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Hot spots in circular and rectangular holes of plane-parallel dielectric resonators

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Hot spots are areas of local electromagnetic field enhancement that can be generated in submicron regions between two or more closely spaced metal or dielectric resonators. In this paper, we change the strategy of studying hot spots, demonstrating their occurrence inside dielectric resonators in extremely small air holes. The transformation and disappearance of the hotspot is analyzed numerically when the hole size increases to macroscopic values. Two cases are considered: hot spots in circular and rectangular holes of plane-parallel dielectric resonators.

Keywords: local amplification of electromagnetic field, dielectric structures, Mie resonances, Fabry-Perot resonances.

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1. Introduction

Current trends in the miniaturization of optoelectronic devices imply the development of methods and technologies to enhance the effects caused by the interaction of light with matter at the micro and nanoscale. Therefore, the task of concentrating the electromagnetic field on such a scale is of great interest. The use of so-called hot spots (HS) is one of the ways to create fields with high intensity of magnetic and/or electrical components in nanoscale volumes [1,2]. Previously, HS were detected in single metal nanoobjects, as well as in metal clusters of various shapes, and were successfully used for surface enhancement of Raman scattering (SERS), which eventually allowed one to reach the detection level of individual molecules [3–6]. The electromagnetic field near a plasmonic object is unevenly distributed and can be localized in spatially narrow regions, such as nanoscales between particles or between particles and a substrate, where high-localized electromagnetic fields occur.

Relatively recently, an interest in dielectric photonics, which can win the competition with plasmonic photonics due to the absence of ohmic losses, has sharply increased after the experimental detection of Mie magnetic dipole resonance at visible frequencies in spherical silicon [7,8] and gallium arsenide [9] nanoparticles. The explosive interest in dielectric resonant particles has been called the „era of Mietronics“ [10, 11]. It has been demonstrated that dielectric particles in close proximity to each other in a wide variety of configurations, such as a dimer of two spheres [12] and metasurfaces of dimers ring−disk [13] can generate HS when excited by an electromagnetic wave.

We depart from the traditional approach in this study, which is limited to the study of HS in narrow gaps between resonators or on pointed metal needles [4–7] and study HS that occur in an air hole in a dielectric plate. We consider two cases, namely, the introduction of a circular coaxial hole into a dielectric disk, as well as the introduction of a square hole in the center of a square dielectric plate. The dependences of the HS on the size and shape of the hole and the contrast of the dielectric constant between the plates and the air are studied.

2. Calculation procedure

The aim of the work was to numerically study the transformation of patterns of spatial distribution of electric and magnetic field amplitudes depending on the size and shape of air holes in plane-parallel dielectric resonators. The calculations were performed in the COMSOL Multiphysics software package for TE-polarization. We use the fact that Maxwell's equations for a medium without dispersion of permittivity ε and magnetic susceptibility μ do not include frequency, so our results are valid for any spectral range, both optical and microwave. The extinction and scattering of light by a particle of a given shape depend only on the ratio of the characteristic particle size and the incident wavelength, as well as the dielectric constant ε and the magnetic susceptibility μ at this wavelength. It should be noted that for generality in calculations, we used both the value $\varepsilon = 12$ (a parameter for silicon in the optical range) and $\varepsilon = 100$ (a typical value for ceramic samples in the microwave range). At the same time, all the results of the work are presented depending on the dimensionless reduced frequency normalized to the characteristic size of

