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Efficiency of the distinguishing of a specific wideband signal under a priori parametric uncertainty

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A synthesis and analysis of an algorithm for distinguishing several ultra-wideband quasi-radio signals of arbitrary shape with unknown initial phases and amplitudes, observed with white Gaussian noise, are carried out. The structure and statistical properties of the algorithm for distinguishing are found and an expression is obtained for the average error probability, which characterizes the effectiveness of the synthesized algorithm. The influence of detuning of various signal parameters on the efficiency of the algorithm for distinguishing is studied. A comparison of the effectiveness of the synthesized discriminator and a device for distinguishing narrowband radio signals is made. Recommendations for the use of the synthesized algorithm in practical applications are formulated. Keywords: distinguishing, ultra-wideband, quasi-radio signals, maximum likelihood, average error probability.

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Introduction

A significant number of modern information systems are designed to monitor various technical or natural processes and phenomena, which in fact boils down to processing and analyzing some useful signal. Such systems should be developed taking into account possible parametric a priori uncertainty caused by distortion of the signal during its emission, passage through the medium, the impact of other sources of signals and interference, interference, reflection, etc.

The use of ultra-wideband (UWB) signals is currently a promising field for the development of wireless networks which allows for increased noise immunity, stealth and bandwidth of the communication channel relative to traditional narrowband systems. Th UWB signals are most widely used in wireless personal area networks (WPAN) [1–5]. The main aspects of the development of UWB communication and data transmission systems are described in sufficient detail in Ref. [1] including channel modeling, compatibility with other systems, as well as interference level control and methods of their suppression. The book covers the issues of the physical layer, the level of access to the environment, network and application layers. [2] covers the application of pulse radio technology (Impulse Radio UWB, IR-UWB) which formed the basis for the development of high-precision range and location detection systems and became the most attractive technology due to the development and implementation of the IEEE 802.15.4z standard. The directions of development of the next generation of the IEEE 802.15.4ab standard are considered in Ref. [3]. This standard is widely applied for data transmission, range determination, location, sensing, etc. and is based on the use of UWB signals. The growing popularity of UWB technology for location detection, access control, tracking and tracing of indoor transceivers in real time, as well as its support in new consumer devices such as smartphones, resulted in the development of numerous new UWB radio chips [4,5], which contributes to the even greater spread of this technology in the commercial sector. However, the transition to UWB signals requires solving a large range of fundamental problems related to the fact that the methods of their generation, emission, propagation and reception, as well as methods of signal processing and extracting useful information from them, differ significantly from those used in the case of narrowband signals.

A separate class of UWB quasi-radio signals (QRS) is discriminated among numerous mathematical models of UWB signals [6–9]. They are segments of amplitude-modulated sinusoids with a period comparable to the duration of the signal, so that the condition of relative narrowbandness for such signals is not fulfilled. The QRS model allows describing various types of signals by imposing various conditions — narrowband, broadband, UWB radio and video signals, which greatly simplifies the comparative analysis of various processing algorithms and increases its correctness. The use of a harmonic multiplier in the formation of the UWB QRS makes it possible to increase the signal propagation range compared with short pulses that do not have a harmonic carrier.

Algorithms for detection [6–9] and estimation of the parameters [9] of UWB QRS under conditions of various a priori uncertainty against the background of noise and/or interference are currently considered in the literature. The task of distinguishing signals due to its high practical importance is solved to a great extent for video signals and narrowband radio signals, and algorithms for distinguishing them are actively implemented in practical applications [10,11], but they are quasi-optimal for UWB QRS. The theoretical study of the effectiveness of functioning and the practical implementation of algorithms for distinguishing UWB QRS is currently very relevant taking into account the active introduction of UWB devices in radio communication systems, range finding, navigation, etc.

An algorithm for distinguishing several UWB QRS with different modulating functions, unknown initial phases and amplitudes is synthesized and analyzed in this paper. The effect of the detuning of various signal parameters on the efficiency of the synthesized discrimination algorithm is studied. The efficiency of distinguishing of UWB QRS and narrowband radio signals was compared analytically and on the basis of statistical modeling.

1. Problem formulation

Let us consider the problem of distinguishing several UWB QRS with unknown amplitudes a and initial phases φ. The following implementation is received at the receiver input at time interval $t \in [0, T]$

$$\xi(t) = s_i(t, a_{0i}, \varphi_{0i}) + n(t), \tag{1}$$

representing an additive mixture of one of n possible signals $s_j(t, a_j, \varphi_j)$, $j = \overline{1, \ldots, i \ldots, n}$, and Gaussian white noise $n(t)$ with a one-sided spectral density N_0. The index i indicates the number of the signal that is actually present in the observed implementation, and a_{0i}, φ_{0i} are true values of its amplitude and initial phase. The signals to be discriminated are UWB QRS of the type [6–9]:

$$s_j(t, a_j, \varphi_j) = \begin{cases} a_j f_j(t)\cos(\omega_j t - \varphi_j), & 0 \le t \le \tau_j, \\ 0, & t < 0,\ t > \tau_j, \end{cases} \quad j = \overline{1, n} \tag{2}$$

and they may differ by *a priori* known modulating functions $f_j(t)$, the frequencies of the forming harmonic oscillation ω_j, and also by the durations τ_j. The amplitude a_j and the initial phase φ_j of the discriminated signals are assumed to be unknown due to the peculiarities of signal propagation.

Let us assume that the modulating functions $f_j(t)$ are continuous and differentiable, and can also be zero on an interval having a zero measure. Let us use $\Delta\omega_j$ to denote the signal bandwidths (2) determined by one of the generally accepted methods, for example, at the level of 0.707 from the maximum of the spectral density modulus. If the condition

$$\Delta\omega_j \ll \omega_j, \tag{3}$$

is met then the signals (2) are narrowband radio signals, and $f_j(t)$ are their envelopes [10,12–14]. Similarly [6–9], let us will assume that if the condition (3) does not hold, then the formula (2) describes the UWB QRS. In this case, the random phases φ_j are considered mutually independent and evenly distributed in the interval $[-\pi, \pi]$. Therefore, having the accepted realization of (1), it is necessary to optimally determine which of the n possible signals is present at the input of the receiving device.

