 1). FFO — flux-flow oscillator based on long Josephson junction. Load — matched load made of molybdenum.

the procedure for searching for working points must be performed anew.

To ensure the possibility of continuous frequency tuning without loss of power, it is necessary to implement smoothing of the Fiske steps, which will increase the value of the differential resistance on them and, consequently, lead to a broadening of the generation line, but at the same time expand the range of operation range.. This problem can be solved by reducing the amplitude of the standing waves in the junction.

The paper in Ref. [19] describes in detail the approach to the solution based on increasing the attenuation parameter near the non-emitting end of the junction, as well as the proposed modifications and the results obtained. At the same time, the wave traveling towards the detector is also subject to increased attenuation, which leads to a decrease in the generation power. To preserve the generation power, it is desirable to use an approach in which the effect is carried out exclusively on standing waves, the amplitudes of which affect the magnitude of the Fiske steps. This can be achieved by reducing the reflection coefficient from the non-emitting end of the LJJ. A matched load in the form of a Chebyshev transformer is capable of absorbing part of the radiation that has reached the edge of the junction. This solution meets all the above requirements.

In the presented work, two types integrated structures were designed and manufactured: with a standard LJJ without a load, as well as an identical sample with the addition of a load in the form of a matched Chebyshev transformer made of molybdenum with a thickness of about 100 nm at the non-emitting end of the LJJ. The transformer parameters were selected in accordance with the results of numerical calculations. Figure 3 schematically shows standard LJJ and a modified LJJ.

A matched load in the form of a Chebyshev transformer is a natural solution for the problem of matching the impedances of the LJJ and the external environment. Moreover, due to the attenuation of electromagnetic waves in the microstrip line, the impedances of the LJJ and the last section of the transformer with the upper electrode made of normal metal are matched.. An impedance transformer with a Chebyshev (and not the most flat) characteristic allows for maximum matching between the elements at a given length. The literature mentions smooth junctions [20], which allow for better matching. This approach will be explored in the future.

3. Modeling in Ansys HFSS

The calculation of the matched load to reduce the reflection coefficient from the non-radiating end of the LJJ, and, consequently, to smooth the Fiske steps on its IVC, was performed in the Ansys HFSS program [21].

Ansys HFSS provides the ability to numerically model structures and determine their parameters, such as propagation and attenuation constants, as well as the wave impedance of transmission lines, by calculating electromagnetic fields at each point of the structure. However, there is no built-in module in Ansys HFSS that allows taking into account the superconducting properties of materials. Therefore, in order to calculate the Josephson microstrip transmission line, which is essentially a LJJ, boundary conditions of a special type were set in the model at the boundary of objects in the superconducting state [17,22]. These boundary conditions were calculated using expressions from the Mattis-Bardeen theory [23]. More complex models, which take into account the finite free path of quasi-particles, introduce insignificant differences for films produced by magnetron sputtering due to the small free path length.

Such boundary conditions make it possible to take into account not only the losses that are present in superconducting films at high frequencies and increase sharply beyond the gap frequency, but also the penetration of a magnetic field into superconductors, and as a result, changes in the propagation constant and wave impedance, which, according to simpler analytical models, have the following form [24]:

$$Z' = Z_0\sqrt{1+\lambda_1/H+\lambda_2/H},$$

$$\gamma' = \gamma_0\sqrt{1+\lambda_1/H+\lambda_2/H},$$

where Z_0, γ_0 is the wave impedance and propagation constant in an ideal microstrip line; λ_1 and λ_2 is the London penetration depth of the magnetic field into the lower and upper electrodes; H is the dielectric thickness. A more

