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Dispersion of localized plasticity autowaves in active deformable mediums

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The feasibility of the elastoplastic invariant and the evolution of dispersion laws for successive stages of strain hardening (linear, parabolic strain hardening and the pre-fracture stage) are considered. A uniform description of the plastic flow process at different stages of the deformation process is proposed. Basic model concepts are formulated that connect microscopic mechanisms of dislocation deformation with the properties of an active deformable medium capable of generating corresponding autowave modes of localized plastic flow.

Keywords: deformation, plasticity, autowaves, dispersion, active medium.

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1. Introduction

The problem of multilevel description of plastic flow processes arises from the need to align the microscopic scales of traditional dislocation process models with macroscopic features such as Luders bands and fracture necks. The variety of approaches used for these purposes has given rise to many attempts to find a relation between scales. For example, the analysis of multiple scales in the case of impacts on materials is well known [1]. Successful attempts to solve such problems within the framework of the gradient theory of plasticity were made in Ref. [2], and the author of Ref. [3] considered the structural aspects of the multiscale plastic flow process. In most cases, it was possible to establish the necessary relations between the scales of the process only on the basis of sufficiently strong assumptions about the nature of the flow processes.

The autowave mechanics of inhomogeneous plastic strain [4] makes it possible to explain the main patterns of plastic flow. It is based on the idea that the deformable medium is active, that is, it contains energy sources distributed over the volume [5]. For this reason, the medium becomes capable of generating a variety of autowave deformation processes (autowave deformation modes) [6], which serve as mechanisms for its self-organization. The modes are characterized by experimentally determined spatial (length λ and time period ϑ) scales and are closely related to the stages of strain hardening based on the dependence of deforming stress on deformation $\sigma(\varepsilon)$ [7]. The existing one-to-one correspondence between the stages of strain hardening and autowave modes of localized plasticity, called the *Conformity Principle*, was introduced and analyzed earlier [4]. The Principle is based on changes in the structure and properties of the deformable medium that generates these autowaves.

Together with the Conformance Principle, the most important laws of the autowave theory of plasticity are the *Elastoplastic Invariant* and the *Dispersion Relation* for autowave modes [4]. Both of these laws have so far been studied only for the stage of linear strain hardening. The purpose of this work is to clarify the possibilities of their use at other stages and scales of the plastic flow process, taking into account the properties and structure of deformable media and the ability to generate autowave modes of localized plastic strain.

2. Elastoplastic invariant of autowave deformation

The Elastoplastic Invariant has the role of the basic equation in the autowave plasticity model [4]. For the stages of linear strain hardening and light sliding in single crystals, when $\sigma \sim \varepsilon$, the invariant is defined as the ratio

$$\frac{\lambda V_{aw}}{\chi V_t} = \hat{Z} \approx \frac{1}{2}. \tag{1}$$

which relates the characteristics of elastic waves (interplane distance λ and the velocity of transverse sound V_t) to the characteristics of autowaves of plastic flow localization (length λ and propagation velocity V_{aw}). The consequences of the invariant describe the basic patterns of plastic flow. The importance of the invariant (1) necessitates the discussion of the possibility of its application for other stages of the deformation process, in particular, for parabolic strain hardening and pre-fracture.

The main difficulty of applying the invariant (1) for the stage of parabolic strain hardening, where $\sigma \sim \varepsilon^n$, $n = 1/2$ consists in the fact that the autowave mode characteristic of this stage is a stationary dissipative structure

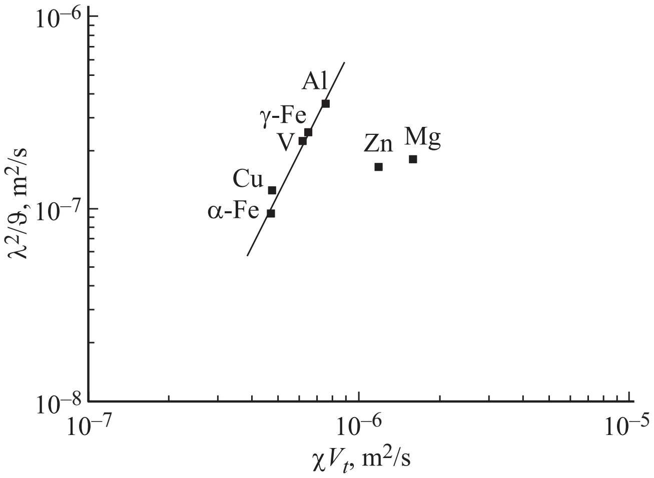

Figure 1. Checking of the invariance for the stage of parabolic strain hardening.