It is possible to define the formulation of the discrimination problem in terms of the theory of statistical hypothesis testing [10,12–14]. Let us introduce hypotheses H_j corresponding to the presence in the observed implementation of a signal with the number $j = \overline{1, n}$, as well as $p_j = P(H_j)$ — a priori probabilities of these hypotheses. An algorithm for distinguishing several UWB QRS based on the analysis of the accepted realization (1) should discriminate in favor of one of the hypotheses.

2. Synthesis of the discrimination algorithm

Let's synthesize an algorithm for distinguishing several UWB QRS using the generalized maximum likelihood (ML) method [10,14], which is known in English literature as GLRT (Generalized Likelihood Ratio Test). The likelihood ratio functional logarithm (LRFL) depends on two unknown parameters provided that there is a signal $s_j(t, a_j, \varphi_j)$ in the accepted implementation and is determined by the expression

$$L_j(a_j, \varphi_j) = \frac{2}{N_0}\int_0^{\tau_j} \xi(t) s_j(t, a_j, \varphi_j)dt - \frac{1}{N_0}\int_0^{\tau_j} s_j^2(t, a_j, \varphi_j)dt. \tag{4}$$

The decision γ_i in favor of the presence of a signal $s_i(t, a_i, \varphi_i)$ is made based on a pairwise comparison with the threshold of the likelihood ratio in the following case

$$L_i > L_j, \quad j = 1, \ldots, i-1, i, i+1, \ldots, n, \tag{5}$$

where

$$L_j = \sup_{a_j, \varphi_j} L_j(a_j, \varphi_j) \tag{6}$$

— the absolute (highest) maximum of the decisive statistics (4) [10,15].

Let us substitute an explicit type of UWB QRS (2) with number j in (4). Then the expression for the LRFL will have the following form

$$L_j(a_j, \varphi_j) = a_j(X_j \cos\varphi_j + Y_j \sin\varphi_j) - \frac{a_j^2}{2}(Q_j + P_{cj}\cos 2\varphi_j + P_{sj}\sin 2\varphi_j), \tag{7}$$

where the following notation is introduced:

$$X_j = \frac{2}{N_0}\int_0^{\tau_j} \xi(t) f_j(t)\cos\omega_j t\, dt,$$

$$Y_j = \frac{2}{N_0}\int_0^{\tau_j} \xi(t) f_j(t)\sin\omega_j t\,dt,$$

$$Q_j = \frac{1}{N_0}\int_0^{\tau_j} f_j^2(t)dt,$$

$$P_{cj} = \frac{1}{N_0}\int_0^{\tau_j} f_j^2(t)\cos 2\omega_j t\,dt,$$

$$P_{sj} = \frac{1}{N_0}\int_0^{\tau_j} f_j^2(t)\sin 2\omega_j t\,dt.$$

In the case of solving the problem of distinguishing narrowband radio signals in expression (7) integrals can be discarded from functions oscillating with double frequency due to their smallness [7], as a result of which $P_{cj} \approx 0$, $P_{sj} \approx 0$, the expression (7) is significantly simplified and takes the form similar to that given in [11].

The solution statistics (6) is found by analytical maximization of the LRFL (7) in amplitude and initial phase which requires composing and solving a system of likelihood equations

$$\left.\frac{\partial L_j(a_j,\varphi_j)}{\partial a_j}\right|_{\hat a_j,\hat\varphi_j} = 0, \quad \left.\frac{\partial L_j(a_j,\varphi_j)}{\partial \varphi_j}\right|_{\hat a_j,\hat\varphi_j} = 0$$

and then substituting the found solutions in (7) to obtain

$$L_j = \sup_{a_j,\varphi_j} L_j(a_j,\varphi_j) = \frac{X_j^2(Q_j - P_{cj}) + Y_j^2(Q_j + P_{cj}) - 2X_jY_jP_{sj}}{2(Q_j^2 - P_{cj}^2 - P_{sj}^2)}. \quad (8)$$

The expressions (5), (8) define the structural diagram of the synthesized discriminator of several UWB QRS with different modulating functions, as well as unknown initial phases and amplitudes. A solution with the signal number i is formed at the output of the device, which was most likely present in the adopted implementation $\xi(t)$.

3. Analysis of the discrimination algorithm

The analysis of the effectiveness of the synthesized discriminator (5), during the operation of which processing errors may occur, requires finding the average probability of error. Let us use $P_{ij} = P(\gamma_j|H_i)$ to denote the probability that a decision was made in favor of the hypothesis H_j with the validity of the hypothesis H_i. The values P_{ij} are elements of the matrix $\|P_{ij}\|$, the diagonal elements of which represent the probabilities of correct decisions, and the rest are conditional error probabilities. In this case, the sum of the probabilities in the row satisfies the normalization condition

$$\sum_{j=1}^{n} P(\gamma_j|H_i) = \sum_{j=1}^{n} P_{ij} = 1.$$

Provided that the a priori probabilities of hypotheses $p_i = P(H_i)$ are known, the average probability of error will be determined by the expression

$$p_e = 1 - \sum_{i=1}^{n} P(H_i)P(\gamma_i|H_i) = 1 - \sum_{i=1}^{n} p_i P_{ii}.$$

Let us assume that the a priori probabilities of the hypotheses match ($p_1 = \ldots = p_n = 1/n$), then the average probability of error has the form

$$p_e = 1 - \frac{1}{n}\sum_{i=1}^{n} P_{ii}.$$

Let us use $W(l_1,\ldots,l_n|H_i)$ to denote the conditional joint probability density of random variables L_j with the validity of the hypothesis H_i. Then, the following expression is valid according to [10], for the probability of a correct decision on the presence of a signal $s_i(t,a_i,\varphi_i)$ in the accepted implementation

$$p_{ii} = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{l_i}\ldots\int_{-\infty}^{l_i} W(l_1,\ldots,l_n|H_i) \times dl_1\ldots dl_{i-1}dl_{i+1}\ldots dl_n\right)dl_i. \quad (9)$$

