Technical Physics, 2025, Vol. 70, No. 6

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Non-isochronicity of ferromagnetic nanoparticles of various shapes in a magnetic field

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Received December 18, 2024
Revised February 19, 2025
Accepted February 20, 2025

The possibility of controlling the non-isochronicity of ferromagnetic nanoparticles using an external magnetic field is investigated. Analytical expressions characterizing the nonlinear frequency shift of resonant magnetization oscillations for the diagonal tensor of demagnetizing factors are obtained by the method of Hamiltonian formalism. It is shown that when rotating and changing the magnitude of the vector of the external field of the bias magnetic field, it is possible to achieve a restructuring of the resonant oscillation frequency by changing the amplitude of the applied alternating magnetic field for both spherical and cylindrical the sample in the presence of uniaxial anisotropy. In the isotropic case, for a spherically symmetric sample, the non-isochronicity coefficient is zero due to the symmetry of the demagnetization tensor of the sphere, whereas for a cylindrical sample it is nonzero. The results obtained can be used to construct new types of essentially nonlinear microwave spintronics and magnonics devices.

Keywords: non-isochronicity, non-isochronicity coefficient, Hamiltonian formalism, demagnetization tensor.

DOI: 10.61011/TP.2025.06.61381.461-24

Introduction

Magnetic materials are used in spintronic devices for transmitting and processing of information. The possibility of generating ultrahigh-frequency dynamics of magnetization in various magnetic samples is being actively investigated [1,2]. Traditionally, radio engineering elements based on films or spheres are considered for practical applications [3,4]. For example, spheres made of iron-yttrium garnet are widely used in microwave electronics as frequency-selective elements [4]. Single-cascade filters and resonators are suggested to be fabricated [5]. Research is also being conducted to study the physics of interactions between ferromagnetic spheres and superconductors [6]. Much scientific focus is made on application of linear and nonlinear dynamics of magnetization in ferro- and antiferromagnetic films in spintronic devices, as well as on the control of the properties of such structures [1,7]. Magnetic films can be used to solve a wide range of tasks: from data storage [8] and magnetometry [9] to spin current generation [10] and neuromorphic computing [11]. Currently, the study of magnetization dynamics in magnetic samples, the geometric shapes of which are different from spheres and films, is not given so much scientific attention. Nevertheless, magnetic cylinders are proposed to be used as non-planar solutions for signal transmission problems in spintronics [12,13]. It should be stressed that the ability to control the non-isochronous oscillations, possibly caused by the shape anisotropy, is essential in the development of spintronic devices, since it determines the ratio between the frequency of magnetization oscillations and their amplitude [14,15]. In addition, with a sufficiently large non-isochronism, phase noise will be amplified by amplitude noise due to amplitude-phase conversion [3].

Earlier, in [14], the dynamics of magnetization oscillations in a sample with a thin film geometry was considered given the nonlinear frequency shift of vibrations, and in [15], the influence of external magnetic field on the nonlinear frequency shift in this sample was investigated. The purpose of this work is to obtain and study the nonlinear coefficients of magnetization oscillations frequency shift (non-isochronism coefficients) for spintronic ferromagnetic samples with a geometry in the form of a sphere and a thin cylinder when exposed to external magnetic field.

1. Theoretical analysis

The dynamics of magnetization in a ferromagnetic material can be described using the Landau–Lifshitz equation, which has the form [14,16]:

$$\frac{d\mathbf{m}}{dt} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{d\mathbf{m}}{dt}, \tag{1}$$

where γ — modulus of gyromagnetic ratio of an electron, $\mathbf{m}$ — magnetization vector normalized to the saturation magnetization M_s, $\mathbf{H}_{\text{eff}}$ — effective magnetic field, α — Hilbert coefficient [17]. Note that in this paper we are investigating the effect of an external magnetic field on the nonlinearity of a ferromagnetic sample, so it is enough to consider only conservative part of equation (1). In this case, further analysis may not take into account Hilbert attenuation. The effective magnetic field is related to the magnetic energy E as a variational derivative of its functional [14,18]:

$$\mathbf{H}_{\text{eff}} = -\frac{1}{\mu_0 M_s}\frac{\delta E}{\delta \mathbf{m}}. \tag{2}$$