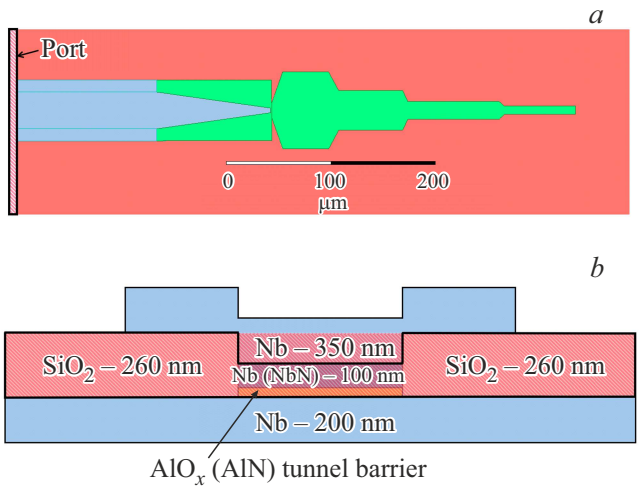

Figure 4. Image of the simulated structure in Ansys HFSS from above (*a*) and from the front (*b*). The location of the wave port is shown in red with shading.

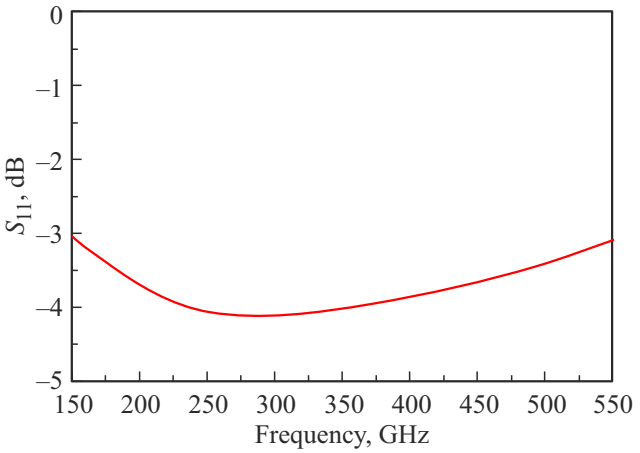

Figure 5. The ratio of the reflected wave amplitude to the incident wave amplitude obtained for the structure calculated in Ansys HFSS.

rigorous analytical model for Z' and γ' in a microstrip line is described in Ref. [25].

The thickness of the tunnel layer in the LJJ has a value of the order of 1 nm, however, when carrying out the procedure for splitting the structure into tetrahedra in HFSS, the length of the partition cannot be less than 5 nm. In this regard, in order to correctly describe the simulated line, a structure with a width and length of approximately $\sqrt{5}$ times greater than on the sample was modeled in HFSS. This change is made in order to ensure the same values of wave impedance and propagation constant as in the original structure.

The process of excitation of electromagnetic waves in the simulated structure in Ansys HFSS was carried out using a waveguide port. This port selection avoids reflections on the boundary between the port and the structure. An image of the simulated structure with modified dimensions and a waveguide port is shown in Figure 4.

The result of calculating the parameter S_{11} (the ratio of the reflected wave amplitude to the incident wave amplitude) for the waveguide port is shown in Figure 5. In the range from 200 to 500 GHz, the ratio of the amplitudes of the reflected signal to the original signal does not exceed −3.5 dB. It should be noted that the characteristic value of the impedance of the Josephson transmission line, which is the LJJ, is about 0.3 Ω. The wave impedance of the first stage of the Chebyshev transformer of microstrip lines, whose width is 76 μm, is approximately 1.5 Ω, which leads to a large impedance mismatch and a large reflection coefficient. A further increase in the width of the first section of the transformer is impossible due to the excitation of transverse modes.

Nevertheless, a decrease in the amplitude of the wave when reflected from the non-emitting edge of the LJJ, even by 3.5 dB, made it possible to achieve significant suppression of the Fiske steps in the experiment in terms of the operating range and a reduction in the space between the steps.