for which $V_{aw} = 0$ [4]. The effective velocity $V_{aw}^{(ef)} \neq 0$ can be introduced in this case to use the ratio (1). For that end let us write down the right side of the equation (1) as $1/2\,\chi V_t = \chi^2\omega_D$ using the relations $V_t \approx 2\chi\omega_D$ and $\hbar\omega_D = k_B\theta_D$, in which k_B is the Boltzmann constant, and θ_D is the Debye parameter (Debye temperature), and ω_D is the Debye frequency and we obtain

$$\lambda V_{aw} \approx \frac{1}{2}\chi V_t \approx \frac{\chi^2 k_B\theta_D}{\hbar}, \qquad (2)$$

where the effective velocity is

$$V_{aw}^{(ef)} = \frac{(\lambda V_{aw})}{\lambda} \approx \frac{k_B\theta_D}{2\hbar}\frac{\chi^2}{\lambda} \approx \omega_D\frac{\chi^2}{\lambda}. \qquad (3)$$

It describes an increase of deformation inside the focus of active plastic flow for the stage of parabolic strain hardening and the formation of a stationary dissipative structure due to an increase of the density of defects in it without macroscopic displacement of the boundaries of the focus.

$V_{aw}^{(ef)} \approx 2\cdot 10^{-3}$ m/s follows from the equation (3), which, after substitution in expression (1) gives $\hat{Z} = \lambda V_{aw}^{(ef)}/\chi V_t \approx 1.2$. The coincidence of this value with the value of the invariant (1) indicates the possibility of its application, including for the stage of parabolic strain hardening. The values of the elastoplastic invariant for several metals studied earlier are listed in Table 1 [4]. The results shown in Figure 1 lead (after excluding the drop-out data for Zn and Mg) to the value $\hat{Z} = \lambda V_{aw}^{(ef)}/\chi V_t \approx 0.4$, which is close to the value $\hat{Z} \approx 1/2$ discussed above.

For the pre-fracture stage ($\sigma \sim \varepsilon^n$, $n < 1/2$), where the autowave of localized plasticity collapses, the analysis of $X{-}t$ diagrams also confirmed the validity of the elastoplastic invariant and its applicability at this stage of the process. This is confirmed by the data in Table 2, from which it follows that for this stage the value is $\hat{Z} = \lambda V_{aw}/\chi V_t = 0.49 \pm 0.16 \approx 1/2$, which coincides with the value set for the stage of linear strain hardening.

Table 1. Checking of the applicability of an invariant (1) at the parabolic stage of strain hardening

Metal	λ	ϑ	$\lambda^2\vartheta^{-1}$	χV_t	$\lambda^2\vartheta^{-1}/\chi V_t$
	$\times 10^3$, m	s	$\times 10^7$ m^2/s		
V	7.5	250	2.25	6.2	0.36
γ-Fe	5	100	2.5	6.5	0.38
α-Fe	4	170	0.94	4.7	0.2
Cu	5	200	1.25	4.8	0.26
Al	7	140	3.5	7.5	0.46

Table 2. Checking the invariant for the pre-destruction stage

Characteristics of invariant		Mg	D1	Al*	Zr	Ti	V	Al**	α-Fe
λV_{aw}	$\times 10^7$ m^2/s	7.0	11.3	5.5	3.2	1.1	2.4	9.8	1.7
χV_t		15.8	7.5	7.5	11.9	7.9	6.2	7.5	4.7
$\lambda V_{sw}/\chi V_t$		0.45	1.5	0.7	0.25	0.15	0.4	0.1	0.35

Note. Al* — coarse-grained, Al** — fine-grained, D1 — duralumin.