Here μ_0 is the magnetic constant. Full magnetic energy is written as

$$E = \int \varepsilon_V \, dV, \tag{3}$$

where ε_V — volumetric density of magnetic energy. We use the macrospin approximation because we do not study wave propagation in ferromagnets or the dynamics of inhomogeneous magnetization states [19]. This means that the energy contribution of the exchange interaction can be ignored. Then the magnetic energy density may be written as

$$\varepsilon_V = -\mu_0 M_s \mathbf{H}_0 \mathbf{m} - K_u(\mathbf{m}\mathbf{e}_u)^2 + \frac{\mu_0 M_s^2}{2}\,\mathbf{m}\,\widehat{D}\,\mathbf{m}. \tag{4}$$

In the expression (4) the first term in the right-hand side arises due to the application of external magnetic field to the ferromagnetic sample $\mathbf{H}_0$ and is called Zeeman volumetric energy density. The second term describes the contribution of uniaxial magnetic anisotropy, which may occur due to a violation of crystal symmetry during the growth of [20], with a constant K_u and a direction vector $\mathbf{e}_u$. In this paper, we do not consider crystallographic anisotropy. However, within the approach used for spherical and cylindrical shapes of a ferromagnetic sample, it can be taken into account in the same way as for thin ferromagnetic films [15]. The third term is the demagnetization field energy density that arises as a result of the dipole-dipole interaction in a ferromagnet. The diagonal demagnetization tensor $\widehat{D}$ may be used to describe the energy contribution of such an interaction. This type of tensor is used to simulate the dynamics of magnetization in a sphere, extended cylinder, or a thin film [21]. Thus, the energy density in the equation (4) will be expressed as

$$\varepsilon_V = -\mu_0 M_s \mathbf{H}_0 \mathbf{m} - K_u(\mathbf{m}\mathbf{e}_u)^2 + \frac{\mu_0 M_s^2}{2}\big(D_{xx}(\mathbf{m}\mathbf{e}_x)^2 + D_{yy}(\mathbf{m}\mathbf{e}_y)^2 + D_{zz}(\mathbf{m}\mathbf{e}_z)^2\big), \tag{5}$$

where D_{xx}, D_{yy}, D_{zz} — diagonal components of the demagnetization tensor $\widehat{D}$. Then, the effective magnetic field may be written as

$$\mathbf{H}_{\text{eff}} = \mathbf{H}_0 + H_u(\mathbf{m}\mathbf{e}_u)\mathbf{e}_u - M_s\big(D_{xx}(\mathbf{m}\mathbf{e}_x)\mathbf{e}_x + D_{yy}(\mathbf{m}\mathbf{e}_y)\mathbf{e}_y + D_{zz}(\mathbf{m}\mathbf{e}_z)\mathbf{e}_z\big), \tag{6}$$

where H_u — uniaxial anisotropy field associated with constant K_u by the relation

$$H_u = 2K_u/(\mu_0 M_s).$$

Equation (1) is a vector nonlinear equation. It is difficult to theoretically analyze such kind of equations. In this case, we can use the method of Hamiltonian formalism [22], which allows us to convert the Landau–Lifshitz equation to Hamiltonian equations. It should be noted that such equations are an effective tool for analyzing nonlinear dynamics and are widely used in various applications. For example, the Hamiltonian formalism is applicable to the study of plasma [23], Bose-Einstein condensate [24] and in hydrodynamics [25]. This method consists in introducing a new complex variable describing the magnetization dynamics, the square of modulus of which delineates the oscillation power [14,22]. One of the simplest ways to introduce such a variable is to transform it by relating this new variable to the amplitude of the magnetization precession. To determine the precession amplitude, it is necessary to find the vector around which the magnetization $\mathbf{m}$ will oscillate. The unit vector $\mathbf{e}_\xi$ of this direction can be found from the equation.

$$-\frac{1}{\mu_0 M_s}\frac{\delta E}{\delta \mathbf{m}} = H\mathbf{e}_\xi, \tag{7}$$

where H — internal magnetic field. In Cartesian coordinate system the vector $\mathbf{e}_\xi$ is expressed as

$$\mathbf{e}_\xi = \big(\cos(\theta)\cos(\varphi), \cos(\theta)\sin(\varphi), \sin(\theta)\big).$$

