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Depairing current and diode effect in a bridge with disordered boundary layers

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The superconducting state of narrow bridges with disordered normal metal layers at the boundaries is considered. The critical current and magnetic field of these structures are calculated within the framework of the Ginzburg-Landau theory. The diode effect in such bridges is studied. The estimates of the depairing current made within the framework of the presented model are in quantitative agreement with the experimental data for Nb and NbN bridges. It is shown that the average diode efficiency in the considered case can reach 18 %.

Keywords: superconducting bridges, critical current, Ginzburg-Landau theory, diode effect.

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1. Introduction

Superconducting structures hold great promises for application in electronics, therefore today they have become the object of many theoretical and experimental papers [1–20]. Their properties are determined by the material that they are made of, dimensions, shape etc. Critical current — is one of the key parameters of superconducting structures. For large-size structures, if there is sufficient strong magnetic field, critical current is determined by movement of Abrikosov vortices. In rather thin and narrow bridges (bridge thickness does not exceed 1.81ξ, where ξ — coherence length of superconductor that the bridge is made of [21]) critical current is related to depairing of Copper pairs, in other words, it is Ginzburg-Landau (GL) depairing current.

For the first time the equation for thin superconducting film depairing current density was obtained in paper [22]. The results of the paper and more recent analytical calculations for depairing current are used to estimate the critical current of superconducting structures with the corresponding sizes. For example, in paper [10] the authors showed that for thin (with thickness of $D \approx \xi(0)$, $\xi(0)$ — coherence length at zero temperature) and narrow superconducting bridges from Nb and NbN the analytical estimates of critical current exceed experimental values by 30−50 %. Such differences was related by the authors to formation of a thin non-superconducting metal layer on the surface of bridges in process of manufacture. In paper [13] the decrease of experimentally determined critical parameters of thin niobium bridges compared to the theoretical estimates was related to disordering of metal layers in the interface of superconductor/substrate.

The accuracy of critical current density estimates is important for correct description of diode effect (DE). The essence of this effect consists in changing the critical current value with the change of its flow direction [15,23,24]. DE is observed both in parallel and perpendicular orientation of external magnetic field relative to the surface of bridges. Several DE mechanisms are assumed in such structures. Some of them are related to electron spins [10,15,16,25]. In most cases the main role is played by spin-orbit coupling of electrons. Nevertheless, in paper [25] they described a situation, when the presence of finite moments in electrons was not due to spin-orbit coupling, but change of density of Cooper pairs by thickness of S-N (superconductor−normal metal) structure. Other mechanisms explain DE by asymmetry of superconducting structures [7,19], related, for example, to change of the surface layers relative to the volume ones. It should be noted that asymmetrical structures demonstrate higher diode efficiency (up to 65 %), compared to those where spin mechanism prevails [19]. It should be noted separately that paper [26] presents a structure based on graphene, which demonstrates DE in absence of the magnetic field.

Disordering of layers on the surface of superconducting structures may play an important role in determination of their critical current and explanation of DE [10,13,19]. This paper presents a model making it possible within the GL theory to estimate the depairing current for thin and narrow superconducting bridges, and transport current flow in adjacent non-superconducting strongly disordered layers is taken into account, too. The proposed model makes it possible to establish interrelation between the thickness of bordering non-superconducting layers and value of critical parameters. The critical state of thin and narrow superconducting bridge being a layered N-S-N (normal metal−superconductor−normal metal) structure, was analyzed. The characteristics of this structure were described in the context of its use as a superconducting diode.

2. Model description

A narrow and thin bridge is considered with thickness of D. The geometry of the problem, as well as the directions of the transport current I_t flowing through the structure and the external magnetic field $\mathbf{H}$ are shown in the insert in Figure 1. A temperature range is considered, where the following conditions are met: bridge thickness $D < 1.81\xi$, and width does not exceed the magnetic field penetration depth λ. In this case the superconducting bridge is in vortex-free state, and superconducting current is distributed homogeneously along its width. In this problem the vector potential is $\mathbf{A} = \mathbf{e}_y A(x)$.

The superconducting bridge is assumed to be inhomogeneous in thickness (along axis x), which complies with real superconducting structures [27]. Due to the difference from the bulk conditions, layers with multiple defects are formed at the boundaries substrate/superconducting material and material/environment [28,29]. This, in its turn, decreases the mean free path (l) when approaching the boundaries (see dependence $l(x)$ in Figure 1). For the purposes of this paper an assumption is made that if the mean free path is less than a certain threshold value l_{norm}, the area, where this condition is met, is in the normal state (grey areas in Figure 1). Thicknesses of disordered non-superconducting layers at the interface of substrate/bridge and bridge/environment are indicated as δ_s and δ_e accordingly.