4. Description of the studied samples

The Josephson junctions (SIS detector and LJJ) are based on the tunnel structures Nb-AlO_x-Nb. The technology for manufacturing superconducting elements based on high-quality Josephson junctions was developed and optimized at the Kotelnikov Institute of Physics and Technology of the Russian Academy of Sciences [26]; this technology was tested in the manufacture of low-noise receivers for THz-band radio astronomy and integrated receivers for atmospheric monitoring and laboratory applications [27,28]. The impedance transformers of the integrated matching structure are made in the form of microstrip line segments based on niobium films with a silicon dioxide insulator SiO_2 250 nm thick. The value of R_nS of Josephson junctions is approximately 30 Ω · μm^2.

The micrographs of the two studied samples are shown in Figure 6. The difference between the sample in Figure 6, *a* and the sample in Figure 6, *b* lies only in the presence of a matched load at the non-radiating end of the LJJ in the second sample. The photos were taken with an optical microscope.

The photographs show the key components of the studied samples: a matched load made of molybdenum (1), an LJJ-based generator (2), an SIS detector (3), elements of an integrated matching structure (4), a critical current suppression line of the SIS detector (5), supply electrodes (6). All components are located on a single silicon substrate.

5. Experimental results

A comparison of the series of IVC samples with an LJJ without a load and with a load at different magnetic field

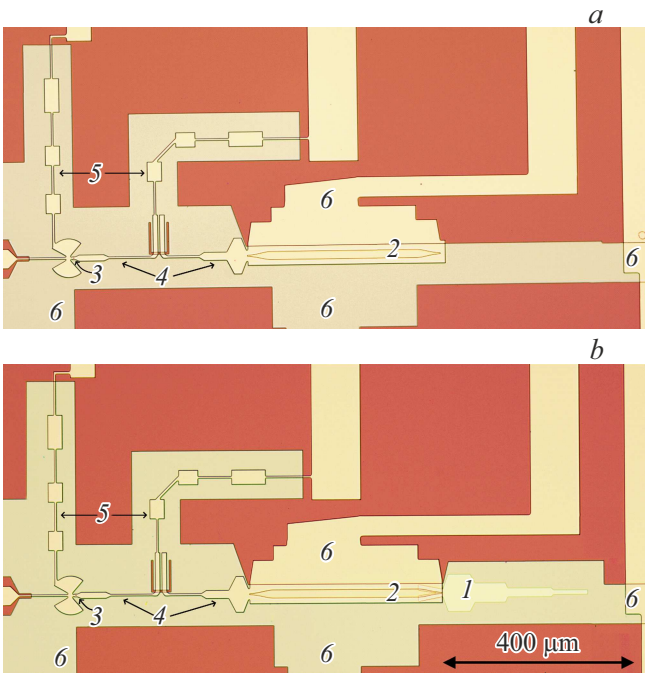

Figure 6. Micrographs of samples: *a* — without a load, *b* — with a load.

control currents is shown in Figures 7, 8. When measuring the dependence of the voltage at the LJJ on the bias current $V_{LJJ}(I_B)$, a certain value of the control current I_{CL} is set. A sharp change in the differential resistance values R_D is observed on the IVC of the LJJ both in the structure with a matched load and without it.

The color indicates the pumping level of the SIS detector I_{pump}. This value is uniquely related to the power absorbed by the SIS detector by the relations from the Tucker-Feldman model [29]. For the LJJ to work as a heterodyne in a superconducting integrated receiver, it is necessary that stable signal generation is possible at the operating point, and at the same time the pumping current of the SIS detector is at least 25 % of the current jump at the gap voltage I_g. The IVC of the SIS detector is shown in Figure 8.

Figure 7, *a* clearly shows sharp steps in the voltage range up to 0.9 mV.

Figure 9 shows the sections of the IVC series with the most pronounced Fiske steps, the frequency range: 314–435 GHz.

A comparison of the differential resistances of the LJJ without load elements and with the addition of a matched load is shown in Figure 10. These data were obtained by measuring the dependencies $V_{LJJ}(I_B)$ at fixed I_{CL}.

