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Comparison of the speed and accuracy of algorithms of optimization of fiber-optic communication lines

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The study addressed the problem of improvement of the quality of the transmitted optical signal which is measured by the magnitude of the excess of the signal-to-noise ratio over its critical value. It boils down to finding the extremum of a function of many variables unique to each fiber-optic communication line. Various optimization algorithms are compared for finding the most suitable one for this family of functions. The applicability of algorithms based on Bayesian optimization to this problem was demonstrated. The time spent on finding a solution using the proposed algorithms is less than when using deterministic global optimization algorithms, and the deviation from the absolute optimum remains at an acceptable level.

Keywords: fiber optic line, signal/noise, optimization algorithm, Bayesian algorithm.

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Introduction

The demand for the capacity and bandwidth of fiber-optic communication lines (fiber optic lines) underlying the Internet backbone networks increases with the growing number of Internet users, IoT (Internet of Things) devices and the rapid development of cloud technologies. It is necessary to develop infrastructure, build new FOCL and modernize existing ones. All this requires preliminary modeling and selection of the optimal set of equipment and its settings (designing) [1].

The accurate consideration of the effects that affect the signal in an optical fiber, including nonlinear effects, is one of the most difficult tasks that arise when designing communication lines. It is necessary to solve the nonlinear Schrodinger equation (NSE) for assessment of their impact on the quality of signal transmission. It should be noted that there is no analytical solution to the problem, and the complexity of the numerical solution increases rapidly with the increase of the accuracy, so this approach is not applicable in practice due to limitations of computing power. Various approximate methods for estimating nonlinear effects are used to speed up calculations, but the GN model obtained from the first order of perturbation theory [3] is still most commonly used for the assessment [2]. As it was shown in Ref. [4], all components of the electromagnetic field acquire the same statistically independent Gaussian distribution for coherent communication channels without dispersion compensation, as they propagate through the fiber, therefore, the effect is considered as additive Gaussian noise in the GN model.

The reception signal-to-noise ratio (SNR) and therefore the quality of signal transmission is affected by other factors depending on various parameters of the fiber optic cable apart from nonlinear effects: the total power at the input of the cable sections and the characteristics of the optical fiber used in them, the types of amplifiers used, their gain factors (GF), frequency equalizer settings and the used signal spectrum pre-distortion. The design process can be represented as finding the global extremum of the signal transmission quality function, which depends on all these parameters. The designing task is simplified by the truncation of the parameters of the objective function when some of them are fixed and their values are replaced by a preliminary estimate. This reduces the number of independent variables and narrows the scope of the optimum search. Various methods of truncating function parameters are used. For example, a method for evaluating the amplifier GF is provided in Ref. [5] which reduces the transmission quality function to a function of one variable — output power of the first amplifier, which significantly simplifies the task of finding the optimal point of operation of the line. But an analysis of the results of optimizing the signal transmission quality function with truncation of parameters to one showed that the deviation of the achieved quality from the global maximum can reach 1 dB [6]. The use of this truncation method could, according to calculations, lead to a degradation to up to 1 dB of reception SNR due to finding a local extremum instead of a global extremum

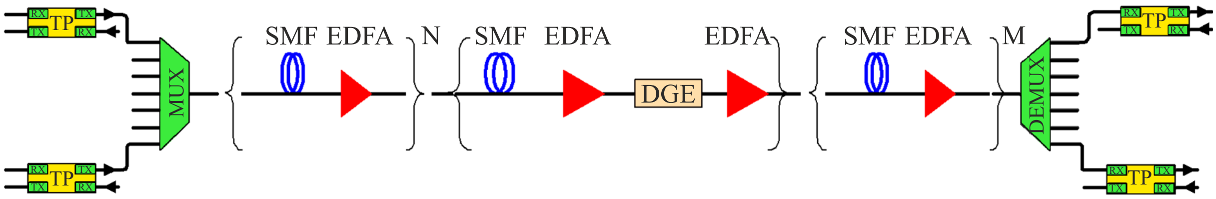

Figure 1. General diagram of the considered FOCL. The spans are marked with curly brackets.

in case of optimization. Thus, the choice of an appropriate truncation of the parameters of the objective function of signal transmission quality is an important part of the fiber optic network designing task.