Change of the boundary layers condition is taken into account via a model distribution of mean free path along the thickness of superconducting layer [30]:

$$l(x) = l_{Cn}\left(1 - 2^{n-2}\eta\left(\frac{x}{D} - \frac{D+\delta_e-\delta_s}{2D}\right)^n\right), \quad (1)$$

where l_{Cn} is mean free path in the center of superconducting layer, and η is parameter showing the difference between values l in the center of the layer and at the boundaries of the bridge. The homogeneous superconducting bridge is compliant with $\eta = 0$ (in this case $l(x) = l_{Cn}$). In other words, η specifies the degree of inhomogeneity of the superconducting structure. The parameter of degree n sets the sharpness of change $l(x)$ near the structure boundaries. The calculations will use values of parameters $\eta = 3.9$ and $n = 8$, and the mean free path will change slightly in the center of the bridge and decrease sharply when approaching the boundaries (Figure 1).

In case of strong disordering of the layers at the bridge boundaries their critical temperature may decrease. It should be noted here that Anderson theorem states the absence of impact at critical temperature from non-magnetic scattering in superconductor with s-type of pairing. At the same time, this statement has certain additional restrictions [31–34]. The critical temperature of the superconductor may vary if there are impurities present in case of very strong disordering [31,32] and in thin superconducting films and bridges [31]. In the last case several mechanisms of impurities impact at the critical temperature of the structures depending on their thickness are considered. The described factors are directly related to the problem considered in this paper. A situation is considered, when surface layers will be in the normal state (due to lower critical temperature), and the central layer will be superconducting.

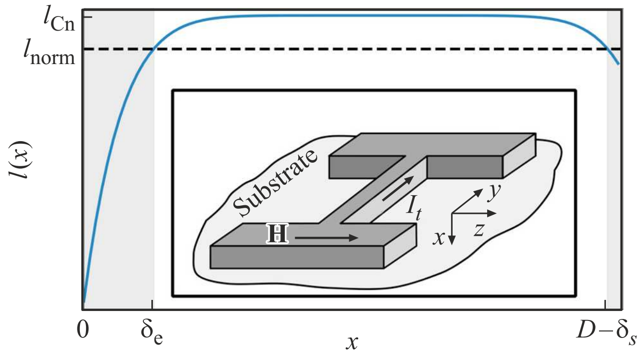

Figure 1. Distribution of mean free path $l(x)$ along bridge thickness. Insert — problem geometry.

The free energy functional for the studied structure is:

$$F \propto \int_0^D \Bigg[-a_1(T)|\Psi(x)|^2 + \frac{a_2}{2}|\Psi(x)|^4 + b(x)\left(\left|\frac{\partial\Psi}{\partial x}\right|^2 + \frac{4e^2}{c^2}A(x)^2|\Psi(x)|^2\right) + \frac{(\partial A/\partial x - H)^2}{8\pi}\Bigg]dx, \quad (2)$$

where Ψ is order parameter, e is electron charge, c is speed of light, T is temperature of superconducting bridge, a_{1-2} and b are coefficients in the expansion of the free energy functional. Temperature dependence $a_1(T)$ is as follows:

$$a_1 = \alpha_1\left(1 - \frac{T}{T_{CM}}\right), \quad (3)$$

where T_{CM} is critical temperature of massive superconductor, from which the corresponding layer is made. In turn, the expansion coefficient $b(x)$ in the „dirty limit" $l \ll \xi_0$, where ξ_0 is the coherence length in a pure superconductor in the BCS theory is proportional to the mean free path (l) [35]. Taking into account (1), dependence $b(x)$ will be as follows:

$$b(x) = b_{Cn}\left(1 - 64\cdot\eta\left(\frac{x}{D} - \frac{D+\delta_e-\delta_s}{2D}\right)^8\right), \quad (4)$$

where b_{Cn} is coefficient not depending on x.

It should be noted that since the critical temperature of superconducting central layer (T_{CMs}) and surface disordered layers (T_{CMn}) differs, the values of coefficient $a_1(T)$ in these areas will differ due to different T_{CM}.