The random variables L_j depend on X_j, Y_j, which are Gaussian because they represent linear transformations of Gaussian white noise $n(t)$. Therefore, the random variables X_j, Y_j are completely described by the first two moments. Let us represent random variables X_j, Y_j as sums of deterministic and random components, provided that the hypothesis H_i is valid, i.e. when the signal $s_i(t,a_i,\varphi_i)$ is present in the observed implementation:

$$X_j = S_{xj}^{(i)} + N_{xj}, \quad Y_j = S_{yj}^{(i)} + N_{yj}, \quad (10)$$

where the following notation is introduced:

$$S_{xj}^{(i)} = \frac{2}{N_0}a_{0i}\int_0^{\min(\tau_j,\tau_i)} f_j(t)f_i(t)\cos(\omega_i t - \varphi_{0i})\cos\omega_j t\,dt = a_{0i}\left(R_{ccj}^{(i)}\cos\varphi_{0i} + R_{csj}^{(i)}\sin\varphi_{0i}\right),$$

$$S_{yj}^{(i)} = \frac{2}{N_0}a_{0i}\int_0^{\min(\tau_j,\tau_i)} f_j(t)f_i(t)\cos(\omega_i t - \varphi_{0i})\sin\omega_j t\,dt = a_{0i}\left(R_{scj}^{(i)}\cos\varphi_{0i} + R_{ssj}^{(i)}\sin\varphi_{0i}\right), \quad (11)$$

$$N_{xj} = \frac{2}{N_0} \int_0^{\tau_j} n(t) f_j(t) \cos\omega_j t \, dt,$$

$$N_{yj} = \frac{2}{N_0} \int_0^{\tau_j} n(t) f_j(t) \sin\omega_j t \, dt,$$

$$R^{(i)}_{ccj} = \frac{2}{N_0} \int_0^{\min(\tau_j,\tau_i)} f_j(t) f_i(t) \cos\omega_j t \cos\omega_i t \, dt,$$

$$R^{(i)}_{ssj} = \frac{2}{N_0} \int_0^{\min(\tau_j,\tau_i)} f_j(t) f_i(t) \sin\omega_j t \sin\omega_i t \, dt,$$

$$R^{(i)}_{csj} = \frac{2}{N_0} \int_0^{\min(\tau_j,\tau_i)} f_j(t) f_i(t) \cos\omega_j t \sin\omega_i t \, dt,$$

$$R^{(i)}_{scj} = \frac{2}{N_0} \int_0^{\min(\tau_j,\tau_i)} f_j(t) f_i(t) \sin\omega_j t \cos\omega_i t \, dt.$$

After completing the averaging, we find the mathematical expectations

$$\langle X_j | H_i \rangle = S^{(i)}_{xj}, \quad \langle Y_j | H_i \rangle = S^{(i)}_{yj}$$

and the correlation matrix of random variables X_j, Y_j, which is represented in the block form

$$\mathbf{K} = \begin{pmatrix} \mathbf{K_{XX}} & \mathbf{K_{XY}} \\ \mathbf{K_{YX}} & \mathbf{K_{YY}} \end{pmatrix}, \tag{12}$$

where

$$\mathbf{K_{XX}} = \{K_{X_iX_j}\} = \{\langle N_{xi} N_{xj} \rangle\},$$
$$\mathbf{K_{XY}} = \{K_{X_iY_j}\} = \{\langle N_{xi} N_{yj} \rangle\},$$
$$\mathbf{K_{YX}} = \mathbf{K}^T_{\mathbf{XY}} = \{K_{Y_iX_j}\} = \{\langle N_{yi} N_{xj} \rangle\},$$
$$\mathbf{K_{YY}} = \{K_{Y_iY_j}\} = \{\langle N_{yi} N_{yj} \rangle\},$$

$$K_{X_iX_j} = \begin{cases} Q_j + P_{cj}, & i = j, \\ R^{(i)}_{ccj}, & i \neq j, \end{cases}$$

$$K_{Y_iY_j} = \begin{cases} Q_j - P_{cj}, & i = j, \\ R^{(i)}_{ssj}, & i \neq j, \end{cases}$$

$$K_{X_iY_j} = \begin{cases} P_{sj}, & i = j, \\ R^{(i)}_{scj}, & i \neq j, \end{cases}$$

$$K_{Y_iX_j} = \begin{cases} P_{sj}, & i = j, \\ R^{(i)}_{csj}, & i \neq j. \end{cases}$$

Let's introduce similarly [7] in the expression (8) replacing variables

$$U_{2j-1} = -P_{sj} X_j + (Q_j + P_{cj}) Y_j, \quad U_{2j} = g_j X_j,$$

$$g_j^2 = Q_j^2 - P_{cj}^2 - P_{sj}^2, \quad j = \overline{1, n}, \tag{13}$$

which allows representing the decisive statistics (8) in the form

$$L_j = \frac{U^2_{2j-1} + U^2_{2j}}{2g_j^2 (Q_j + P_{cj})}. \tag{14}$$

According to (13), random variables U_{2j-1}, U_{2j} are Gaussian, since they are linear transformations of Gaussian random variables X_j and Y_j. Then their mathematical expectations have the following form

$$m^{(i)}_{2j} = \langle U_{2j} | H_i \rangle = g_j \langle X_j | H_i \rangle = g_j S^{(i)}_{xj},$$

and the elements of the correlation matrix $\mathbf{K}_U$ are defined by the expression

$$K_{U\,kq} = \Big\langle \big(U_k - \langle U_k \rangle\big)\big(U_q - \langle U_q \rangle\big) \Big\rangle, \ k, q = \overline{1, 2n}. \tag{15}$$

Taking into account the found moments, the joint Gaussian probability density of random variables $U_1 \dots U_{2n}$ is determined by the formula

$$W_U(u_1, \dots, u_{2n} | H_i) = \frac{1}{(2\pi)^n \sqrt{D}} \times \exp\left\{ -\frac{1}{2} \sum_{k=1}^{2n} \sum_{q=1}^{2n} d_{kq} (u_k - m^{(i)}_k)(u_q - m^{(i)}_q) \right\}. \tag{16}$$

Here d_{kq} are elements of the matrix $\mathbf{D} = \mathbf{K}_U^{-1}$, the inverse correlation matrix (15), $D = \det \mathbf{K}_U$, $k, q = \overline{1, 2n}$.