A new method for truncating the parameters of the objective function is proposed in this paper. The power at the input of each cable section was taken as the main parameters. The amplifier GF was calculated for calculating the quality of signal transmission based on the selected capacities and the amplifiers optimal in terms of noise factor were selected, taking into account the limitations on input and output capacities. This allows finding the best overall noise factor for each individual case for a given general set of amplifier types. Formulas for calculating the quality of signal transmission are given in section 1.

The resulting objective function of signal transmission quality does not have a general analytical form and is essentially piecewise due to a limited set of amplifier types. Its gradient is not defined at all points, so methods based on gradient descent are not applicable to find the extremum of this function. A comparison of various optimization methods is presented in Ref. [7], including a comparison of global and local optimization in relation to the FOCL parameters and it is shown that heuristic and stochastic algorithms, machine learning, analytical and other approaches will be used to solve this problem. The choice of the algorithm determines the truncation and allows considering the task either as a global optimization of input power profiles in fiber sections, or as a search for the optimal configuration of the operating points of amplifiers. The purpose of this work was to select the optimal method for searching for the global extremum for the truncation of the signal transmission quality function described above.

The derivation of the truncated transmission quality function used is described in sec. 1 of this paper. The optimization algorithms that were used to find the extremum of this objective function are given in sec. 2. The tasks used to compare the algorithms are described in sec. 3. The results of the comparison and recommendations on the choice of the algorithm are covered in sec. 4

1. FOCL model

The point-to-point topology of the fiber optic network is considered, the scheme of which is shown in Fig.1. It consists of receiving and transmitting devices and spans containing cable sections and equipment between them — amplifiers, equalizers.

The main effects that degrade the quality of signal transmission in coherent FOCL are the noise of amplified spontaneous emission of erbium-doped amplifiers (ASE) and nonlinear signal distortion in the fiber due to the effects of phase self-modulation and cross-modulation, which in the GN model are considered as nonlinear noise. Both types of noise are considered Gaussian and additive in the framework of the GN model, [2], so that the total noise in the line can be represented as the sum of two components:

$$P_{\Sigma} = P_{ASE} + P_{NL}. \tag{1}$$

Then the following inequality is the criterion of line operability

$$P_{\Sigma} \leq P_{\mathrm{cr}},$$

where P_{cr} is the critical noise level at reception in the transceiver, at which demodulation of the received signal is possible. If the left and right sides of the inequality is divided by the signal strength, then it is possible to write the expression (1) in terms of signal-to-noise ratio. It is more convenient to calculate the performance criterion in this form, since it does not require taking into account the evolution of capacities along the communication line:

$$\frac{1}{OSNR_L} + \frac{1}{OSNR_{NL}} \leq \frac{1}{OSNR_{BtB}},$$

where $OSNR_L$ — ratio of signal power to ASE noise power in the 0.1 nm band; $OSNR_{NL}$ — ratio of signal power to nonlinear noise power in the 0.1 nm band; $OSNR_{BtB}$ — the minimum allowable optical signal-to-noise ratio at which demodulation of the signal with a given level of noise is possible.

It is convenient [1] to introduce the concept of the required optical signal-to-noise ratio $OSNR_R$, which sets the minimum optical ratio of signal power to ASE noise power required for signal reception in the presence of nonlinear line noise:

$$\frac{1}{OSNR_R} = \frac{1}{OSNR_{BTB}} - \frac{1}{OSNR_{NL}}. \tag{2}$$

The OSNR margin $(OSNR_M)$ is used for characterizing the excess of the signal transmission quality level over the minimum required value which is defined by the formula