Variation of free energy functional (2) by order parameter and vector potential makes it possible to obtain the following

GL equations:

$$\psi - \frac{p(T/T_{CMs})}{p(T/T_{CMs,n})}\left[\psi^3 - g(x_\xi)\frac{\partial^2\psi}{\partial x_\xi^2} - \frac{\partial g(x_\xi)}{\partial x_\xi}\frac{\partial\psi}{\partial x_\xi} + \frac{U^2}{\kappa_{Cn}^2}g(x_\xi)\psi\right] = 0, \tag{5}$$

$$\frac{\partial^2 U}{\partial x_\xi^2} - \frac{\psi^2}{\kappa_{Cn}^2}g(x_\xi)U = 0, \tag{6}$$

where $p(T/T_{CMs}) = (1 - T/T_{CMs})$, $p(T/T_{CMs,n}) = (1 - T/T_{CMs,n})$ ($T_{CMs,n} = T_{CMs}$ in the superconducting layer and $T_{CMs,n} = T_{CMn}$ in the surface layers),

$$g(x_\xi) = \left(1 - 64\cdot\eta\left(\frac{x_\xi}{d} - \frac{d + \delta_{e\xi} - \delta_{s\xi}}{2d}\right)^8\right),$$

ψ is normalized order parameter: $\psi=\Psi/\Psi_0$, $\Psi_0=\sqrt{a_1/a_2}$ — order parameter in massive superconductor in absence of external magnetic field (Ψ_0 complies with the central superconducting layer), κ_{Cn} — GL parameter in the center of the superconducting layer. Besides, the dimensionless values are introduced:

$$x_\xi = \frac{x}{\xi_{Cn}},\ d = \frac{D}{\xi_{Cn}},\ \delta_{s\xi} = \frac{\delta_s}{\xi_{Cn}},\ \delta_{e\xi} = \delta_e/\xi_{Cn},$$

$$U = \frac{2\pi\kappa_{Cn}\xi_{Cn}}{\phi_0}A,$$

where ϕ_0 is magnetic flux quantum, ξ_{Cn} is GL coherence length in the center of the superconducting layer, and $\xi_{Cn} = \sqrt{b_{Cn}/a_1(T/T_{CMs})}$. All further values of length and thickness are presented as units of coherence length in the center of the superconducting layer ξ_{Cn}. When deriving the equations, the calibration of the vector potential $\mathrm{div}\mathbf{A} = 0$ was used.

In this paper the calculations are made for temperature $T = 0.95T_{CMs}$. In turn, the average over the thickness critical temperature of the disordered layers is $T_{CMn} = 0.9T_{CMs}$. Since the critical temperature of disordered layers is less than the bridge temperature ($T > T_{CMn}$), they are in normal state.

Additionally a case is considered, when the disordered surface layers are in superconducting state ($T_{CMn} = T_{CMs}$). This makes it possible to separately analyze the effect of inhomogeneity of superconducting layers and proximity effect occurring when the layers are in normal state.

Let us discuss the boundary conditions to equations (5) and (6). Since the transport current I_t in the bridge creates a magnetic field:

$$H_I = \frac{2\pi}{c}I_t, \tag{7}$$

then the full field near the surfaces of the bridge (at the interface with the substrate and the environment) is equal to $H \pm H_I$, and boundary conditions for equation (6) are as follows:

$$\left.\frac{\partial U}{\partial x}\right|_{sub} = h + h_I,\quad \left.\frac{\partial U}{\partial x}\right|_{env} = h - h_I, \tag{8}$$

where

$$h = \frac{H}{H_\xi},\quad h_I = \frac{H_I}{H_\xi},\quad H_\xi = \frac{\phi_0}{2\pi\kappa_{Cn}^2\xi_{Cn}^2}.$$

At the interface of superconducting layer/disordered layers the boundary conditions to equation (6) will meet the condition of continuity of the magnetic field and vector potential. In its turn the order parameter and its derivative will also be continuous at the boundary between the superconducting and non-superconducting layers [36]. This determines the boundary conditions to equation (5).

All the below values of magnetic field are given in units H_ξ (see (8)). The current values within the model are represented in terms of H_I (7) and therefore, like the magnetic field, are expressed in units of H_ξ. The iterative procedure for solving the system of equations (5), (6) with boundary conditions is similar to that given in paper [37].

3. Results of numerical calculations

Let us consider the dependences of critical current (I_c) on external magnetic field (h) for a superconducting bridge with thickness of $d = 1.5$ (Figure 2, a). Note that solid lines comply with the structure with disordered layers on the surface, and a blue line (indicated as I_{NL+inh}) corresponds to the non-superconducting disordered layers, and the orange one — to the superconducting ones (indicated as I_{inh}). Thicknesses of inhomogeneous surface layers for boundaries of bridge/environment and bridge/substrate are equal to $0.13\xi(0)/0.029\xi(0.95T_{CMs})$ and $0.03\xi(0)/0.007\xi(0.95T_{CMs})$ accordingly, which corresponds to the experimentally derived values for niobium films [10,13]. A black dashed line (indicated as I_{dep}) shows the estimate of the depairing current for the structure of homogeneous thickness (in this case $\eta = 0$, and surface non-superconducting layers are not available). The analysis shows that accounting for only inhomogeneity of surface layers (orange curve in Figure 2, a) results in the decrease of the critical current value, and the value of the critical magnetic field will increase at the same time (h_c, point of curves crossing $I_c(h)$ with axis h). If you additionally take into account the fact that the disordered surface layers are in normal state (blue line in Figure 2, a), this will result to considerable decrease of the critical current. In its turn h_c will become a bit smaller than in the case of disordered superconducting surface layers.