To quantify the result, the percentage of the total width of the areas where there are no working points is compared to the total width of the selected range. For a generator without adding a matched load (see Figure 10, solid dots), this value is more than 49 %. For a generator with the addition of a matched load (see Figure 10, dotted points), the areas of absence of operating points occupy only 19 % of the allocated range. Thus, the range in which stable signal generation is possible at the control line current $I_{CL} = 30$ mA has been increased by 30 %.

As a result, the dependence of differential resistance on frequency for samples with an added load has a significantly less rugged character, and the areas of jumps (absence

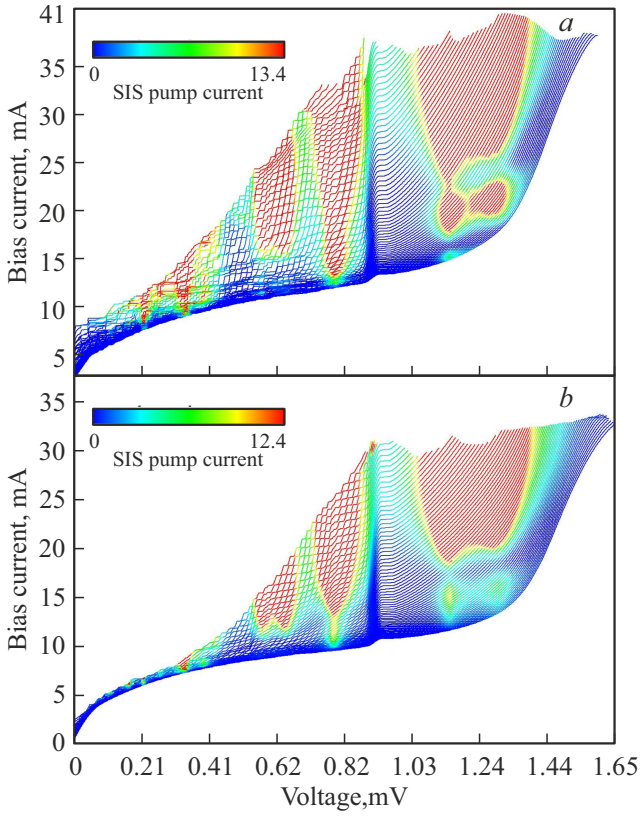

Figure 7. The family of high-voltage characteristics of the LJJ: *a* — without a load, *b* — with a load.

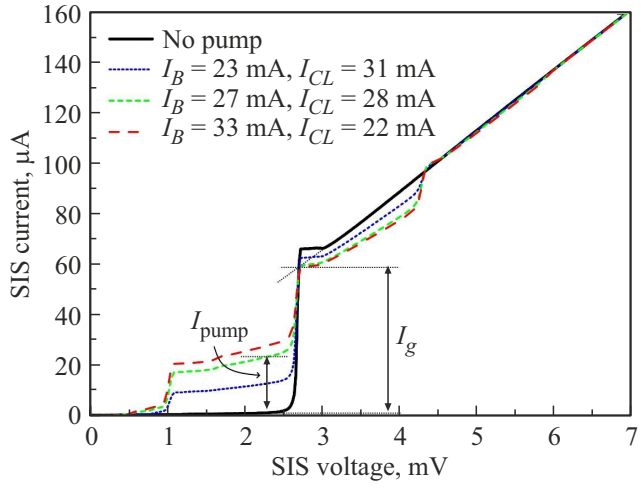

Figure 8. The voltage characteristic of the SIS detector under the influence of a 400 GHz frequency signal at several operating points. I_{pump} is the value of the pumping current, I_g is the value of the current jump at the gap voltage.