$$OSNR_M = \frac{OSNR_L}{OSNR_R}. \tag{3}$$

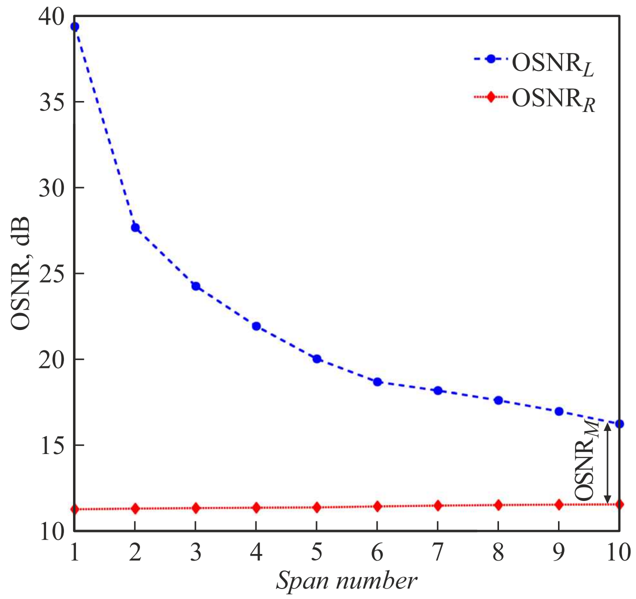

Figure 2. Change of $OSNR_L$ and $OSNR_R$ as the signal propagates over the spans. The difference between them in dB is the value $OSNR_M$.

Figure 2 shows the evolution of the required and linear OSNR in the FOCL. The condition for the operability of the communication line is the absence of intersection of these curves, it can also be written as an inequality

$$OSNR_M \geq 1. \tag{4}$$

Also, a transition to dB is used to determine $OSNR$ then the formula (4) is written as

$$osnr_M = 10\log_{10}(OSNR_M) \geq 0. \tag{5}$$

Thus, the $OSNR$ margin is a metric reflecting the quality of signal transmission, and the optimization of the operation of the fiber optic system can be represented as finding the maximum $OSNR_M$ in case of variation of the parameters of the optical line. It is necessary to know $OSNR_L$, $OSNR_{NL}$ and $OSNR_{BtB}$ to calculate the margin as follows from formulas (2) and (3). Let's take a closer look at the calculation of linear and nonlinear $OSNR$, since $OSNR_{BtB}$ is a characteristic of the transceiver and does not depend on the FOCL parameters.

$OSNR_L$ of a signal for each channel that has passed through several intermediate spans can be calculated using the formula of inverse $OSNR$:

$$\frac{1}{OSNR_L^{\Sigma}} = \sum_i \frac{1}{OSNR_L^i},$$

where $OSNR_L^{\Sigma}$ — total $OSNR_L$, and $OSNR_L^i$ — linear $OSNR$ of the ith span, which by definition is equal to the ratio of signal power P_s and power P_{ASE}, or

$$OSNR_L^i = \frac{P_s}{P_{ASE}} = \frac{G^i P_{in}^i}{(F_{EQ}^i G^i - 1)h\nu\Delta\nu},$$

where G^i — full GF of span, $h\nu\Delta\nu$ — quantum noise power in the band $\Delta\nu$, P_{in}^i — span input power, F_{EQ}^i — equivalent noise-factor of the span. The Friis formula is used to calculate the equivalent channel noise-factor of the entire span

$$F_{EQ} = F_1 + \sum_{i=2}^{N} \frac{F_i - 1}{\Pi_{j=1}^{i-1} G_j},$$

where F_{EQ} — the equivalent noise factor of the entire group of elements, F_i — noise factor of each of the elements and G_j — GF of elements. The noise factor of the amplifier was calculated in this paper using a piecewise function depending on the input power P_{in}^i , this allowed obtaining the noise factor values close to those measured in the experiment.

A modified GNCFF model was used to calculate $OSNR_{NL}$ [8]. The following formula was applied for each span

$$OSNR_{NL}^i = \frac{1}{\eta^i (P_{in}^i)^2},$$

where η^i — the nonlinearity coefficient of the corresponding cable section. The total value for receiving a signal was calculated using the coherent summation formula

$$\frac{1}{OSNR_{NL}^{\Sigma}} = \left[\sum_i \left(\frac{1}{OSNR_{NL}^i}\right)^{\frac{1}{1+\varepsilon}}\right]^{1+\varepsilon},$$

where ε — noise coherence coefficient. This noise calculation model was experimentally verified [9].