The described results expanded the field of application of the elastoplastic invariant, as the basic equation of the autowave theory of plasticity, to other stages of plastic flow. The physical nature of the invariant is discussed in detail in Ref. [4] and will not be considered here, but it is interesting to focus on another version of the explanation initiated by study in Ref. [8]. It was shown that the extreme values of the physical characteristics of materials can be estimated using the scales of the natural system of units of D. Hartree. This allows expressing the coefficients of the basic equations in a physically meaningful way. So, for example, using the Bohr radius of a hydrogen atom, which serves as a natural length scale in the Hartree system

$$a_0 = \frac{\hbar^2}{me^2} = 5.291\cdot 10^{-11}\ \text{m}, \qquad (4)$$

It is possible to associate such important characteristics of a solid body as the limit speed of sound with universal constants

$$V_s = \frac{e^2}{\hbar}\left(\frac{m}{2M}\right)^{1/2} \qquad (5)$$

and the Debye frequency

$$\omega_D \approx \frac{E}{\hbar}\left(\frac{m}{M}\right)^{1/2}. \qquad (6)$$

$\hbar = h/2\pi$ is reduced Planck's constant in equations (2), (3) and (4), e and m are the charge and mass of the electron. The atomic mass M and the bond energy E characterize the deformed medium.

Table 3. Stages of plastic strain (according to data from Figure 2)

№	Stage of the strain process	Dependence of strain stress on the strain
I	Yield plateau (Luders deformation)	$\sigma = \text{const}$
II	Linear strain hardening	$\sigma \approx \theta_{\text{II}}\varepsilon \sim \varepsilon$
III	Parabolic strain hardening	$\sigma \approx \theta_{\text{III}}\varepsilon^{1/2} \sim \sqrt{\varepsilon}$
IV	Pre-fracture	$\sigma \approx \theta_{\text{IV}}\varepsilon^{n} \sim \varepsilon^{n}$; $n < 1/2$
V	Ductile fracture	$\sigma \to 0$

This opened the way to expressing many characteristics of continuous media through universal physical constants. So, in case of replacement $\chi \to a_0$ and $V_t \to V_s$ in the equation (1) the minimum value of the characteristic of the autowave λV_{aw}, called the plasticity parameter in Ref. [4] and included in the elastoplastic invariant (1), will be

$$\lambda V_{aw} = \frac{\chi V_t}{2} \approx \frac{\hbar}{2(mM)^{1/2}}. \quad (7)$$

The value $\lambda V_{aw} \approx 10^{-6}\,\text{m}^2/\text{s}$ calculated using the formula (7) is close to the experimentally found values of this value for the materials studied so far [4] and can be interpreted as the minimum value of the kinematic viscosity of a plastically deformable medium. The ratio (7) is promising for further analysis of the nature of the elastoplastic invariant.

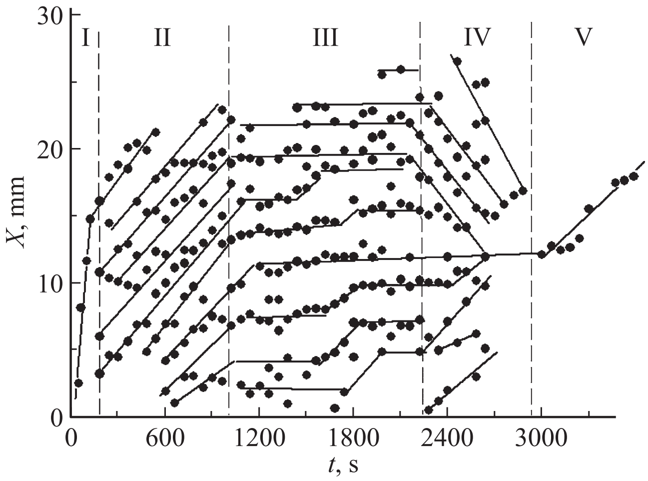

Figure 2. $X{-}t$ diagram of plastic strain of an hydrogen-saturated polycrystalline alloy Cr–Ni austenite. Stages: I — Luders strain; II — linear strain hardening stage; III — parabolic strain hardening stage; IV — pre-fracture stage; V — ductile fracture stage.