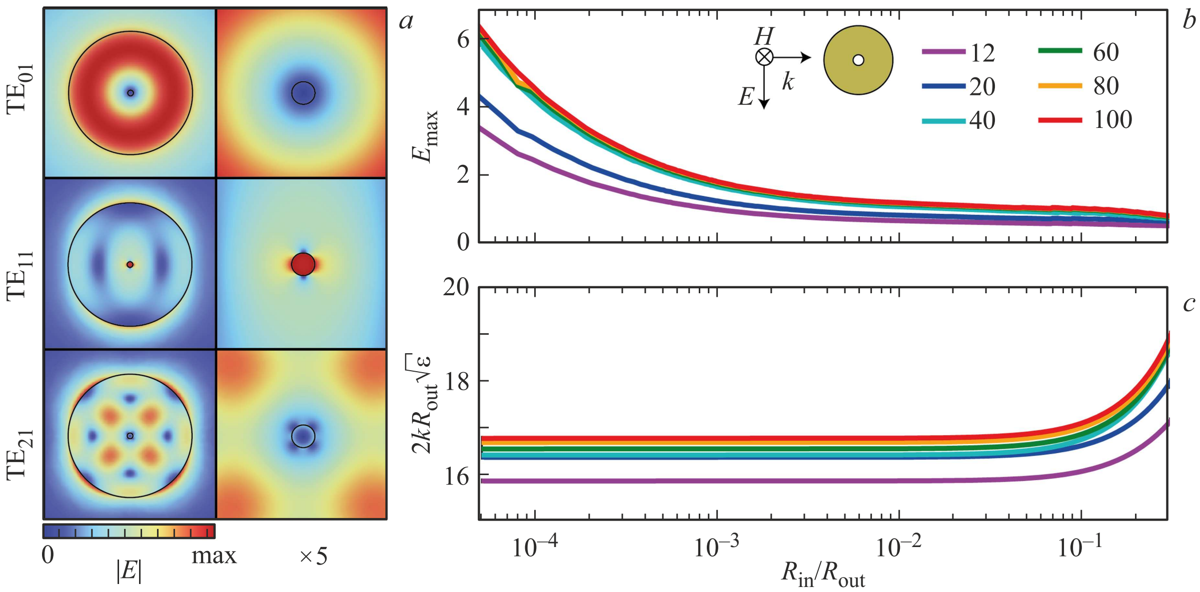

Figure 1. *a*) Electric field amplitude distribution patterns for an ring resonator with a hole with a ratio of $R_{\mathrm{in}}/R_{\mathrm{out}} = 0.05$, permittivity of $\varepsilon = 40$ (on the right — the central area of the hole is magnified fivefold). The patterns for the zero (TE_{01}), first (TE_{11}) and second (TE_{21}) azimuthal modes are presented; the designation TE_{mr} is used, where m and r — azimuthal and radial indexes, respectively. *b*) Dependence of the amplitude of the electric field in the center of the hole on the size of the hole for the first azimuthal harmonic ($m = 1$) at different dielectric permittivity of the resonator $\varepsilon = 12$; 20; 40; 60; 80; 100. *c*) The given natural frequencies $2kR_{\mathrm{out}}\sqrt{\varepsilon}$ of TE_{11} mode for the dielectric permittivity shown on the panel (*b*).

the structure, which in the case of a disk is $2kR_{\mathrm{out}}\sqrt{\varepsilon}$, and $ka_{\mathrm{out}}\sqrt{\varepsilon}$ for a square plate, where k is the wave the number of the incident electromagnetic wave, R_{out} and a_{out} is the radius of the disk and the side of the square plate, respectively.

The parameters of dielectric resonators in the form of a disk and a square plate were determined in our calculations by the ratio of the diameter of the disk to the height $2R_{\mathrm{out}}/h$ and the ratio of the outside of the square plate to its height a_{out}/h. This parameter was selected the same way in both cases: $2R_{\mathrm{out}}/h = a_{\mathrm{out}}/h = 6.25$. A coaxial hole in the shape of a circle into a disk and a coaxial square hole into a square plate were inserted into the resonators and gradually enlarged. The hole sizes varied from $R_{\mathrm{in}}/R_{\mathrm{out}} = 5 \cdot 10^{-5}$ to $R_{\mathrm{in}}/R_{\mathrm{out}} = 0.6$ (ratio of inner to outer radius) for the disc and from $a_{\mathrm{in}}/a_{\mathrm{out}} = 1 \cdot 10^{-5}$ to $a_{\mathrm{in}}/a_{\mathrm{out}} = 0.6$ (the ratio of the hole side to the outside of the square) for the square plate.

3. Results and discussion

Intense HS was observed in the air hole in case of excitation by a plane wave in TE polarization when a small coaxial hole was inserted into a dielectric disk resonator ($R_{\mathrm{in}}/R_{\mathrm{out}} = 5 \cdot 10^{-5}$). This effect can be explained using the decomposition of the field into vector cylindrical harmonics for a multilayer cylinder (an extension of the Lorenz–Mie method [14,15]). The radial E_r and angular E_φ components of the electric field are expressed in our case using the Bessel functions and Lorenz–Mie coefficients with the index m. Taking into account that all Bessel functions of integer nonzero order are zero at the origin, it can be shown that E_r and E_φ in the center ($r = 0$) of a two-layer (dielectric and air) infinite cylinder are nonzero only for the azimuthal harmonic with the index $m = 1$, and for all other harmonics with $m \neq 1$ are equal to zero. Thus, the HS that occurs in the air coaxial opening of the disk is determined by the resonance of the entire disk with the order $m = 1$. The increase of the electric field strength in the hole is caused by a jump of its component normal to the interface between two media with different values of ε.