$OSNR_L^{\Sigma}$, $OSNR_{NL}^{\Sigma}$ and $OSNR_M$ are functions of the input powers in each span. A special case of the reserve function of the input powers in the case of only two spans is shown in Fig. 3. In general, the dimension of the function can be higher, since it is determined by the number of spans.

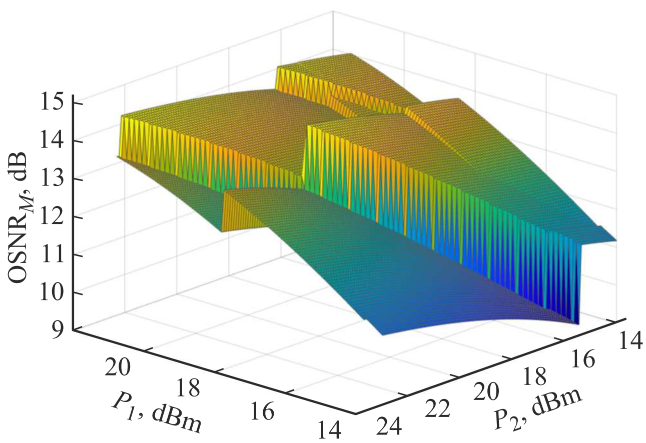

Figure 3. The $OSNR$ margin function for a fiber optic cable containing two fiber spans. P_1 — input power of the first cable section, P_2 — input power of the second section. Gaps in function are associated with the transition to other types of amplifiers due to their limitations in input and output power.

2. Extremum search algorithms

The considered target function of the *OSNR* margin, depending on the input power in the spans, is multidimensional. Each input power value is subject to a limitation related to the permissible output power of the amplifiers. The function gradient is not defined at all points due to the presence of discontinuities in the function, which means that it is desirable to use direct methods for its optimization, that do not use a gradient in calculations. The use of a predefined type of amplifier in each span may be another optimization method, then there will be no gaps in the function definition area. But such a method will require additional iteration of the amplifiers and, in fact, will introduce the need for additional optimization with the amplifier types being the arguments. The first approach allows finding the optimal amplifiers and settings in one optimization, for this reason it was chosen in the study. Thus, the algorithm of multidimensional, conditional, direct optimization is suitable for the previously defined task of optimizing the margin from the input powers of the spans.

Since the task of this study is to choose the optimal optimization algorithm, rather than writing or modifying an existing one, the available implementations of algorithms were used for comparison that satisfied the necessary conditions for the multidimensionality and non-smoothness of the function, the presence of restrictions, as well as, if possible, the search for a global minimum. Algorithms from the SciPy, BayessOpt and HyperOpt, AX packages available for Python were used. 4 algorithms were selected for a more detailed comparison after preliminary comparison of the results for one of the calculation tasks.

The first algorithm from the BayessOpt package is a development of random search, which allows reducing the number of iterations in the search for the optimum — this is the Bayesian optimization algorithm [10]. It is based on a probabilistic model and a data collection function. The first one is used to approximate the optimized problem, which allows reducing the number of calculations of the initial function and, as a result, reducing the time to find the optimum. The value of the calculated function is found at a certain point at each optimization step, and the probabilistic model is updated based on it. Then, the next point is determined using the data collection function for calculating the optimized function. Optimization is completed after reaching the iteration limit, which is set by the user. Several different modifications of the algorithm were used in this paper that differed in the probabilistic model, one from the AX package, and the second from the BayessOpt package.