3. Dispersion of autowaves of localized plasticity

It is natural to expect that each autowave mode of plastic flow that occurs during the deformation process will correspond to a specific dispersion law $\omega(k)$, that is, a certain form of dependence of the frequency of the autowave $\omega = 2\pi/\vartheta$ on the wavenumber $k = 2\pi/\lambda$. The autowave modes occurring in case of deformation can be experimentally identified on $X{-}t$ diagrams that determine the dependence of the position of localized plasticity foci on time, as shown in Figure 2 [9], which demonstrates the alternation of five different stages of plastic flow in accordance with the change of the acting law of strain hardening, which is illustrated in Table 3. The values of frequencies and wavenumbers necessary for constructing dispersion equations are found from vertical and horizontal sections of $X{-}t$ diagrams.

The nature of the dispersion of autowaves of localized plasticity is related to the existence of its own spatial scales in a deformable medium, determined by its defective microstructure. For this reason, the form of the dispersion law is an important source of information about the structure of the medium and the kinetics of deformation phenomena in it. Taking into account the ability of a deformable medium to consistently generate autowave modes [6] in case of deformation at a constant rate, it is very important for the development of the autowave theory of plastic strain to find dispersion ratios $\omega(k)$corresponding to each stage of the plastic flow.

At the stage of the elastoplastic transition, the process of plastic flow is often realized through the development of a Luders deformation, that is, a localized transition from an elastic to a plastic state [10] on the Luders band front moving with a constant velocity. In this case, it is possible to assume that the phase $V_{\text{ph}} = \omega/k$ and group $V_{gr} = d\omega/dk$ velocities are equal, and

$$V_{aw} = \frac{\omega}{k} = \frac{d\omega}{dk} = \text{const} = a_1. \quad (8)$$

It follows thence that the simple linear law of dispersion $\omega(k) = a_1 k$ is relevant for this stage of the process, where $a_1 \equiv V_{aw}$ is the velocity of the Luders front.

The law of autowave dispersion of localized plasticity follows for the stage of linear strain hardening from equation (7). Writing the plasticity parameter as $\lambda V_{aw} = \lambda^2/\vartheta$,