Let ξ be the angle in OXY plane between the vector of anisotropy $\mathbf{e}_u$ and axis OX. Let's denote by θ_0 and φ_0 the polar and azimuthal angles of the external magnetic field H_0. Then the orientation angles of vector $\mathbf{e}_\xi$ and the internal magnetic field H can be found from the system of equations

$$\begin{aligned}
(H + M_s D_{xx})\cos(\theta)\cos(\varphi) &= H_0\cos(\theta_0)\cos(\varphi_0) \\
&\quad + H_u(\mathbf{e}_\xi\mathbf{e}_u)\cos(\xi), \\
(H + M_s D_{yy})\cos(\theta)\sin(\varphi) &= H_0\cos(\theta_0)\sin(\varphi_0) \\
&\quad + H_u(\mathbf{e}_\xi\mathbf{e}_u)\sin(\xi), \\
(H + M_s D_{zz})\sin(\theta) &= H_0\sin(\theta_0).
\end{aligned} \tag{8}$$

Let's introduce two unit vectors $\mathbf{e}_\xi$ and $\mathbf{e}_\eta$ complementing $\mathbf{e}_\xi$ to the right triple

$$\begin{aligned}
\mathbf{e}_\xi &= \sin(\theta)\cos(\varphi)\mathbf{e}_x + \sin(\theta)\sin(\varphi)\mathbf{e}_y - \cos(\theta)\mathbf{e}_z, \\
\mathbf{e}_\eta &= -\sin(\varphi)\mathbf{e}_x + \cos(\varphi)\mathbf{e}_y.
\end{aligned} \tag{9}$$

Hamiltonian equations are written with respect to variables called canonical [22]. Depending on the specific task, such variables can be complex conjugate, as well as depend on coordinates and time. When describing the dynamics of magnetization, canonical variables are most often associated with the deviation of the vector $\mathbf{m}$ from its equilibrium position [14,15]. For spin waves, the canonical transformation was first introduced by Holstein and Primakov [26]. In the considered case, such a transformation may be written as

$$a = \frac{m_\xi - i m_\eta}{\sqrt{2(1 + m_\xi)}}. \quad (10)$$

Here m_ξ, m_η, m_ξ — Projections of vector $\mathbf{m}$ on the respective coordinate axes. The dimensionless variable a describes the amplitude of the magnetization precession. The dynamic equation for this variable in the studied case under will have the form

$$\frac{\partial a(\mathbf{r}, t)}{\partial t} = -i \frac{\partial H_a}{\partial a^*}, \quad (11)$$

where

$$H_a = \gamma E/(\mu_0 M_s)$$

— normalized magnetic energy depending on the complex amplitude a. We operate in the macrospin approximation and neglect the spatial inhomogeneities of — magnetization $\mathbf{m}$. In this case $a = a(t)$, which means that in equation (11) on the left side, it's possible to write the full, not the partial derivative. To switch from the Landau–Lifshitz equation (1) to its Hamiltonian notation (11), it is necessary to replace the basis $\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$ with a new $\mathbf{e}_\xi, \mathbf{e}_\xi, \mathbf{e}_\eta$ and take into account the inverse transformation to (10)

$$m_\xi = (1 - 2|a|^2),$$

$$m_\xi = \sqrt{1 - |a|^2}\,(a + a^*),$$

$$m_\eta = -i\sqrt{1 - |a|^2}\,(a - a^*). \quad (12)$$

Expressions (12) can be obtained directly from (10), given that the length of the magnetization vector is preserved $m_\xi^2 + m_\xi^2 + m_\eta^2 = 1$. The energy H_a may be expanded into a series by a dimensionless variable a near the minimum of the magnetic energy. Let's write down such a series, taking into account terms not higher than 4 of the order of smallness.

$$H_a = A|a|^2 + \left(\frac{1}{2}Ba^2 + \frac{1}{2}B^*a^{*2}\right) + \left(V|a|^2 a + V^*|a|^2 a^*\right)$$
$$+ U_1|a|^4 + \left(U_2|a|^2 a^2 + U_2^*|a|^2 a^{*2}\right) + \ldots \quad (13)$$

The expressions obtained for the coefficients of the series (13) are presented in Appendix A. Using the Hamiltonian H_a, it is possible to describe the dynamics of magnetization of ferromagnetic samples of various geometric shapes. The resulting Hamiltonian (13) is very bulky to analyze, so it is necessary to apply some transformations. Let's note that its quadratic part is not diagonal. It is possible to simplify the quadratic part using Bogolyubov transformation [27]. In the considered case, such a transformation may be written as

$$a = ub - vb^*, \quad (14)$$

where

$$u = \sqrt{\frac{A + \omega_0}{\omega_0}},$$

$$v = \sqrt{\frac{A - \omega_0}{\omega_0}},$$

$$\omega_0 = \sqrt{A^2 - |B|^2}.$$

In such a replacement the series for the Hamiltonian will look as follows

$$H_b = \omega_0|b|^2 + \left(W_1|b|^2 b + W_1^*|b|^2 b^* + W_2 b^3 + W_2^* b^{*3}\right)$$
$$+ T|b|^4 + \ldots \quad (15)$$