To clarify the described phenomena, let us consider the change of the order parameter $\psi(x_\xi)$ under action of transport current I_t and external magnetic field h. In the absence of the external magnetic field, when the transport current flows (Figure 2, b), the order parameter of the structure with disordered superconducting surface layers (curve indicated as I_{inh}) will be lower than in the bridge of homogeneous thickness (curve indicated as I_{dep}). If the surface layers are in normal state, this will result in decrease of the order parameter (see curve I_{NL+inh}). Therefore, the

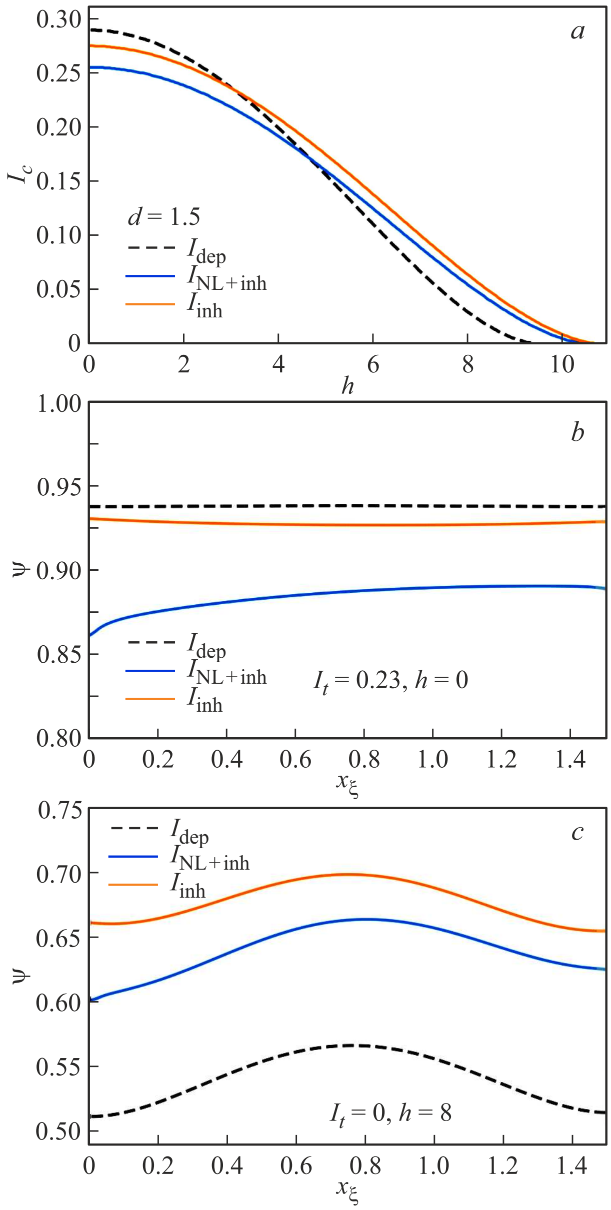

Figure 2. *a* — dependences of critical current I_C on external magnetic field h for bridge with thickness of $d = 1.5$. Solid lines comply with the structure with the disordered layers on its surface. The blue line corresponds to the non-superconducting disordered layers, and the orange one — to the superconducting ones. The black dashed line shows the estimates of the depairing current for the structure of homogeneous thickness. *b–c* — distribution of order parameter ψ along the bridge thickness ($d = 1.5$), obtained for various values of transport current I_t and external magnetic field h. For all figures the GL parameter in the center of the superconducting layer is $\kappa_{Cn} = 2$. Other parameters of calculations are shown in the figures. The grey area corresponds to disordered layers.

order parameter in the structure with the surface disordered layers is suppressed stronger by the transport current than in the homogeneous structure. Besides, if the surface layers are in normal state, then ψ is suppressed stronger. As a result the critical current in the bridge with disordered layers is lower than the depairing current (I_{dep}) and reaches the least value, if the disordered surface layers are in non-superconducting state.