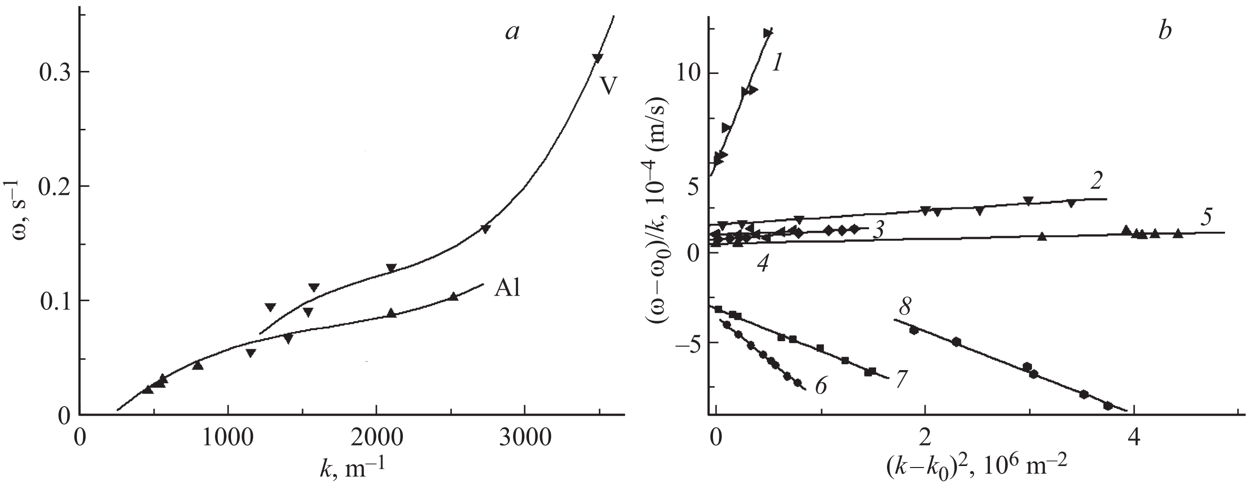

Figure 3. Dispersion in case of the collapse of autowave of localized plasticity. The initial data for V and Al (*a*). The same in functional coordinates: *1* — D1, *2* — V, *3* — Ti, *4* — Mg, *5* — Al (coarse-grained), *6* —Zr, *7* —FeSi, *8* — Al (fine-grained) (*b*).

we obtain

$$\lambda V_{aw} = \frac{\lambda^2}{\vartheta} = \frac{(2\pi/k)^2}{2\pi/\omega} = 2\pi\,\frac{\omega}{k^2} \approx \frac{\hbar}{\sqrt{mM}}, \qquad (9)$$

which leads to the quadratic dispersion equation for this stage of the deformation process

$$\omega(k) = \frac{\hbar}{2\pi\sqrt{mM}}\,k^2 \sim k^2. \qquad (10)$$

Experimental verification of the dispersion equation (10), performed on polycrystalline Al and austenite Cr−Ni single crystals [1], confirmed its validity. The same form of dispersion dependence was demonstrated by FCC single crystals at the stages of light sliding [4], for which $\sigma \sim \varepsilon$, too. The fulfillment of the quadratic dispersion law was later confirmed under conditions of plastic strain of Cr−Ni-austenite polycrystals at low temperatures [11]. The experimentally obtained dispersion equation for the linear stage of the process has the following form in all these cases

$$\omega(k) = a_2k^2 + a_1k + \omega_0 = \alpha(k - k_0)^2 + \omega_0, \qquad (11)$$

where ω_0, k_0 and a are empirical constants depending on the grade of the material.

The stage of parabolic strain hardening is characterized by similarity of the corresponding autowave structures of localized plasticity with those observed at the stage of linear strain hardening, but in this case $V_{aw} = 0$. The dispersion relation for the parabolic stage is obtained from equation (11) if $k = k_0$ is used in it. In this case $\omega = \omega_0$, which reduces to the obvious condition $\omega = 0$ if the observed pattern is stationary. The same ratio can be obtained by equating the phase and group velocity of the autowave at this stage to zero, that is, by writing

$$\frac{d\omega}{dk} = \frac{\omega}{k} = 0. \qquad (12)$$

This equality holds if $\omega = 0$, and then the condition $\omega = 0$ can be considered the law of dispersion of the autowave of localized plasticity at the stage of parabolic strain hardening [12].

For the stages of collapse of the autowave of localized plasticity at the pre-fracture stage, as well as for stepwise plastic strain, the dispersion law acquires the most complex cubic character, that is, $\omega(k) \sim k^3$. In the case of collapse, the data for the law of dispersion were obtained by graphical processing of $X{-}t$ diagrams for various metals and alloys presented in Ref. [4,13] and shown in Figure 3, *a, b*.

The dispersion in case of *serrated yielding* deformation was studied according to the data presented in Figure 4, *a*, which shows the dependence of the periodicity of localization fronts on the total deformation of polycrystalline Ni, which has an exponential form. Since the width of the front of the localized deformation bands is almost constant during loading, the distance between the single fronts, equal to the length of the working part of the sample, can be considered the length of the autowave λ.

The possibility of independent measurement of the elongation of the sample and the time period during stretching allowed finding the form of the law of dispersion of autowaves of localized plasticity for this stage of plastic strain. Figure 4, *a, b* shows a change of the period and frequency of the localization fronts movement process with an increase of the overall strain. The law of variance in this case has the form $\omega(k) \sim k^3$ as shown in Figure 4, *c*. Experimental data obtained during studies of the deformation of Ni polycrystals made it possible to construct the dependences of the phase and group velocities of the studied excitation autowaves on the wavenumber shown in Figure 4, *d*. $V_{\text{ph}} > V_{gr}$ in the studied range of wavenumbers.