Let's introduce the replacement of variables: $V_{2j-1} = L_j$ (14) and $V_{2j} = \mathrm{arctg}(U_{2j}/U_{2j-1})$, with $V_{2j-1} \geq 0$, $V_{2j} \in [-\pi, \pi]$. The reverse transition to U_{2j-1}, U_{2j} has the form

$$U_{2j-1} = \sqrt{2g_j^2 (Q_j + P_{cj}) V_{2j-1}} \cos V_{2j} \equiv \Psi_{2j-1}(V_{2j-1}, V_{2j}),$$

$$U_{2j} = \sqrt{2g_j^2 (Q_j + P_{cj}) V_{2j-1}} \sin V_{2j} \equiv \Psi_{2j}(V_{2j-1}, V_{2j}).$$

Using the rules for replacing variables in probability densities [14], let's write an expression for the joint probability density of random variables V_{2j-1}, V_{2j}:

$$W_V(\nu_1, \dots, \nu_{2n} | H_i) = W_U\Big(\Psi_1(\nu_1, \nu_2), \Psi_2(\nu_1, \nu_2), \dots, \Psi_{2n-1}(\nu_{2n-1}, \nu_{2n}) | H_i\Big) \, |J|, \tag{17}$$

where the Jacobian of the transformation is $|J| = |\partial U_i / \partial V_j|$. Random variables L_j (12) coincide with random variables V_{2j-1} having an odd number. Substituting $\Psi_{2j-1}(\nu_{2j-1}, \nu_{2j})$, $\Psi_{2j}(\nu_{2j-1}, \nu_{2j})$ into the expression (17), we find the joint probability density $W_V(\nu_1, \dots, \nu_{2n} | H_i)$, integrating which with even-numbered variables ν_{2j}, we obtain a conditional probability density of random variables (14):

$$W_L(l_1,\ldots,l_n|H_i)=\int\limits_{-\pi}^{\pi}\ldots\int\limits_{-\pi}^{\pi}W_V(l_1,\nu_2,l_2,\nu_4,l_3,\nu_6,\ldots,l_n,\nu_{2n}|H_i)d\nu_2\ldots d\nu_{2n}. \tag{18}$$

We find the average probability of error in the following general form by substituting the probability density (18) into formula (9)

$$p_e=1-\frac{1}{n}\times\sum_{i=1}^{n}\int\limits_{-\infty}^{\infty}\Bigg(\int\limits_{-\infty}^{l_i}\ldots\int\limits_{-\infty}^{l_i}W_L(l_1,\ldots,l_n|H_i)dl_1\ldots dl_n\Bigg)dl_i. \tag{19}$$

The resulting expression (19) allows for analytical calculation of the exact value of the average probability of error in distinguishing any number n of signals transmitted to the input of the receiving device. Calculations using formula (19) are very cumbersome, since they contain n-fold integral. Let us further specify the results obtained for the special case of distinguishing two UWB QRS.

4. Discrimination of two UWB QRS

Let us consider the discrimination between two UWB QRS with a rectangular modulating function $f_j(t)=1$, $j=\overline{1,2}$, $n=2$ as an example. Let's assume that the discriminated signals have the same initial phase $\varphi_j=0$ and may differ in the frequency of the forming harmonic oscillation ω_j, amplitude a_j and duration τ_j. Then only two hypotheses H_1, H_2 require testing, the discrimination algorithm (5) is reduced to comparing two random variables L_1, L_2 (8):

$$L_1 \underset{H_2}{\overset{H_1}{\gtrless}} L_2, \tag{20}$$

and the values have the following form in the expression (8) in the case of a rectangular modulating function

$$X_j=\frac{2}{N_0}\int\limits_0^{\tau_j}\xi(t)\cos\omega_j t\,dt,\quad Y_j=\frac{2}{N_0}\int\limits_0^{\tau_j}\xi(t)\sin\omega_j t\,dt,$$

$$Q_j=\rho_j^2,\ P_{cj}=\rho_j^2\frac{\sin(4\pi\kappa_j)}{4\pi\kappa_j},\ P_{sj}=\rho_j^2\frac{1-\cos(4\pi\kappa_j)}{4\pi\kappa_j},$$

where $\rho_j^2=\tau_j/N_0$ — the values characterizing the signal-to-noise ratio (SNR) at the output of the ML receiver for single amplitude signals, and $\kappa_j=\omega_j\tau_j/2\pi$ — the narrowband parameters of the distinguishable signals, which are equal to the number of harmonic fill oscillation periods that fit within the UWB QRS duration. If $\kappa_i\to\infty$, then UWB QRS acquire the properties of narrowband radio signals, and UWB QRS coincide with video pulses at $\kappa_i=0$, the shapes of which are described by the functions $f_j(t)=1$.

Let us introduce into consideration the value $\Delta_\kappa=\kappa_2/\kappa_1$, which characterizes the difference in signals in the narrowband parameter, which can be achieved by both the difference in durations and frequencies of harmonic filling. Let us use $\kappa=\sqrt{\kappa_1\kappa_2}$ to denote the averaged narrowband parameter and represent the narrowband parameters of the discriminated signals as $\kappa_1=\kappa/\sqrt{\Delta_\kappa}$, $\kappa_2=\kappa\sqrt{\Delta_\kappa}$. Thus, the discriminated signals differ in values κ_1,κ_2 in opposite directions relative to κ, and this difference is the greater the stronger the value Δ_κ differs from one. It should be noted that the discriminated signals at $\Delta_\kappa\neq 1$ may have different energy

$$E_j=a_{0j}^2\frac{N_0}{2}(Q_j+P_{cj})=\frac{a_{0j}^2\tau_j}{2}\left(1+\frac{\sin 4\pi\kappa_j}{4\pi\kappa_j}\right).$$