In the area of a thicker surface layer (at the boundary with the environment, at x_ξ close to 0, see Figure 1) there is a fracture of the order parameter (blue line I_{NL+inh} in Figure 2, *b*). It is caused by the fact that the surface layer is in the normal state. In the area of a thinner surface layer (at the boundary with the substrate, at x_ξ near 1.5, Figure 1) no such fracture is observed, which is related to a very small thickness of the layer.

Under the action of the external magnetic field in absence of the transport current the situation will be the opposite: at the same values h the order parameter $\psi(x_\xi)$ of the bridge with the surface inhomogeneous layers will be higher than in the homogeneous structure (Figure 2, *c*). Therefore, the superconducting state of the bridge with the disordered surface layers is more resistant to suppression with the external magnetic field. In other words, for the inhomogeneous bridge the value of the critical magnetic field will be higher than for the homogeneous one. This statement is true both for superconducting and non-superconducting disordered layers. At the same time in the case of non-superconducting layers h_c will be slightly lower than in the case of superconducting ones.

Let us analyze the dependences of the critical current on the external magnetic field for the superconducting bridges of various thickness ($d = 0.25$, 0.5 and 1). Figure 3 presents the comparison of the calculation results I_c for the bridges with the disordered non-superconducting surface layers with the estimates of the depairing current for the homogeneous bridges. The calculations have shown that the accounting of the surface layers results in reduction of the critical current relative to the depairing current values by 12–44 % (Figures 2, *a* and 3). Besides, the thinner the bridge, the higher is that reduction. This measurement coincides with the one determined experimentally for the bridges from niobium and niobium-based materials [10]. Therefore, despite the small thickness of disordered surface layers in respect to the bridge, they impact substantially the critical current of the structure.

With the growth of the magnetic field, the difference between I_c of the bridges with surface disordered layers and the depairing current for the homogeneous structures becomes less and less, and at certain value of h they may even become equal (Figure 3, *b*). Critical magnetic field h_c near the bridges with the non-superconducting surface layers may be higher (case $d = 1.5$ and $d = 1$), may coincide ($d = 0.5$) or be lower ($d = 0.25$) than in the homogeneous bridges. Previously papers [38,39] studied the impact of the proximity effect (for the case of normal layers with thickness of at least the coherence length ξ) and inhomogeneity at the critical parameters of the superconducting bridges. It turned out that both factors suppress the critical current of the bridge substantially, and affect the critical magnetic field in a different manner: inhomogeneity increases it,

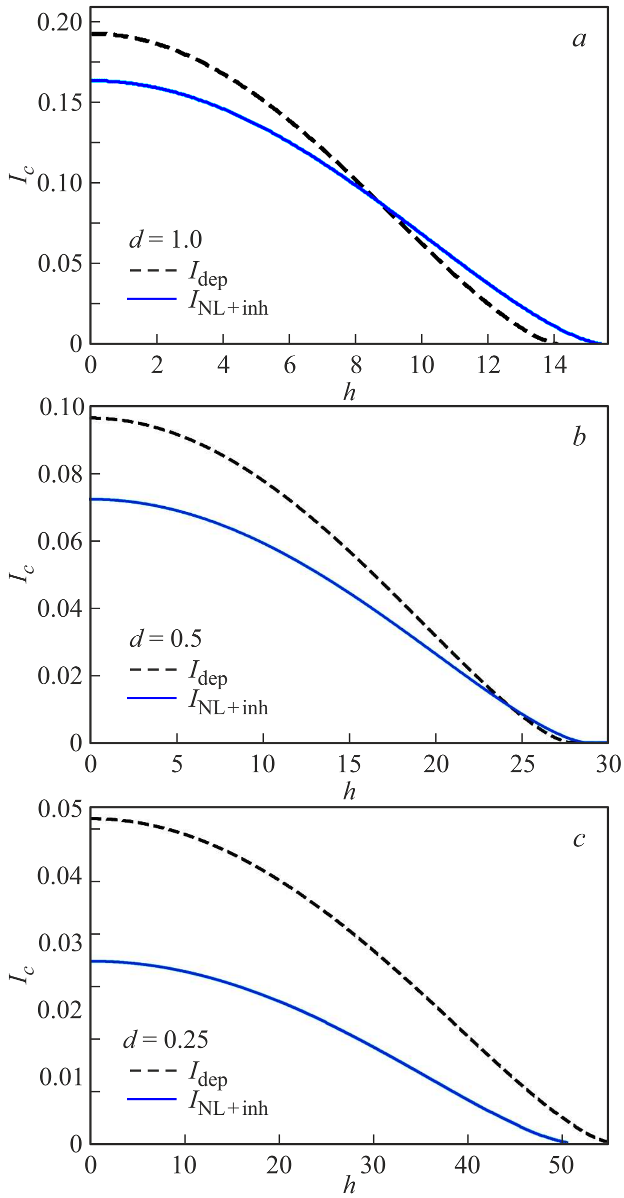

Figure 3. Dependences of critical current I_C on external magnetic field h for bridges with disordered non-superconducting layers on their surface (solid blue lines). The black dashed lines show the estimates of the depairing current for the structures of homogeneous thickness. Thicknesses of bridges are indicated in the figures ($d = 1$ (a), $d = 0.5$ (b) and $d = 0.25$ (c)). The GL parameter in the center of the superconducting layer is $\kappa_{Cn} = 2$.