The deformation at the pre-fracture stage during the collapse of an autowave of localized plasticity resembles in some of its details what is observed in case of a stepwise deformation. For example, it was possible to see a solitary

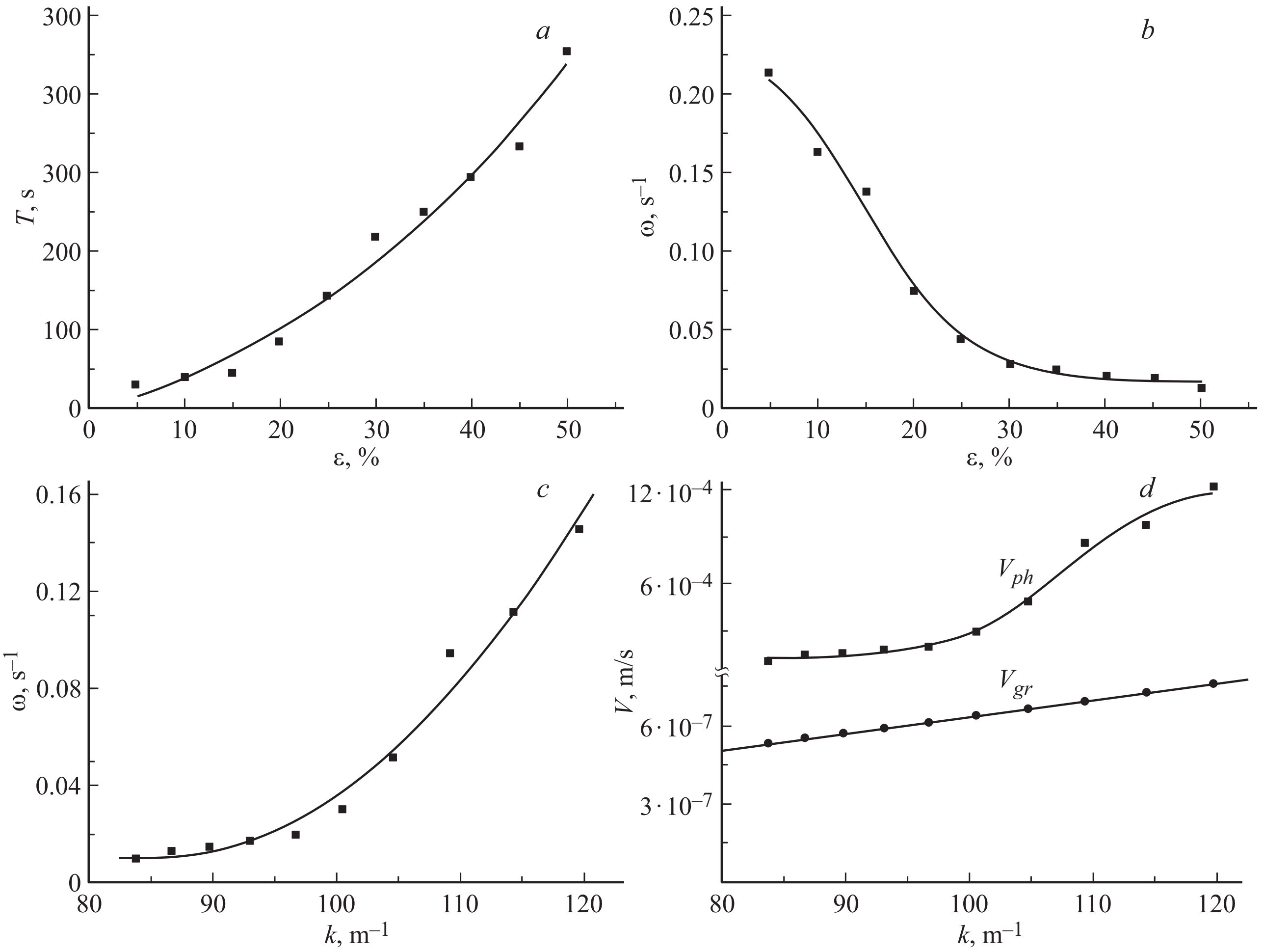

Figure 4. Dependence of the time period of the strain fronts (a) and frequency (b) on the total strain. The law of dispersion of autowaves in case of a serrated yielding deformation (c); dependence of the phase and group velocity of propagation of autowaves on the wavenumber (d).

zone of localized plasticity moving at a constant speed at the end of this stage (Figure 2), which serves as an analog of an individual moving front in case of a deformation step.