Let us use Δ to denote the value characterizing the difference in UWB QRS energies:

$$\Delta=\frac{E_2}{E_1}=\frac{a_{02}^2\tau_2}{a_{01}^2\tau_1}\left(1+\frac{\sin 4\pi\kappa\sqrt{\Delta_\kappa}}{4\pi\kappa\sqrt{\Delta_\kappa}}\right)\Big/\left(1+\frac{\sin 4\pi\kappa/\sqrt{\Delta_\kappa}}{4\pi\kappa/\sqrt{\Delta_\kappa}}\right).$$

It should be noted that the SNR at the output of the ML receiver for UWB QRS with amplitude a_{0j}, duration τ_j and narrowband parameter κ_j [7.10] turns out to be different for the discriminated signals

$$z_j^2=\frac{2E_j}{N_0}=z_{rj}^2\left(1+\frac{\sin 4\pi\kappa_j}{4\pi\kappa_j}\right), \tag{21}$$

where

$$z_{rj}^2=2a_{0j}^2\tau_j/N_0,\quad j=\overline{1,2}$$

— SNR at the output of the ML receiver for a video signal with amplitude a_{0j} and duration τ_j [6,12].

The substitution of variables (13) in relation to two discriminated signals has the following form

$$U_1=-P_{s1}X_1+(Q_1+P_{c1})Y_1,\quad U_2=g_1X_1,$$
$$U_3=-P_{s2}X_2+(Q_2+P_{c2})Y_2,\quad U_4=g_2X_2,$$
$$g_1^2=Q_1^2-P_{c1}^2-P_{s1}^2,\quad g_2^2=Q_2^2-P_{c2}^2-P_{s2}^2. \tag{22}$$

Random variables (22) are Gaussian with mathematical expectations

$$m_{\{1,3\}}^{(i)}=\langle U_{\{1,3\}}|H_i\rangle=-P_{s\{1,3\}}S_{x\{1,3\}}^{(i)}+(Q_{\{1,3\}}+P_{c\{1,3\}})S_{y\{1,3\}}^{(i)},$$
$$m_{\{2,4\}}^{(i)}=\langle U_{\{2,4\}}|H_i\rangle=g_{\{2,4\}}S_{x\{2,4\}}^{(i)} \tag{23}$$

and the correlation matrix, which is the same for both hypotheses

$$\mathbf{K}_U=\|K_{U\,kq}\|=\left\langle\left(U_k-m_k^{(1)}\right)\left(U_q-m_q^{(1)}\right)\right\rangle=\left\langle\left(U_k-m_k^{(2)}\right)\left(U_q-m_q^{(2)}\right)\right\rangle,\ k,q=\overline{1,4}, \tag{24}$$

$$K_{U11} = K_{U22} = g_1^2(Q_1 + P_{c1}),$$
$$K_{U33} = K_{U44} = g_2^2(Q_2 + P_{c2}),$$
$$K_{U34} = K_{U43} = 0, \quad K_{U12} = K_{U21} = 0,$$
$$K_{U24} = K_{U42} = g_1 g_2 R_{cc1}^{(2)},$$
$$K_{U14} = K_{U41} = g_2(Q_1 + P_{c1})R_{sc1}^{(2)} - P_{s1} g_2 R_{cc1}^{(2)},$$
$$K_{U23} = K_{U32} = g_1(Q_2 + P_{c2})R_{sc2}^{(1)} - P_{s2} g_1 R_{cc1}^{(2)},$$
$$K_{U13} = K_{U31} = P_{s1}P_{s2}R_{cc1}^{(2)} + (Q_1 + P_{c1})(Q_2 + P_{c2})R_{ss1}^{(2)}$$
$$- P_{s1}(Q_2 + P_{c2})R_{cs1}^{(2)} - P_{s2}(Q_1 + P_{c1})R_{cs2}^{(1)}.$$

Taking into account the obtained statistical characteristics (23), (24), let us write the probability density of the random variable U_j (16) in the following form:

$$W_U(u_1, u_2, u_3, u_4|H_i) = \frac{1}{(2\pi)^2\sqrt{D}}$$
$$\times \exp\left\{-\frac{1}{2}\sum_{k=1}^{4}\sum_{q=1}^{4} d_{kq}(u_k - m_k^{(i)})(u_q - m_q^{(i)})\right\}, \ k, q = \overline{1, 4}.$$

Let's introduce the substitution of variables

$$V_1 = L_1 = \frac{U_1^2 + U_2^2}{2g_1^2(Q_1 + P_{c1})}, \ V_2 = \operatorname{arctg}\frac{U_2}{U_1},$$
$$V_3 = L_3 = \frac{U_3^2 + U_4^2}{2g_2^2(Q_2 + P_{c2})}, \ V_4 = \operatorname{arctg}\frac{U_4}{U_3}. \qquad (25)$$

In this case, the reverse transition to U_1 will have the following form

$$U_1 = \sqrt{2g_1^2(Q_1 + P_{c1})V_1}\cos V_2 \equiv \Psi_1(V_1, V_2),$$
$$U_2 = \sqrt{2g_1^2(Q_1 + P_{c1})V_1}\sin V_2 \equiv \Psi_2(V_1, V_2),$$
$$U_3 = \sqrt{2g_2^2(Q_2 + P_{c2})V_3}\cos V_4 \equiv \Psi_3(V_3, V_4),$$
$$U_4 = \sqrt{2g_2^2(Q_2 + P_{c2})V_3}\sin V_4 \equiv \Psi_4(V_3, V_4).$$

Using the rules for replacing variables in probability densities [14], let us write the expression for the joint probability density of random variables V_1, V_2, V_3, V_4 as

$$W_V(v_1, v_2, v_3, v_4|H_i) =$$
$$= W_U\big(\Psi_1(v_1, v_2), \Psi_2(v_1, v_2), \Psi_3(v_3, v_4), \Psi_4(v_3, v_4)\big)|J|,$$

where the Jacobian of the transformation is

$$|J| = |\partial U_i/\partial V_j| = g_1^2 g_2^2(Q_1 + P_{c1})(Q_2 + P_{c2}).$$

The random variables L_1, L_2 (25) coincide with V_1 and V_3, respectively. Therefore, for the joint probability density of random variables L_1, L_3 (18) it is possible to write

$$W_L(l_1, l_2|H_i) = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} W_V(l_1, v_2, l_2, v_4|H_i)dv_2 dv_4. \qquad (26)$$