and the proximity effect reduces it. Boundary disordered layers impact the critical parameters of bridges both due to proximity effect and the structure inhomogeneity it creates. The changes of the critical magnetic field described in this paper show that both factors must be taken into account in the case above. At the same time, the thicker bridges demonstrate prevalence of structure inhomogeneity and in the thinner ones — the proximity effect.

The structure with disordered boundary layers is not symmetrical. Besides, the magnetic field developed by the transport current on one of the surfaces matches the direction of the external magnetic field and is directed oppositely on the other one (8). In this case one should expect that the change in the direction of the transport current may cause the change of the critical current value. Therefore, a diode effect should be observed for the considered structure. Figure 4 shows the dependence $I_c(h)$ for a bridge with disordered surface layers at different directions of the transport current. Current I_+ flows along the positive direction of axis y, I_- — to the opposite side. The structure is considered with both superconducting (orange and red lines I_{inh} in Figure 4) and non-superconducting (blue and pink lines I_{NL+inh} in Figure 4) disordered layers. As the direction of the current changes in the magnetic field h, the critical current value changes. Let us introduce the diode efficiency $\varepsilon = (I_- - I_+)/I_+$, characterizing the change of the critical current upon the direction change. The average diode efficiency for the considered structure with thickness of $d = 0.25$ is 18 %. At the same time, the average diode efficiency practically does not differ from the surface superconducting and non-superconducting layers. For the bridges of larger thickness the diode effect is also observed, but the value ε will be smaller.

The reason for change in the critical current value is asymmetry of the bridge and the difference of boundary conditions to the magnetic field (8) at its boundaries. In one of the boundaries the magnetic field created by the transport current increases the external magnetic field h, and in the other one — decreases it. In a special case of the external magnetic field absence the boundary conditions (8) in the boundaries of the bridge are same. Therefore, I_+ and I_- are not different at $h = 0$.

Figure 5, $a-c$ shows distributions of the order parameter by thickness of bridge $\psi(x_\xi)$ at various values of the

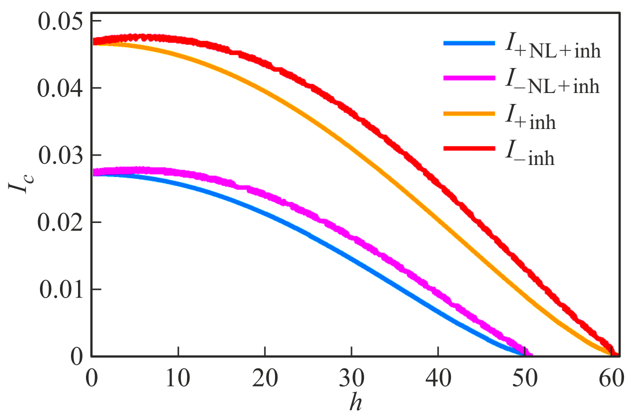

Figure 4. Dependences of critical current I_C on external magnetic field h for bridge with disordered superconducting (orange and red lines I_{inh}) and non-superconducting (blue and pink lines I_{NL+inh}) layers on its surface. Blue and orange lines correspond to the current flowing in the positive direction of axis y (I_+), pink and red ones correspond to the current in the opposite direction (I_-). Calculations were made for the bridge with thickness $d = 0.25$. The GL parameter in the center of the superconducting layer is $\kappa_{Cn} = 2$.