Thus, summing up the analysis of the dispersion laws for possible autowave modes of localized plastic strain, it is possible to say that a well-defined power law of dispersion corresponds to each stage of the plastic shaping process. In particular, $\omega(k) \sim k$ at the stage of Luders strain, $\omega(k) \sim k^2$ at the stage of linear strain hardening, $\omega(k) \sim k^3$ at the pre-fracture stage (collapse of an autowave of localized plasticity). The same (cubic) dependence is characteristic of a stepwise deformation. The exception here seems to be the stage of parabolic strain hardening, for which $\omega(k) = 0$.

4. Auto-wave dispersion and active deformable media

Starting the discussion of the obtained results, it is necessary to remind that the activity of the medium is a condition for the generation of autowave processes in it [14,15]. Let's clarify the meaning of this concept using the definition provided in Ref. [15], according to which „active media are characterized by a continuous dispersed influx of energy from an external source and its dissipation". The activity of deformable media is ensured by the formation of dislocations and dislocation ensembles of varying degrees of complexity [7] in case of plastic flow, which serve as stress concentrators. The inhomogeneous elastic fields of these concentrators, which evolve during deformation, provide the medium with energy sources dispersed over the volume.

Two important circumstances should be borne in mind when discussing the meaning of the laws of dispersion for autowave plastic flow processes in an active deformable medium. First, it comes to taking into account the role of microscopic (dislocation) mechanisms of deformation processes, which have been well studied to date [7]. Secondly, it is necessary to take into account that deformation kinetics can be adequately described only within the framework of the theory of nonlinear processes in active media [15], which, unfortunately, has so far found almost no direct application in the physical theory of plasticity.

Based on the analysis of the dispersion relations obtained for different stages of the flow process, it is possible to express the law of dispersion of macroscopic autowaves of localized plasticity in the following generalized polynomial form

$$\omega(k) = \omega_0 + a_1k + a_2k^2 + a_3k^3 = \sum_{i=0}^{i=3} a_ik^i, \qquad (13)$$

describing the dispersion at all stages of the plastic flow. In the general case, the coefficients of the polynomial (13) can be defined as

$$a_i = \lambda^{i-1}V_{aw} = \frac{\lambda^i}{\vartheta}, \qquad (14)$$

where $V_{aw} = \lambda/\vartheta$, and $i = 0, 1, 2, 3$. These coefficients, judging by their experimentally estimated magnitude and dimension, can be written at the first stage as $a_1 \approx V_{aw} \approx 10^{-4}$ m/s, $a_2 \approx \lambda V_{aw} \approx \hbar/\sqrt{mM} \approx 10^{-6}\,\mathrm{m^2/s}$ and $a_3 \approx \lambda^2 V_{aw} \approx 10^{-8}\,\mathrm{m^3/s}$. These data indicate that each form of the law of dispersion for different stages corresponds to the existence of active deformable media of different nature at the corresponding stages of the deformation process. It can be assumed that the latter is determined by dislocation microscale mechanisms of plastic strain [7].

Thus, the elastoplastic transition (Luders deformation) is realized at the yield plateau stage ($i = 1$) as the transformation of an elastically deformable medium into a plastically deformable medium. This transformation consists in a rapid increase of the density of mobile dislocations, associated, for example, with the massive release of dislocations from blocking impurities. Together with the linear dispersion $\omega(k) = \omega_0 + a_1k$, this allows considering the Luders front *a switching autowave* [15], occurring in a bistable medium from interconnected bistable elements with two stable states. This role is played by dislocations that initially have an immobile (metastable) state that changes to a mobile (stable) state after unblocking at the beginning of the plastic flow. The kinetics of the deformation switching autowave is characterized by a constant velocity of its front along the sample and in this respect is similar to the kinetics of the development of the front of the phase transformation of the 1st kind, as noted in Ref. [14].