Here

$$W_1 = -v^{*2}V^*v + 2u(Vv - u^*V^*)v^* + u^*Vu^2,$$

$$W_2 = V^*v^{*2} - Vu^2V^*,$$

$$T = \left(u^2U_1 - 3uU_2^*v^*\right)u^{*2} - 3\left(u^2U_2 - \frac{4}{3}uU_1v^* + v^{*2}U_2^*\right)vu^*$$
$$-3v^*U_2uv^2 + v^{*2}U_1v^2. \quad (16)$$

Note that the diagonalization (14) is a canonical transformation, i.e., preserves the Hamiltonian form of the equations. In this case, the conservative dynamic equation for the complex amplitude b can be written as follows:

$$\frac{db}{dt} = -i\frac{\partial H_b}{\partial b^*}. \quad (17)$$

In (15), terms proportional to the third power of the complex amplitude b can be excluded. However, it is necessary to use a quasi-canonical nonlinear substitution for this. This means that the Hamiltonian form will be preserved only if the high-order terms are removed during the decomposition of the Hamiltonian. As mentioned earlier [14,15], the following nonlinear replacement may be used:

$$b = c + \frac{1}{\omega_0}\left(W_1c^2 - 2W_1^*|c|^2 - W_2^*c^{*2}\right). \quad (18)$$

Under this transformation, the Hamiltonian takes the simple form

$$H_c = \omega_0|c|^2 + \frac{N}{2}|c|^4, \quad (19)$$

where non-isochronism coefficient is determined by the expression

$$N = 2\left(T - 3\frac{|W_1|^2 + |W_2|^2}{\omega_0}\right). \quad (20)$$

The dynamic equation for a complex amplitude c with Hamiltonian (19) takes the form

$$\frac{dc}{dt} = -i\omega\left(|c|^2\right)c, \quad (21)$$

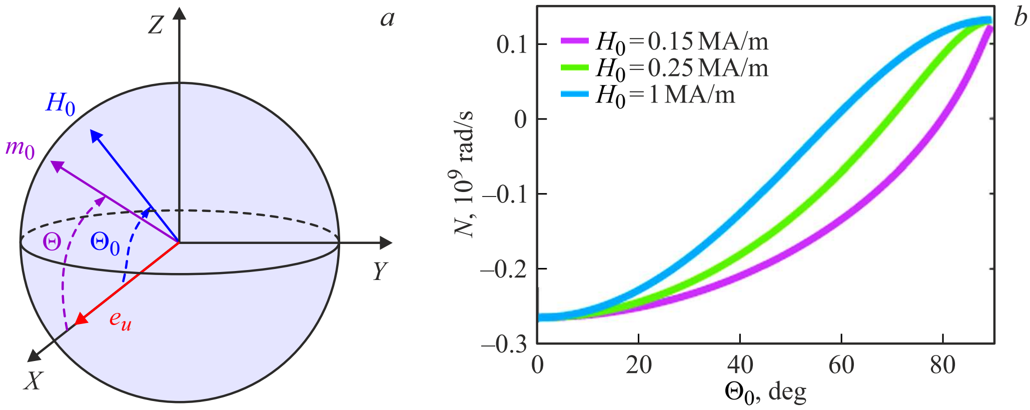

Figure 1. *a* — schematic view of a spherical ferromagnetic sample with a magnetization $\mathbf{m}$ when exposed to the magnetic field $\mathbf{H}_0$. The uniaxial anisotropy vector $\mathbf{e}_u$ is directed along OX axis. *b* — non-isochronism coefficient of a sphere versus polar angle Θ_0 at which the external magnetic field is applied.

where the precession frequency is expressed as

$$\omega\left(|c|^2\right) = \omega_0 + N|c|^2. \tag{22}$$

Here ω_0 — eigen frequency, N — nonlinearity coefficient. Thus, the nonlinearity coefficient determines the relationship between the frequency and power $|c|^2$ of magnetization precession in a ferromagnetic material. Note that a change of $|c|^2$ is possible when an ultrahigh-frequency alternating magnetic field is applied to the sample, which excites the magnetization precession [28]. In this case, at $N \neq 0$, it is possible to control the resonant frequency of a ferromagnetic nanoparticle.