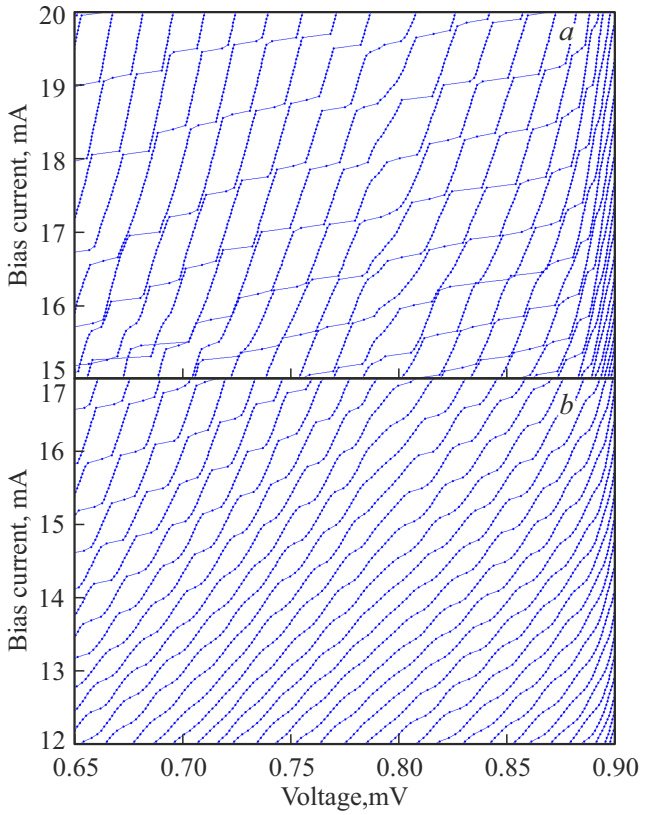

Figure 9. The family of high-voltage characteristics of the LJJ: *a* — without a load, *b* — with a load. All measured points are displayed. Information about the pumping of the SIS detector is not displayed for readability.

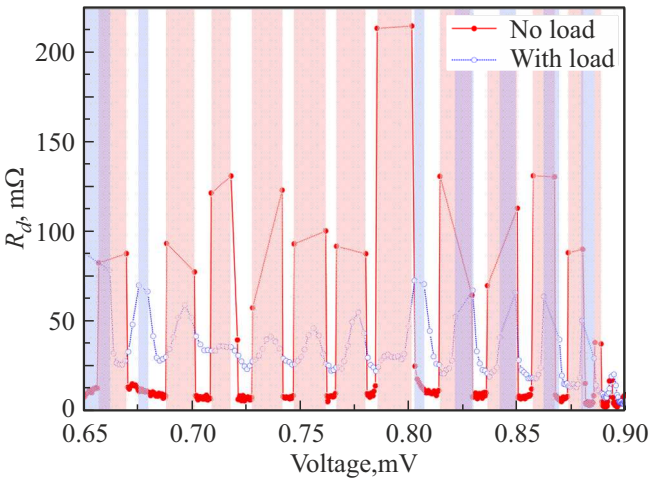

Figure 10. Frequency dependence of the differential resistance for LJJ without load elements (solid dots) and LJJ with matched load (empty dots) at current $I_{CL} = 30$ mA.

of operating points) have either noticeably narrowed or disappeared.

The equipment of the UNU #352529„Kriointegral“ was used for fabrication and studying the samples.

6. Conclusion

By adding a matched load in the form of a Chebyshev transformer made of molybdenum with a thickness of about 100 nm to the non-emitting end of the LJJ, it was possible to smooth out the Fiske steps on the LJJ. This has increased the voltage range at which it is possible to generate a signal of the required power. A comparison of the frequency dependences of the differential resistances for samples differing only in the presence of a load at the non-emitting end of the junction showed the potential of using such loads in improving sub-terahertz generators based on LJJ.

Further studies of this modification will help improve the results obtained and will allow us to get closer to ensuring smooth tuning of the signal generation frequency over the entire range by a long Josephson junction in a resonant mode of operation with minimal loss of radiation power. A possible direction for optimizing the suppression and smoothing of Fiske steps is to combine the methods of adding a matched load to the non-emitting end of the LJJ to lower its parameter S_{11} and applying an additional absorber along the edges of the LJJ to increase the attenuation parameter α inside the junction. This will completely ensure a smooth frequency adjustment without significant loss of radiation power.

Conflict of interest

The authors declare that they have no conflict of interest.

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