The quadratic law of dispersion $\omega(k) = \omega_0 + a_1k + a_2k^2$, which operates at the linear strain hardening stage ($i = 2$), corresponds to the solutions of the nonlinear Schrodinger equation [12]. The latter describes the course of the process of self-organization in a deformable medium, and in this case, a sequence of thermally activated elementary acts of plasticity at this stage of the process can be used as a microscopic basis [16]. The self-similarity of the emerging deformation structures indicates the self-oscillatory nature of the medium at this stage, and the corresponding autowave mode is a *phase autowave* for which the condition of phase constancy $\omega t - kx = \mathrm{const}$ is valid.

The most complex cubic form of the law of dispersion of autowave processes is found at the stages of collapse of autowaves of localized plasticity ($i = 3$) or in case of a stepwise deformation. The cubic law of dispersion (13) corresponds to the well-known Korteweg–de Vries equation [12], which successfully describes the propagation of solitary excitation pulses in active excitable media [15]. The movement of the stepwise deformation front in case of a plastic flow can be considered at the macroscopic scale level of plastic flow as an equivalent of the propagation of such an impulse. It is possible that the movement of the deformation front formed towards the final stage of pre-fracture, as shown in Figure 2 (Stage V), is an event of this kind.

Finally, the parabolic strain hardening stage, when the autowave of localized plastic strain takes the form of a stationary dissipative structure, is characterized by a dispersion law of the form $\omega(k) = \omega_0 = 0$ corresponding to $i = 0$, which also satisfies the equation (13). It is clear from its shape that there is no oscillatory component of the autowave process. It can be explained at the dislocation level by the fact that microscopic deformation mechanisms, comprising the development of transverse sliding dislocations at this stage, are capable of ensuring a local increase and spatial alignment of defect density in deformable zones of the material [7]. The growth of the total deformation in this case does not require a contribution associated with the macroscopic displacement of the boundaries of these zones, as is necessary, for example, for the linear strain hardening stage.

5. Conclusion

The study establishes a relation between dislocation and autowave concepts of the nature of plastic flow. A thermally activated nature of the plastic flow was the main feature used for such a comparison [16]. For this reason the plastic strain is usually considered as a sequence of relaxation decays of local stress concentrators, accompanied by their subsequent occurrence in case of the development of plastic flow. In fact, this model, which is common for dislocation approaches, ensures the activity of the deformable medium due to the occurrence of distributed energy sources, which serve as elastic fields of concentrators. The occurrence of an active medium of one nature or another, in turn, makes it possible to generate autowaves of localized plasticity in it, and the evolution of autowaves determines the kinetics and dynamics of the evolution of the deformable active medium. The interconsistency of the processes of formation of the active medium and the generation of autowaves of localized plasticity in it explains the nature of the Conformity Principle [4].

Thus, it is the activity of the deformable medium caused by the existence of dislocation-type elastic stress concentrators in it that becomes a significant feature of the deformable medium. The activity of the deformable medium makes it possible to align two alternative points of view on the nature of plastic flow. The alignment procedure is based on the belief that dislocation and autowave views on the nature of the plasticity phenomenon are mutually complementary to each other. Mutual complementarity forms a new view of the nature of plasticity, which consists in the fact that dislocation mechanisms of deformation ensure the occurrence of active elements of the deformable medium. Their interaction results in the generation of autowave processes in the medium, and autowaves of localized plasticity, in turn, form macroscopic heterogeneity in the spatial distribution and kinetics of deformation processes.

It is clear that the same deformation process is visualized in the form of well-known dislocation substructures [7] of microscopic scale within the framework of such assumptions when observed by high-resolution electron microscopic methods, and it is recorded as a macroscale autowave structure — a pattern of localized plasticity when observed by such methods as digital speckle photography [4,6]. The stated point of view on the nature of the deformation process makes it possible to align the micro- and macroscales of the process.

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Conflict of interest

The authors declare that they have no conflict of interest.

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