2. Calculations

Yttrium-iron garnet $Y_3Fe_5O_{12}$ was chosen as the material from which ferromagnetic samples of various shapes were made. The material in question is considered one of the best among solid-state magnets, since it can be used to study the dynamics of high-frequency magnetization due to its extremely low Hilbert coefficient α, which can reach about $5 \cdot 10^{-5}$ [29]. A sphere and a circular cylinder were used as the studied geometries (Fig. 1, *a* and 2, *a*) of the ferromagnetic samples. Components of the demagnetization factor tensor for the sphere: $D_{xx} = D_{yy} = D_{zz} = 1/3$; circular cylinder: $D_{xx} = D_{yy} = 1/2$, $D_{zz} = 0$. The following magnetic parameters of the material were used for calculations: $M_s = 1.3 \cdot 10^5$ A/m, $H_u = 1.2 \cdot 10^3$ A/m, $\mathbf{e}_u = (1, 0, 0)$.

Because at different magnitudes and directions of the external magnetic field vector applied to the ferromagnetic nanoparticle, the direction of the ground state $\mathbf{m}_0$ also turns out to be different, in order to determine the non-isochronism coefficient N for a given $\mathbf{H}_0$ it is necessary first to solve the system (8) relative to the angles φ, Θ and only then use the formulae (16), (20), (A1). Therefore, when plotting the graphs shown in Fig. 1, *b*, 2, *b* and 2, *c*, for each point the ground state was found separately using the numerical solution (8).

3. Results

Let's consider how the nonlinearity of ferromagnetic samples will change at different orientations and magnitudes of the external magnetic field. Fig. 1, *b* shows the dependence of N for the sphere on the polar angle of external magnetic field $\mathbf{H}_0$. Note that N rearrangement in this case is possible only due to the presence of uniaxial magnetic anisotropy. In a homogeneous isotropic sphere, the magnetic energy at

$$\mathbf{H}_{01} = \left(H_{0_{x1}}, H_{0_{y1}}, H_{0_{z1}}\right), \quad \mathbf{m} \parallel \mathbf{H}_{01}$$

doesn't differ from the energy in state

$$\mathbf{H}_{02} = \left(H_{0_{x2}}, H_{0_{y2}}, H_{0_{z2}}\right), \quad \mathbf{m} \parallel \mathbf{H}_{02}$$

and is determined only by the applied external magnetic field. Hence, the coefficients energy expansion in terms of amplitude $|c|$ do not differ either. Moreover, it turns out that for the sphere $A = \gamma H_0$, and all other coefficients of the series for H_a (12) are zero, which means $N = 0$. In this case, when precession $\mathbf{m}$ is excited, there will be no amplitude-phase conversion of noise in the ferromagnetic sphere and a shift in the resonant frequency with a change in the amplitude of magnetization oscillations, which makes spherical elements promising frequency-selective elements for use in spintronics devices. Yet, if there is magnetic anisotropy in a spherical ferromagnetic sample, then it will be possible to achieve a zero non-isochronism coefficient using a magnetic field applied at a certain angle to the anisotropy direction vector $\mathbf{e}_u$. Figures 2, *b*, c show the dependences of N for a straight circular cylinder on the polar and azimuthal angles of the external magnetic field $\mathbf{H}_0$, respectively, at different values of this field. Let's consider the nonlinearity coefficient versus azimuth angle φ_0 at $\Theta_0 = 0$. As H_0 rises, the $N(\varphi_0)$ graph