TPE (Tree-structured Parzen Estimator) is the second algorithm used in this paper [11]. It is also a stochastic direct algorithm, but unlike the previous one, it performs several additional iterations of random search before optimization. They are necessary to form two distributions for „successful“ and „unsuccessful“ points. Some percentage of the best optimization metrics is considered as „successful“ for example, 10% of the sets of capacities at the entrance to all spans for which the best *OSNR* margin is obtained. A new point is chosen by maximizing the expected improvement function, or the ratio of the probability of finding a new point in each of the two distributions. The iteration limit is also the completion criterion.

DIRECT algorithm was one of the last selected algorithms [{]12İt is a deterministic optimization algorithm based on Schubert optimization. The maximum slope coefficient of the function is set, i.e. Lipschitz constant, for the work of the Schubert algorithm, the point of the minimum value of the function is determined using the Lipschitz constant and the values of the function at the boundaries, and the domain of definition is divided into two parts and the algorithm is repeated for each of these parts. The algorithm stops when the area becomes less than the specified value after the division, or the number of iterations does not exceed the critical value.

Table 1. List of test task parameters

№	Length FOCL, km	Lengths of spans, km	Number of spans	Format of modulation
1	1000	100	10	DP-QPSK
2	1000	70–130	10	DP-QPSK
3	1000	70–130	10	DP-QPSK
4	500	55–115	5	DP-16QAM

3. Tasks for testing

It was decided to select several typical designing tasks for multi-span fiber optic cables for comparing the speed and accuracy of optimization algorithms and communication lines with different span lengths, their number and modulation format were considered in these designing tasks. Table 1 shows the parameters of the four tasks used for calculations, the second and third tasks differ only in the distribution of fiber lengths in the spans.

The first configuration is a model line containing only 10 identical spans of 100 km with 20 dB attenuation. The second and third configurations are more close to reality, they have the same total line length, but a random distribution of length between spans, the fiber attenuation coefficient matches the first configuration — 0.2 dB/km. The latter configuration contains five spans with randomly distributed length, but a different modulation format is modeled for the transceivers, DP-16QAM.

Calculations in all tasks were performed for 80 coherent DWDM channels. The maximum span length was 130 km, it was limited by the amplifiers used in the calculation. Equalizers were used to compensate for the effects of waveform distortion due to power transfer and uneven fiber