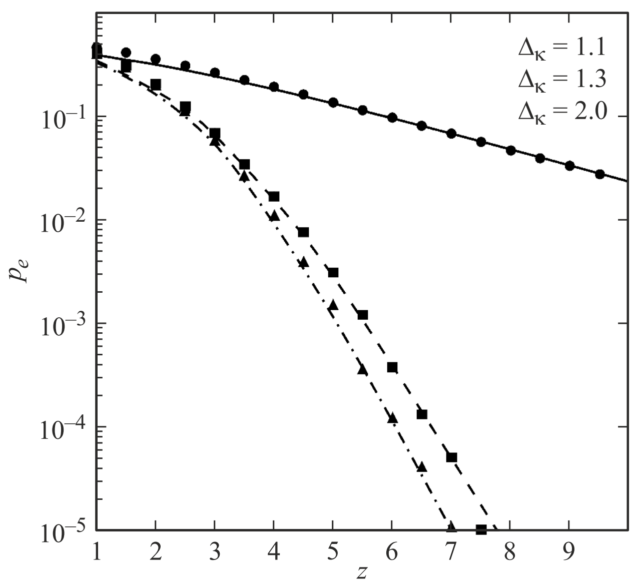

Figure 1. The dependence of the average probability of discrimination error on SNR for different narrowband parameters.

Substituting the function (26) into formula (19), we find the average probability of a discrimination error in the form

$$p_e = 1 - \frac{1}{2}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{l_1} W_L(l_1, l_2|H_1)dl_2\right)dl_1$$
$$- \frac{1}{2}\int_{-\infty}^{\infty}\left(\int_{-\infty}^{l_2} W_L(l_1, l_2|H_2)dl_1\right)dl_2. \qquad (27)$$

Let's consider as an example the most difficult case when the discriminated signals have the same energy $E_1 = E_2$. Therefore, $\Delta = 1$, and also SNR (21) are the same for both signals $z_1 = z_2 = z$.

Figure 1 shows the dependences of the average probability of discrimination error p_e (27) on the SNR z for signals of the same duration $\Delta_\tau = \tau_1/\tau_2 = 1$ with different narrowband parameters κ_1, κ_2 at $\kappa = \sqrt{\kappa_1\kappa_2} = 4$. The solid curve corresponds to $\Delta_\kappa = 1.1$, dashed curve corresponds to $\Delta_\kappa = 1.3$, dash and dot curve corresponds to $\Delta_\kappa = 2$.

Figure 1 shows that a slight difference of the narrowband parameter of the discriminated signals ($\Delta_\kappa = 1.1$ and $\Delta_\kappa = 1.3$) leads to a sharp decrease of the average error probability (at $z = 7$ more than 10^3 times). A further increase of Δ_κ does not lead to a significant increase of the efficiency of discrimination at $\Delta_\kappa > 2$, despite the fact that the signals do not differ in other parameters. In fact, a change of the narrowband parameters with equal duration indicates that the two signals have different carrier frequency, and the parameter Δ_κ characterizes the separation of carrier frequencies. The average probability of error decreases with an increase of the SNR, and it is equal to $p_e = 0.5$ at $\Delta_\kappa = 1$, which indicates the correct operation of the algorithm.

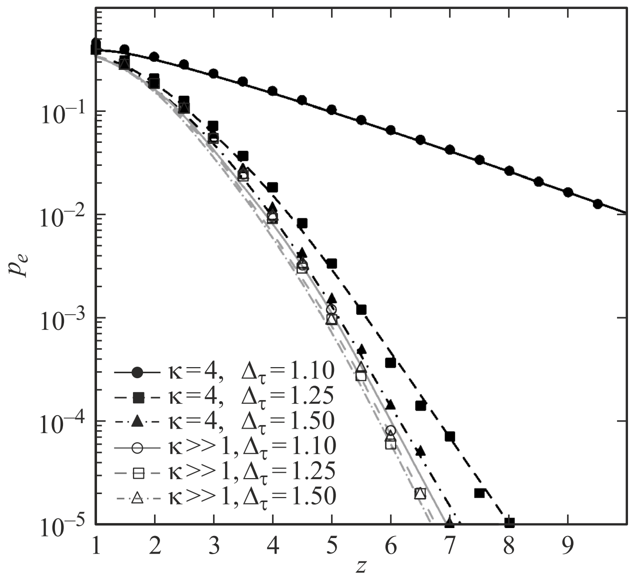

Figure 2. Dependence of the average probability of discrimination error on SNR at different durations for UWB QRS and narrowband radio signal.

Figure 2 shows the dependences of the average probability of discrimination error p_e (27) on SNR z provided that the discriminated signals differ in durations and frequencies, but have the same narrowband parameter $\Delta_\kappa = 1$. The black curves correspond to the case of discrimination of UWB QRS with $\kappa = 4$, gray curves correspond to discrimination of narrowband radio signals at $\kappa \gg 1$ $(\kappa = 100)$. Solid curves are plotted for $\Delta_\tau = 1.1$, dashed curves are plotted for $\Delta_\tau = 1.25$, dash and dot curves are plotted for $\Delta_\tau = 1.5$.

The dependencies in Fig. 2 have a character similar to the character of dependencies shown in Fig. 1 with equal narrowband parameters and different durations and frequencies. A significant improvement of the efficiency of discrimination is already observed with small differences in the duration of the signals. The UWB QRS discriminator has a much higher average error probability, differing in up to 10^5 times with the SNR $z = 7$ in case of high degree of correlation of the discriminated signals, when the informative parameters of the signals practically do not differ. The difference of the duration of the discriminated signals actually indicates a different bandwidth of these signals, and the simultaneous presence of a delay in duration and the same values of narrowband parameters indicates that the discriminated signals may have different carrier frequencies.