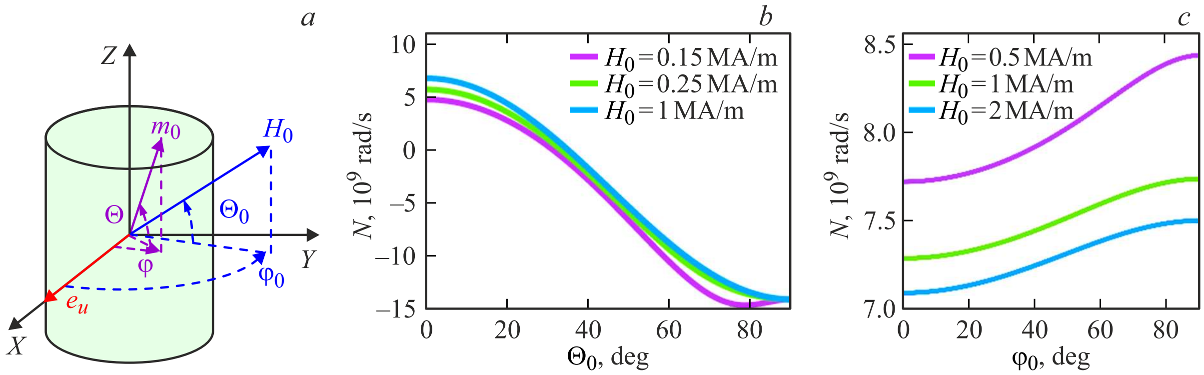

Figure 2. *a* — schematic view of a cylindrical ferromagnetic sample with a magnetization $\mathbf{m}$ when exposed to the magnetic field $\mathbf{H}_0$. The uniaxial anisotropy vector $\mathbf{e}_u$ is directed along OX axis. *b* — non-isochronism coefficient of N cylinder versus polar angle Θ_0 of the applied external magnetic field at azimuth angle $\varphi_0 = 0$. *c* — N versus azimuth angle φ_0 of external magnetic field at $\Theta_0 = 0$.

becomes flatter and non-isochronism rearrangement area $\Delta N = N_{\max} - N_{\min}$, found as the difference between the maximum and minimum values N, decreases. This is explained by the fact that at large external fields, the contribution to energy from magnetic anisotropy becomes small compared to Zeeman energy. And hence, magnetic energy in case when strong external magnetic field is applied along OX axis, i.e. $\mathbf{H}_0 = (H_0, 0, 0)$ doesn't much differ from the magnetic energy when this field is applied along OY axis i.e. when $\mathbf{H}_0 = (0, H_0, 0)$. Now let's consider the dependence $N(\Theta_0)$ at $\varphi_0 = 0$ (Fig. 2, *b*). Note that achieving zero non-isochronism for such a cylinder is also possible if an external magnetic field $\mathbf{H}_0$ is applied at a certain angle to the plane of the cylinder base. If Θ_0 is nonzero, then, unlike a spherical sample, the rearrangement of the nonlinearity coefficient is possible even in the absence of uniaxial magnetic anisotropy. This is due to the fact that the states $\mathbf{m} = (0, 0, 1)$ and $\mathbf{m} = (m_x, m_y, 0)$ differ in energy E due to the presence of anisotropic contribution from the dipole-dipole interaction to the total magnetic energy. Indeed, the tensor of demagnetizing factors $\widehat{D}$ for a ferromagnetic cylinder is non-diagonal.

Conclusion

In this paper, an analytical expression was obtained for the non-isochronism coefficient N of spherical and cylindrical ferromagnetic nanoparticles where the energy of the dipole-dipole interaction may be described using a diagonal tensor of demagnetizing factors. Taking into account uniaxial magnetic anisotropy in this expression allows us to assert that if there is magnetic anisotropy in a spherical ferromagnetic sample, then it is possible to achieve a zero coefficient of nonlinearity when applying an external magnetic field $\mathbf{H}_0$ at some angle to the direction vector of this anisotropy. In addition, it was shown that the magnetization fluctuations in a homogeneous isotropic sphere are isochronous. For a sample having a straight circular cylinder shape, the analysis of the expression obtained showed that in such a geometry it is possible to achieve a zero coefficient of nonlinearity when an external magnetic field is applied at a certain angle to the plane of the cylinder base.

Funding

The study was supported financially by the Ministry of Science and Higher Education of the Russian Federation (agreement № 075-15-2024-538).

Conflict of interest

The authors of this paper declare that they have no conflict of interest.

References

[1] S.A. Nikitov, A.R. Safin, D.V. Kalyabin, A.V. Sadovnikov, E.N. Beginin, M.V. Logunov, M.A. Morozova, S.A. Odintsov, S.A. Osokin, A.Yu. Sharaevskaya, Yu. P. Sharaevsky, A.I. Kirilyuk, UFN, **190** (10), 1009 (2020). DOI: 10.3367/UFNr.2019.07.038609

[2] M.E. Seleznev, Yu.V. Nikulin, V.K. Sakharov, Yu.V. Khivintsev, A.V. Kozhevnikov, S.L. Vysotsky, Y.A. Filimonov, ZhTF, **91** (10), 1504 (2021). DOI: 10.21883/JTF.2021.10.51363.136-21

[3] B.A. Kalinikos, A.B. Ustinov, S.A. Baruzdin, *Spin-volnovye ustroistva i ekho-protsessory* (Radiotekhnika, M., 2013) (in Russian).