Figure 1, *a* shows the calculated patterns of the electric field amplitude distribution for the Mie modes with indices $m = 0$, 1, 2 in the resonator plane for the hole with $R_{\mathrm{in}}/R_{\mathrm{out}} = 0.05$. It can be seen that the maximum field in the center of the resonator ($r = 0$) exists only at $m = 1$. As the hole increases from the initial value $R_{\mathrm{in}}/R_{\mathrm{out}} = 5 \cdot 10^{-5}$, the intensity of the HS field amplitude rapidly decreases, and when the hole size reaches the order of $R_{\mathrm{in}}/R_{\mathrm{out}} = 0.2$, the field of the Mie mode with $m = 1$ in the center of the hole becomes comparable to the field of the other modes and the HS completely disappears.

A study of the dependence of the amplitude of the electric field in the center of the resonator on the size of the hole for the first azimuthal harmonic $m = 1$ at different permittivity (Figure 1, *b*) showed that HS is observed in a wide range, including the value $\varepsilon = 12$, corresponding to silicon. The electric field strength in the hole increases as the dielectric constant of the ring increases, and the HS becomes brighter. It is important to note that the presented

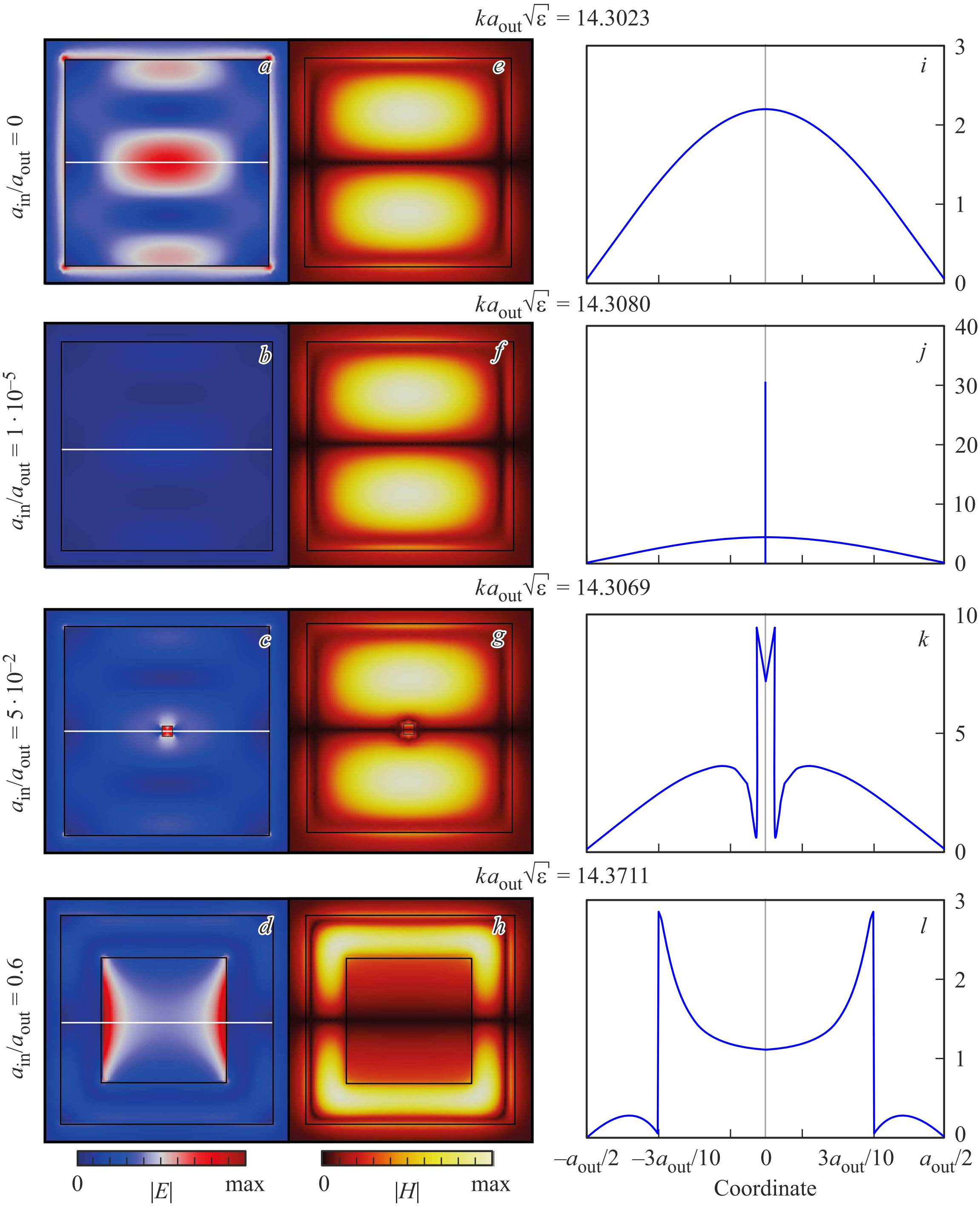

Figure 2. Amplitude distribution patterns of electric $(a-d)$ and magnetic $(e-h)$ fields for mode $\mathrm{TE}_{xy} = \mathrm{TE}_{12}$ in a square plate resonator; permittivity $\varepsilon = 40$. $i-l$) Electric field distribution along the straight line shown by the white line on $(a-d)$ and passing through the center of the resonator. The distributions are presented both for a plate without a hole (a, e, i) and for different hole sizes with $a_{in}/a_{out} = 1 \cdot 10^{-5}$ (b, f, j), $5 \cdot 10^{-2}$ (c, g, k), 0.6 (d, h, l). The reduced frequency $ka_{out}\sqrt{\varepsilon}$ is specified for each hole size.

modes are the first low-frequency resonances, and the resonant wavelength is comparable to the length of the resonator, Figure 1, *c*.

HS is also observed only for some modes with TE polarization for a square dielectric plane-parallel plate with a small square hole in its center. Similar to the case of a ring resonator, where the contribution to the effect is made by the Mie modes with the azimuth index $m = 1$, having a maximum electric field in the center of the disk, the HS appears in the hole of the square plate for Fabry-Perot modes with a maximum electric field at the center of the plate. Using the standard notation of modes in resonators with rectangular cross-section, we will write down such modes as TE_{xy}, where one or both indices x, y are even. The indices x, $y \geq 1$ and are numerically equal to the number of half-waves fitting into the size of the square plate along the corresponding coordinate, which for TE-polarization is also equal to the number of maxima of the magnetic field along the axis x or y.