It should be noted that the value $\kappa \gg 1$ corresponds to narrowband signals, i.e. expressions (19), (27) also allow calculating the characteristics of discrimination of narrowband signals and comparing them with UWB QRS. The results obtained for narrowband signals, in particular, are consistent with the results obtained in Ref. [10]. It can be seen from the dependencies shown in Fig. 2 that the efficiency of discrimination of narrowband radio signals practically does not depend on the values of the different signal parameters. In turn, the narrowband parameter has a significant impact on the processing characteristics for the UWB QRS [6–9] and, as a result, the change of the signal parameters associated with the narrowband parameter results in a significant impact on the discrimination efficiency. The difference of the parameters of the discriminated signals makes it possible to achieve the efficiency of distinguishing UWB QRS which is similar to the efficiency of discrimination of narrowband signals. For example, the average probability of error of discrimination of the UWB QRS and a narrowband radio signal by the corresponding receivers practically does not differ already at $\Delta_\tau = 1.5$.

It was also additionally found that the difference of the narrowband parameter of the discriminated UWB QRS results in a change of the average probability of error according to the harmonic law, which makes it possible to search for local minima with known parameters of useful and interfering signals and allows to significantly increase the efficiency of signal discrimination. The degree of impact of the narrowband parameter on the efficiency of signal discrimination significantly decreases with an increase of the SNR, and it practically ceases to affect the efficiency of discrimination at $\kappa \geq 8$ with the signal parameters described in the example.

5. Statistical modeling of the algorithm for distinguishing two UWB QRS

The verification of the efficiency of the discrimination algorithm (20) is performed by the method of statistical modeling on a computer. Let's use the method of modeling the signals at the output of the receiving device since it was not possible to form an implementation of Gaussian white noise in the observed realization (1). Random variables L_1, L_2 were repeatedly formed for both hypotheses H_1, H_2 during modeling, since their statistical characteristics are fully known. The correct discrimination event was recorded if the inequality $L_1 > L_2$ held with the validity of the hypothesis H_1, and the inequality $L_1 < L_2$ held with the validity of the hypothesis H_2. The relative frequency of correct decisions was used as the probability of making the correct decision.

It is necessary to generate Gaussian random variables (10) to generate random variables L_1, L_2 according to (8). Let's introduce normalized values

$$\tilde{X}_j = X_j/\rho_j = \tilde{S}_{xj}^{(i)} + \tilde{N}_{xj}, \ \tilde{Y}_j = Y_j/\rho_j = \tilde{S}_{yj}^{(i)} + \tilde{N}_{yj}, \quad (28)$$

where

$$\tilde{S}_{xj}^{(i)} = a_{0i}\big(R_{ccj}^{(i)} \cos\varphi_{0i} + R_{csj}^{(i)} \sin\varphi_{0i}\big)/\rho_j,$$

$$\tilde{S}_{yj}^{(i)} = a_{0i}\big(R_{scj}^{(i)} \cos\varphi_{0i} + R_{ssj}^{(i)} \sin\varphi_{0i}\big)/\rho_j$$

— mathematical expectations,

$$\tilde{N}_{xj} = N_{xj}/\rho_j, \quad \tilde{N}_{yj} = N_{yj}/\rho_j$$

— random components with a correlation matrix

$$\tilde{\mathbf{K}} = \begin{pmatrix} \tilde{Q}_1 + \tilde{P}_{c1} & \tilde{R}^{(2)}_{cc1} & \tilde{P}_{s1} & \tilde{R}^{(2)}_{cs1} \\ \tilde{R}^{(2)}_{cc1} & \tilde{Q}_2 + \tilde{P}_{c2} & \tilde{R}^{(2)}_{sc1} & \tilde{P}_{s2} \\ \tilde{P}_{s1} & \tilde{R}^{(2)}_{sc1} & \tilde{Q}_1 - \tilde{P}_{c1} & \tilde{R}^{(2)}_{ss1} \\ \tilde{R}^{(2)}_{cs1} & \tilde{P}_{s2} & \tilde{R}^{(2)}_{ss1} & \tilde{Q}_2 - \tilde{P}_{c2} \end{pmatrix}. \quad (29)$$

Here

$$\tilde{Q}_j = Q_j/\rho_j^2, \quad \tilde{P}_{cj} = P_{cj}/\rho_j^2, \quad \tilde{P}_{sj} = P_{sj}/\rho_j^2,$$

$$\tilde{R}^{(i)}_{ccj} = R^{(i)}_{ccj}/\rho_i\rho_j, \quad \tilde{R}^{(i)}_{ssj} = R^{(i)}_{ssj}/\rho_i\rho_j,$$

$$\tilde{R}^{(i)}_{csj} = R^{(i)}_{csj}/\rho_i\rho_j, \quad \tilde{R}^{(i)}_{scj} = R^{(i)}_{scj}/\rho_i\rho_j.$$

Let's use the linear transformation method [16], according to which the vector of statistically related centered Gaussian quantities $\tilde{N}_{x1}$, $\tilde{N}_{x2}$, $\tilde{N}_{y1}$, $\tilde{N}_{y2}$ can be obtained as a linear transformation of a vector of statistically independent Gaussian quantities B_1, B_2, B_3, B_4 with zero mathematical expectations and unit dispersion. Let's choose the transformation matrix of the lower triangular

$$\begin{pmatrix} \tilde{N}_{x1} \\ \tilde{N}_{x2} \\ \tilde{N}_{y1} \\ \tilde{N}_{y2} \end{pmatrix} = \begin{pmatrix} b_{11} & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} \end{pmatrix} \begin{pmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{pmatrix}. \quad (30)$$

We find b_{ij} calculating the correlation moments of random variables (28) and equating them to the corresponding elements of the correlation matrix (29). So, for example, the dispersion of the random variable $\tilde{N}_{x1} = b_{11}B_1$ is equal to

$$\langle \tilde{N}^2_{x1} \rangle = b^2_{11}\langle B^2_1 \rangle = b^2_{11} = (\tilde{Q}_1 + \tilde{P}_{c1}),$$

hence $b_{11} = \sqrt{\tilde{Q}_1 + \tilde{P}_{c1}}$.