[4] V.M. Gevorgyan, V.N. Kochemasov, A.R. Safin. Elektronika: nauka, technologia, biznes, **227** (6), 76 (2023) (in Russian). DOI: 10.22184/1992-4178.2023.227.6.76.89

[5] J. Krupka, B. Salski, P. Kopyt, G. Wojciech. Sci. Rep., **6** (1), 34739 (2016). DOI: 10.1038/srep34739

[6] R. Morris, A. van Loo, S. Kosen, A. Karenowska. Sci Rep., **7** (1), 11511 (2017). DOI: 10.1038/s41598-017-11835-4

[7] D.O. Krivulin, I.Yu. Pashenkin, R.V. Gorev, P.A. Yunin, M.V. Sapozhnikov, A.V. Grunin, S.A. Zakharova, V.N. Leontiev, ZhTF, **93** (7), 907 (2023). DOI: 10.21883/JTF.2023.07.55744.72-23

[8] N.V. Ostrovskaya, V.A. Skidanov, Yu.A. Yusipova. ZhTF, **93** (5), 687 (2023) (in Russian). DOI: 10.21883/JTF.2023.05.55464.250-22
[9] S.L. Vysotsky, A.V. Kozhevnikov, Yu.A. Filimonov. FTT, **63** (9), 1258 (2021) (in Russian). DOI: 10.21883/FTT.2021.09.51249.02H
[10] D.A. Gabrielyan, D.A. Volkov, E.E. Kozlova, A.R. Safin, D.V. Kalyabin, A.A. Klimov, V.L. Preobrazhensky, M.B. Strugatsky, S.V. Yagupov, I.E. Moskal, G.A. Ovsyannikov, S.A. Nikitov. J. Phys. D: Appl. Phys., **57**, 305003 (2024). DOI: 10.1088/1361-6463/ad3f28
[11] A.Yu. Mitrofanova, A.R. Safin, D.P. Egorov, O.V. Kravchenko, N.I. Bazenkov. *2021 Photonics & Electromagnetics Research Symposium (PIERS)* (Hangzhou, China, 2021), p. 2568–2572. DOI: 10.1109/PIERS53385.2021.9694780
[12] C. Navau, J. Prat-Camps, O. Romero-Isart, J. Cirac, A. Sanchez. Phys. Rev. Lett., **112**, 253901 (2014). DOI: 10.1103/PhysRevLett.112.253901
[13] P. Fischer, D. Sanz-Hernández, R. Streubel, A. Fernández-Pacheco. APL Mater., **8** (1), 010701 (2020). DOI: 10.1063/1.5134474
[14] A. Slavin, V. Tiberkevich. IEEE TMAG, **44** (7), 1916 (2008). DOI: 10.1109/TMAG.2008.924537
[15] A.A. Matveev, A.R. Safin, S.A. Nikitov. JMMM, **592** (7), 171825 (2024). DOI: 10.1016/j.jmmm.2024.171825
[16] L.D. Landau, E.M. Lifshitz. Phys. Z. Sowietunion, **8** (1), 153 (1935).
[17] T.L. Gilbert. IEEE Trans. Magn., **40** (6), 3443 (2004). DOI: 10.1109/tmag.2004.836740
[18] Yu.K. Fetisov, A.V. Makovkin. ZhTF, **71** (1), 86 (2001). (in Russian).
[19] V.A. Orlov, V.S. Prokopenko, R.Yu. Rudenko, I.N. Orlova. ZhTF **67** (4), 609 (2022). DOI: 10.21883/JTF.2022.04.52243.273-21
[20] C. Vittoria, H. Lessoff, N. Wilsey. IEEE Transactions on Magnetics, **8** (3), 273 (1972). DOI: 10.1109/TMAG.1972.1067301
[21] A. Newell, W. Williams, D. Dunlop. J. Geophys. Res., **98**, 9551 (1993). DOI: 10.1029/93JB00694
[22] P. Krivosik, C. Patton. Phys. Rev. B, **82** (18), 184428 (2010). DOI: 10.1103/PhysRevB.82.184428
[23] E.A. Kuznetsov. J. Exp. Theor. Phys., **93** (5), 1052 (2001). DOI: 10.1134/1.1427116
[24] V. Zakharov, S. Nazarenko. Physica D, **201** (3), 203 (2005). DOI: 10.1016/j.physd.2004.11.017
[25] Y. Lvov, E. Tabak. Physica D, **195** (1), 106 (2004). DOI: 10.1016/j.physd.2004.03.010
[26] T. Holstein, H. Primakoff. Phys. Rev., **58** (12), 1098 (1940). DOI: 10.1103/PhysRev.58.1098
[27] N.N. Bogoljubov. Il Nuovo Cimento, **7** (6), 794 (1958). DOI: 10.1007/bf02745585
[28] K. Wagner, L. Körber, S. Stienen, J. Lindner, M. Farle, A. Kákay. IEEE Magn. Lett., **12**, 6100205 (2021). DOI: 10.1109/LMAG.2021.3055447
[29] S. Klingler, H. Maier-Flaig, C. Dubs, O. Surzhenko, R. Gross, H. Huebl, S.T.B. Goennenwein, M. Weiler. Appl. Phys. Lett., **110** (9), 092409 (2017). DOI: 10.1063/1.4977423