The first two columns in Figure 2 show the distribution of the amplitude of the electromagnetic field $|E|$ and $|H|$ in a square plate without a hole and with a hole, when Fabry-Perot resonance with symmetry TE_{12} is excited by a plane wave. In addition, the third column shows the distribution of the electric field amplitude along the white line in Figure 2, a–d passing through the center of the plate. It can be seen from Figure 2, a and e that $|E|$ has a maximum for the TE_{12} mode in the center of the plate, and $|H|$ has a minimum, so when a square hole occurs in its center, we can expect the occurrence of „electric" HS. The division into „electric" and „magnetic" HS was discussed earlier in a number of papers [12]. Indeed, when an extremely small square hole $a_{\text{in}}/a_{\text{out}} = 1 \cdot 10^{-5}$ is introduced into the plate, the amplitude of the electric field $|E|$ in this air hole increases by an order of magnitude relative to the field in the center of the dielectric plate without a hole, from the value $|E|/|E_0| \approx 2$ to $|E|/|E_0| \approx 30$, where $|E_0|$ is the amplitude of the incident wave, Figure 2. It should be noted that the size $a_{\text{in}}/a_{\text{out}} = 1 \cdot 10^{-5}$ is so small that HS is not visible in Figure 2, b. The field distribution over the entire plate is shown by blue color in Figure 2, b in the color scale $0 \leq |E| \leq \max$, since red color corresponds to $\max(|E|/|E_0|) \approx 30$, which differs from Figure 2, a, for which the red color corresponds to $\max(|E|/|E_0|) \approx 2$. It is important to note that the „magnetic" HS is not ignited in this case — the color patterns of the magnetic field in Figure 2, e and f completely coincide.

As the hole size increases, the HS area becomes visible in the distribution of field $|E|$ (the red square area in Figure 2, c), while the shape of the distribution of field $|E|$ along the central axis significantly changes, Figure 2, k. The intensity of HS decreases from 30 to 8 arb. units, and the maximum area shifts to the edges of the hole, resulting in a small minimum forming in the very center. The HS completely disappears with a further increase of the size of the hole, and the amplitude of the field in the center $|E|/|E_0| \approx 2$ (Figure 2, l) becomes comparable to the amplitude in the initial plate, Figure 2, i.

4. Conclusion

The occurrence of HS in an extremely small hole in dielectric resonators of various geometries is found when excited by an electromagnetic wave with TE polarization, because of the intrinsic electromagnetic resonances of the structures: Mie resonances for a ring resonator and Fabry-Perot resonances for a rectangular plate. The occurrence and rapid disappearance of bright HS in case of the formation and enlargement of holes in dielectric plates of various shapes can be considered as a „signature" of the transition between two structures with different topologies. The results of the study can be used in the field of sensorics.

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Conflict of interest

The authors declare that they have no conflict of interest.

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