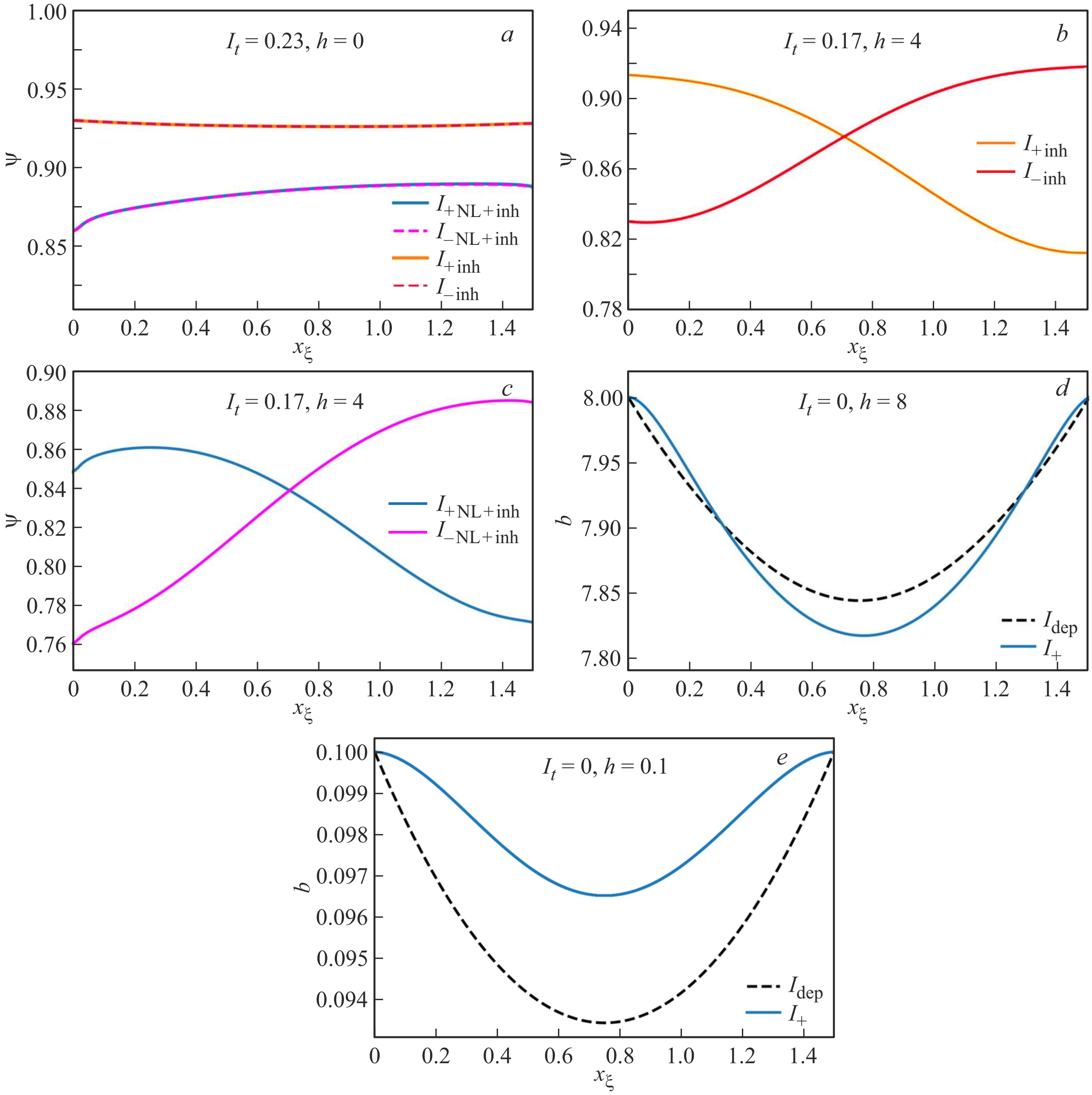

Figure 5. *a–c* — distributions of order parameter ψ along bridge thickness ($d = 1.5$), produced for different values of transport current I_t and external magnetic field *h*. *d–e* — distributions of magnetic field *b* along bridge thickness ($d = 1.5$) for different values I_t and *h*. Figure (*e*) for $n = 2$, $T = 0.99T_{CMs}$ and $T_{CMn} = 0.9T_{CMs}$. The GL parameter in the center of the superconducting layer is $\kappa_{Cn} = 2$. Other parameters of calculations are shown in the figures.

transport current I_t and external magnetic field h for a bridge with thickness of $d = 1.5$. In the absence of the external magnetic field at the same values of I_t the order parameter will be practically same for I_+ and I_- (Figure 5, *a*). This is true both for superconducting and non-superconducting disordered surface layers. Therefore, upon current reversal, the critical current value in the absence of the magnetic field will not change.

Let us consider the suppression of the superconducting state with transport current and external magnetic field in a bridge with inhomogeneous surface layers. The corresponding distributions $\psi(x_\xi)$ for the cases I_+ and I_- are presented in Figure 5, *b* (non-superconducting surface layers) and Figure 5, *c* (superconducting surface layers). The difference between the superconducting and non-superconducting layers is only observed at the interface with the environment and is a fracture of the order parameter. In both cases $\psi(x_\xi)$ for I_- is asymmetric to distributions I_+, which is expressed in the location of the maximum near the other boundary of the bridge. Quantitatively the order parameter differs for I_+ and I_-. This, in turn, is the reason for the difference of critical currents I_+ and I_-.