Translated by T.Zorina

Appendix A

The coefficients of Hamiltonian expansion H_a (13) in a series with respect to the complex amplitude a have the form

$$A = \frac{1}{2}\gamma\big(H_u\cos(\varphi)^2\cos(\theta)^2 - D_{xx}M_s\cos(\varphi)^2\cos(\theta)^2 + M_sD_{yy}\cos(\varphi)^2\cos(\theta)^2 - M_sD_{yy}\cos(\theta)^2 + D_{zz}M_s\cos(\theta)^2 - D_{xx}M_s + M_sD_{yy} + 2H - H_u\big),$$

$$B = \frac{1}{2}\gamma\big(iH_u\sin(2\varphi)\sin(\theta) + iM_sD_{yy}\sin(2\varphi)\sin(\theta) + H_u\cos(\varphi)^2\cos(\theta)^2 - D_{xx}M_s\cos(\varphi)^2\cos(\theta)^2 - iM_sD_{xx}\sin(2\varphi)\sin(\theta) + M_sD_{yy}\cos(\varphi)^2\cos(\theta)^2 - 2H_u\cos(\varphi)^2 + 2D_{xx}M_s\cos(\varphi)^2 - 2M_sD_{yy}\cos(\varphi)^2 - M_sD_{yy}\cos(\theta)^2 + D_{zz}M_s\cos(\theta)^2 + H_u - D_{xx}M_s + M_sD_{yy}\big),$$

$$V = -\gamma\cos(\theta)\big(iH_u\sin(\varphi)\cos(\varphi) - H_u\sin(\theta)\cos(\varphi)^2 - iM_sD_{xx}\sin(\varphi)\cos(\varphi) + D_{xx}M_s\sin(\theta)\cos(\varphi)^2 + iM_sD_{yy}\sin(\varphi)\cos(\varphi) - M_sD_{yy}\sin(\theta)\cos(\varphi)^2 + M_sD_{yy}\sin(\theta) - M_sD_{zz}\sin(\theta)\big),$$

$$U_1 = \frac{1}{2}\gamma\big(-3H_u\cos(\varphi)^2\cos(\theta)^2 + 3D_{xx}M_s\cos(\varphi)^2\cos(\theta)^2 - 3M_sD_{yy}\cos(\varphi)^2\cos(\theta)^2 + 3M_sD_{yy}\cos(\theta)^2 - 3D_{zz}M_s\cos(\theta)^2 - D_{xx}M_s - M_sD_{yy} + 2M_sD_{zz} + H_u\big),$$

$$U_2 = -\frac{1}{4}\gamma\big(iH_u\sin(2\varphi)\sin(\theta) + iM_sD_{yy}\sin(2\varphi)\sin(\theta) + H_u\cos(\varphi)^2\cos(\theta)^2 - D_{xx}M_s\cos(\varphi)^2\cos(\theta)^2 - iM_sD_{xx}\sin(2\varphi)\sin(\theta) + M_sD_{yy}\cos(\varphi)^2\cos(\theta)^2 - 2H_u\cos(\varphi)^2 + 2D_{xx}M_s\cos(\varphi)^2 - 2M_sD_{yy}\cos(\varphi)^2 - M_sD_{yy}\cos(\theta)^2 + D_{zz}M_s\cos(\theta)^2 + H_u - D_{xx}M_s + M_sD_{yy}\big).$$