Let us find the correlation moment for a random value $\tilde{N}_{x2} = b_{21}B_1 + b_{22}B_2$

$$\langle \tilde{N}_{x1}\tilde{N}_{x2} \rangle = \langle b_{11}B_1(b_{21}B_1 + b_{22}B_2) \rangle = b_{11}b_{21}\langle B^2_1 \rangle + b_{11}b_{22}\langle B_1B_2 \rangle = b_{11}b_{21} = \tilde{R}^{(2)}_{cc1}$$

and dispersion

$$\langle \tilde{N}^2_{x2} \rangle = \langle (b_{21}B_1 + b_{22}B_2)^2 \rangle = b^2_{21}\langle B^2_1 \rangle + b^2_{22}\langle B^2_2 \rangle + 2b_{21}b_{22}\langle B_1B_2 \rangle = b^2_{21} + b^2_{22} = \tilde{Q}_2 + \tilde{P}_{c2}.$$

Then

$$b_{21} = \tilde{R}^{(22)}_{cc1}/b_{11}, \quad b_{22} = \sqrt{\tilde{Q}_2 + \tilde{P}_{c2} - b^2_{21}}.$$

Similarly, expressing the remaining correlation moments, we obtain recurrence relations for the elements of the matrix b_{ij}:

$$b_{31} = \tilde{P}_{s1}/b_{11}, \quad b_{32} = \big(\tilde{R}^{(2)}_{sc1} - b_{21}b_{31}\big)/b_{22},$$

$$b_{33} = \sqrt{\tilde{Q}_1 - \tilde{P}_{c1} - b^2_{31} - b^2_{32}},$$

$$b_{43} = \big(\tilde{R}^{(2)}_{ss1} - b_{41}b_{31} - b_{42}b_{32}\big)/b_{33},$$

$$b_{44} = \sqrt{\tilde{Q}_2 + \tilde{P}_{c2} - b^2_{41} - b^2_{42} - b^2_{43}}.$$

Substituting the obtained expressions for b_{ij} in (30), then in (28) and (8), let us write for random variables L_j provided that the hypothesis with number i is valid:

$$L^{(i)}_j = \big(1/2(\tilde{Q}^2_j - \tilde{P}^2_{cj} - \tilde{P}^2_{sj})\big)\Big[z^2_{ri}\big((\tilde{R}^{(i)}_{ccj}\cos\varphi_{0i} + \tilde{R}^{(i)}_{csj}\sin\varphi_{0i}) + \tilde{N}_{xj}/z_{ri}\big)^2(\tilde{Q}_j - \tilde{P}_{cj}) + z^2_{ri}\big((\tilde{R}^{(i)}_{scj}\cos\varphi_{0i} + \tilde{R}^{(i)}_{ssj}\sin\varphi_{0i}) + \tilde{N}_{yj}/z_{ri}\big)^2(\tilde{Q}_j + \tilde{P}_{cj}) - 2z^2_{ri}\big((\tilde{R}^{(i)}_{ccj}\cos\varphi_{0i} + \tilde{R}^{(i)}_{csj}\sin\varphi_{0i}) + \tilde{N}_{xj}/z_{ri}\big)\big((\tilde{R}^{(i)}_{scj}\cos\varphi_{0i} + \tilde{R}^{(i)}_{ssj}\sin\varphi_{0i}) + \tilde{N}_{yj}/z_{ri}\big)\tilde{P}_{sj}\Big]. \quad (31)$$

z_{r1}, z_{r2} were calculated, and random variables (31) were repeatedly generated during the modeling for each value $z = z_1 = z_2$, according to (21). A correct discrimination event was recorded if the inequality $L_1 > L_2$ held at $i = 1$, and the inequality $L_1 \le L_2$ held at $i = 2$. The relative frequency of making the right decision was calculated based on the results of 10^6 tests.

Figures 1 and 2 show the simulation results with shaded markers (for UWB QRS) and empty markers (for narrowband radio signals). The results of the analysis of the discrimination algorithm obtained analytically match the results of statistical modeling to a high degree. The high degree of matching between mathematical and statistical modeling is attributable to the fact that the expression (27) is accurate, and the slight discrepancy between the curves and markers in Fig. 1 and 2 is explained by the error of numerical integration in analytical calculations and the features of multiple formation of random variables in statistical modeling.

Conclusion

The paper synthesizes the most plausible algorithm for distinguishing UWB QRS of arbitrary shape with unknown initial phases and amplitudes observed against a background of white Gaussian noise. The analysis of the synthesized discrimination algorithm allowed for obtaining an accurate analytical expression in general form for the average probability of discrimination error of n UWBQRS, as well as a closed expression for the case of discrimination of two signals. The effect of the bandlimitedness and duration parameter detuning on the efficiency of the discrimination algorithm is studied. It is found that there a slight difference of the bandlimitedness parameter of the signals (from 1 to 2) significantly affects the change of the average probability of discrimination error, and a further increase of the detuning (more 2) has practically no effect on the characteristics of discrimination.

Additionally, the efficiency of the synthesized discriminator and the device for distinguishing narrowband radio signals was compared. The UWB QRS discriminator has a much higher average error probability in the case when the informative parameters of the signals practically do not differ (the detuning is 10–20%). At the same time, a slight difference of the parameters of the discriminated signals (more than 30%) allows the UWB QRS discriminator to achieve efficiency similar to the efficiency of narrowband signal discriminators.

In addition, simpler discriminators of narrowband radio signals, the input of which can receive broadband and UWB signals, will be quasi-optimal when processed, and they will have low efficiency as shown by the example of solving the detection problem [7], the degradation of which is directly proportional to the increase of signal bandwidth. Therefore, it is advisable to use UWB signal discriminators in the absence of a priori information about the bandwidth of the received (distinguishable) signals, which will allow adjusting the discriminator to the desired signal bandwidth by changing the narrowband parameter without reducing the efficiency of signal processing.

Hardware or software implementation of the optimal UWB QRS discrimination algorithm, depending on the number of discriminated signals, may be quite challenging compared to known algorithms for distinguishing narrowband radio signals because the narrowband condition is not met. However, the results obtained in the work can be used for designing new and modernization of existing UWB communication, radar and navigation facilities operating in a complex interference environment, high subscriber density or multipath propagation of signals. It is advisable to use simpler and faster signal detectors in the case of UWB signal processing in conditions of guaranteed absence of signal-like interference [7].

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Conflict of interest

The authors declare that they have no conflict of interest.

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