In the absence of the transport current and at values h, close to the critical field, let us consider the distribution of the magnetic field $b(x_\xi)$ along the structure thickness (Figure 5, d). In vortex-free case, when the field penetrates the homogeneous bridge, it starts decreasing and reaches its minimum value in its center. However, in contrast to the ordinary Meissner effect, the field will not disappear completely, which is due to the relatively small thickness of the structure. If there are boundary non-superconducting disordered layers, the magnetic field will hardly vary therein, as a result of which the distribution $b(x_\xi)$ becomes asymmetric.

According to the numerical calculations, at low values of the magnetic field (let us consider $h = 0.1$) and very thin surface disordered layers ($\delta_e = 0.02\xi(0)$ and $\delta_s = 0.01\xi(0)$) the order parameter ψ hardly varies along the structure thickness. In this case the distribution of the field $b(x_\xi)$ may be obtained using analytical estimates. Let us additionally consider the distribution of mean free path $l(x)(1)$ with $n = 2$. From the qualitative point of view, the dependences $l(x)$ are similar for $n = 2$ and $n = 8$, but in the latter case the mean free path changes more drastically near the boundaries of the structure [30]. Let us also assume that $T = 0.99T_{CMs}$, and $T_{CMn} = 0.9T_{CMs}$. For a homogeneous bridge $b(x_\xi)$ will be determined by the known ratio for a thin plate [40]:

$$b(x_\xi) = \frac{h\big(\mathrm{sh}(\psi d/\kappa_{Cn})\,\mathrm{ch}(\psi x_\xi/\kappa_{Cn}) + + (1 - \mathrm{ch}(\psi d/\kappa_{Cn}))\,\mathrm{sh}(\psi x_\xi/\kappa_{Cn})\big)}{\mathrm{sh}(\psi d/\kappa_{Cn})}$$

$$\approx 0.121608\big(0.822317\,\mathrm{ch}(0.5x_\xi) - 0.294683\,\mathrm{sh}(0.5x_\xi)\big).$$

For a inhomogeneous bridge, equation (6) will look like a parabolic cylinder equation. Distribution $b(x_\xi)$ will be determined by „odd“ solution for $U(x_\xi)$:

$$b(x_\xi) = \frac{\partial U(x_\xi)}{\partial x_\xi}$$

$$\approx 0.096501 \sum_{n=0}^{\infty} \alpha_n \frac{(1.142816x_\xi - 0.857679)^{2n}}{(2n)!},$$

where coefficients $\alpha_0 = 1$, $\alpha_1 = 0.188369$, the other α_n are related by recurrent relations [41]. The produced analytical estimates for the field distribution $b(x_\xi)$ are illustrated in Figure 4, e. The presented results match the obtained numerical calculations.

The results of this paper may be also applied to the structures in the magnetic field perpendicular to the surface. And their dimensions must meet the condition of no vortices in these structures. Additionally the paper considers the disordered surface layers, which are in the normal state, but are superconductors at the same time. A situation is considered, when the bridges are found at temperatures that are higher than the critical temperatures of these layers. GL theory is not applicable for the normal layers that are not superconductors. This is due to the fact that GL theory may only be used in a certain proximity of the critical temperature. Nevertheless, qualitatively the results of this paper may be spread on the normal surface layers that are not superconducting.

To conclude the section, we would like to note that the experimental proof of DE mechanisms caused by electron spins [10,15,16,25] requires development of the structures, where the depairing current will be achievable [42]. In turn, as it is discussed in this paper and in [10], the depairing current as such is sensitive to the structure of superconducting bridges, especially taking into account the fact that the depairing current is achievable in rather thin and narrow bridges. On the other hand, the structural inhomogeneities may themselves contribute to DE. The accounting of such contribution is important in the study of other DE mechanisms, including those related to the electron spins.

4. Conclusion

The paper considers the critical state of the thin and narrow superconducting bridge with the metal layers disordered on the surface. Relying on the experimental research of the structures from niobium and niobium-based materials, a case was considered, when the disordered layers at the interface of substrate/superconductor and superconductor/environment (vacuum, cooling agent) differ by thickness. With the help of the numerical calculations within the GL theory it is shown that the accounting in the model of disordered metal layers makes it possible to produce estimates of the critical current being in quantitative agreement with the experimental data for structures from Nb and NbN. Such bridges demonstrate DE. And the change of the critical current upon reversal is substantial, while the average diode efficiency may reach 18 %.

Conflict of interest

The authors declare that they have no conflict of interest.

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