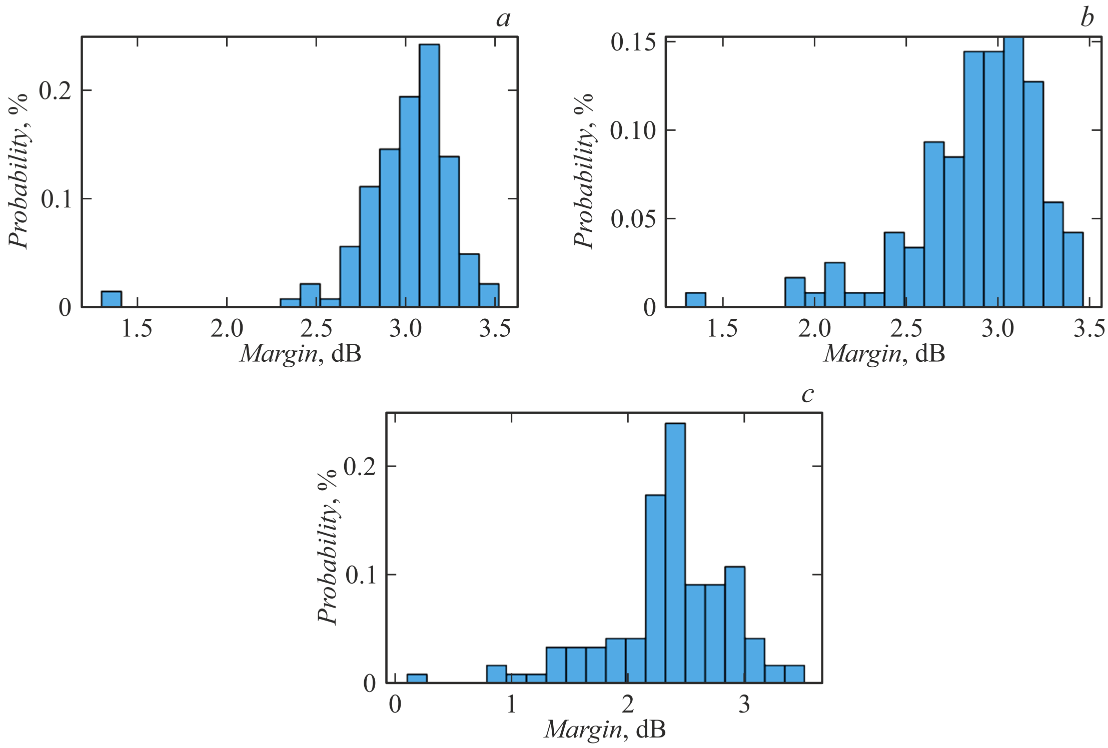

Figure 4. Distribution by margins calculated using various stochastic algorithms for the first problem from Table. 2: *a* — hyperopt, *b* — ax, *c* — bayes-opt.

Table 2. The *OSNR* margin in dB, mean and confidence interval for stochastic algorithms, and value for DIRECT

Task/ Algorithm	Bayesian optimization (bayes-opt)	Bayesian optimization (AX)	TPE (hyperopt)	Direct
1	2.3 ± 1.1	2.9 ± 0.7	3 ± 1	3.7
2	0 ± 2.4	0.4 ± 0.6	0.6 ± 0.3	1.2
3	0.9 ± 1	1.4 ± 2.1	1.6 ± 0.8	2.2
4	0.1 ± 0.5	0.6 ± 0.1	0.4 ± 0.1	0.6

transmission spectrum, which were installed after every third span.

4. Discussion of the results

For each task, the margin of acceptance was calculated by different algorithms in order to compare their applicability and accuracy for different tasks. The DIRECT algorithm is deterministic, so the result of its work is one value of the optimal value, and the rest of the algorithms are stochastic, so the calculation of each of them was carried out 150 times to form a sample of the results.

Table 2 shows the results of comparison of algorithms of optimization of the *OSNR* margin in dB for four different tasks which are described in section 3. Figure 4 shows the probability distributions for the results of calculations of the *OSNR* margin in a problem with 10 spans of equal length using stochastic algorithms.

It can be seen that stochastic algorithms are always inferior to deterministic algorithms on average, but an order of magnitude higher speed of operation is their advantage. The average calculation time by the DIRECT algorithm was 30 min, while stochastic algorithms allowed obtaining result in a minute due to a noticeable reduction of the total number of calculations. The working time is directly related to the total number of calculations of the optimized function, since the calculation time of one iteration is weakly dependent on the input parameters. The criterion for termination of the stochastic algorithm is the completion of the calculation of a fixed number of iterations, so that a flexible adjustment of the required calculation accuracy is possible for them due to the full working time.

The value of the confidence interval weakly depends on the type of task, it was smaller for AX and Hyperopt for tasks with spans of random length, but the largest confidence interval for bayes-opt was obtained for such particular a task. The confidence interval slightly decreases with a decrease of the number of spans, which means that the error of finding the optimum relative to the global minimum for stochastic algorithms may increase with an increase of the number of spans, i.e., an increase of the number of iterations is required for designing longer lines.

Conclusion

Four most effective algorithms were selected and compared for searching the extremum of a function of many variables in the FOCL designing problem. Optimization of the span input power using the DIRECT algorithm made it possible to obtain the largest *OSNR* margin in all the considered problems. The calculation time increases nonlinearly with an increase of the number of spans and is over 30 min for 10 spans. Optimization of power using stochastic algorithms allowed reaching a compromise by reducing the calculation time by an order of magnitude to 1 min, while the degradation of the quality of margin determination was no more than 0.7 dB when taking into account the average value for the TPE algorithm. An increase of the confidence interval of the results was observed with an increase of the number of spans for stochastic algorithms, their applicability for this class of problems is beyond the scope of this study.

Conflict of interest

The authors declare that they have no conflict of interest.

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