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09

Electric pulse area in a layer of a medium with electric conductivity

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An analysis of the transmission and reflection of an extremely short electromagnetic pulse in a layer of a linear medium with electric conductivity has been carried out. It is shown that taking into account the transition boundary layers of the sample does not change the results of considering a problem with sharp boundaries. The conclusions are radically different from those obtained in the unidirectional propagation approximation.

Keywords: extremely short pulses, electric pulse area, electric conductivity.

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Introduction

In connection with the progress in obtaining increasingly shorter laser pulses and the need to reject some approximations that are common in the optics of multicycle radiation, such as the approximation of slowly-varying envelope [1], the issue of developing new approaches to the theoretical description of the maximally short pulses propagation has become relevant. Currently the so called approximation of unidirectional propagation is being widely used [2]. The paper [3] shows that within this approximation the value that is important for the maximally short pulses — electric area of the pulse — is not preserved in the linear media with non-zero conductivity exemplified by plasma. In [4] it is specified that this conclusion is caused specifically by the approximated nature of the unidirectional propagation approach, while within the strict Maxwell equations or the wave equation following from those the electric area will be maintained in those media as well.

A wider selection of the theoretically used approximated approaches for extremely short pulses is analyzed in [5] in respect to the electric area conservation rule. This paper analyzes in more detail the task on the field structure determination in the layer of the linear homogeneous medium that allows for a simple analytical solution.

General relations

The electric area of the pulse is determined in the following manner [6]:

$$\mathbf{S}_E = \int\limits_{-\infty}^{\infty} \mathbf{E}\, dt. \tag{1}$$

Here $\mathbf{E}$ — electric field strength and t — time. Naturally, we assume that the value (1) is finite, since we are interested in the pulses, for which in the fixed point of space the electric field strength is different from zero (apparently exceeds the noise level) only for the finite interval of time. In the monograph [7] and in some subsequent papers this value appears as „integral of the field by time“. Various properties of the electric area of maximally short pulses are summed in the reviews [8–10]. It is essential that the value of the electric area serves as the main criterion for the effectiveness of such pulses interaction with microobjects. Within the plane wave (one-dimensional) approximation, which is used in [4] and below in this paper, from the Maxwell equations it follows that the electric area is conserved for propagation in the non-magnetic media [6]:

$$\frac{d\mathbf{S}_E}{dz} = 0, \tag{2}$$

where z — coordinate along the radiation propagation direction.

To reduce the task to the classic one, we use interpretation of the electric area of the pulse as a zero-frequency spectral component of the field:

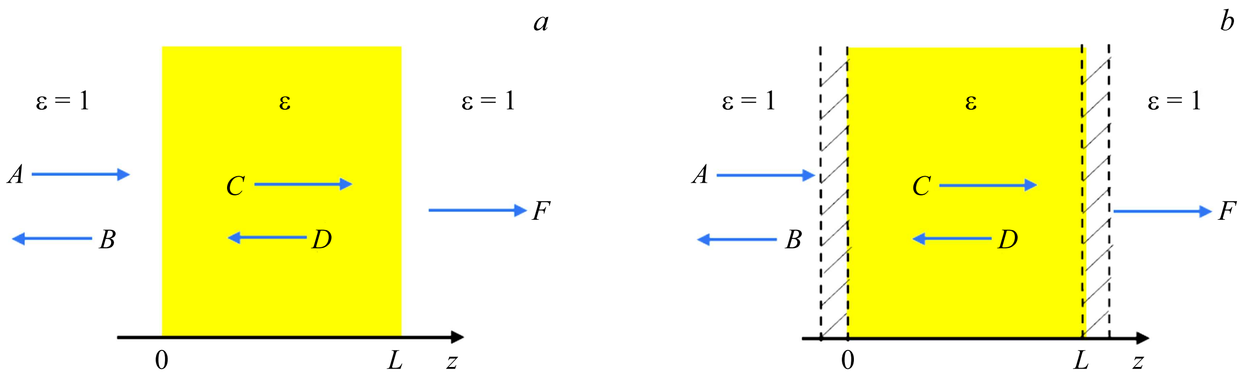
$$\mathbf{S}_E = \lim_{\omega\to 0} \mathbf{S}_\omega, \tag{3}$$

where

$$\mathbf{S}_\omega = \int\limits_{-\infty}^{\infty} \mathbf{E}\exp(-i\omega t)dt$$

— Fourier component of the electric field strength.

Let us consider the reflection and transmission of the plane wave with frequency ω, which is normally incident from the vacuum to the medium layer with (complex) refraction index ε (Fig., a). Omitting the time factor



Transmission and reflection of radiation in the medium layer with sharp boundaries (a) and with the boundary transition present (b, which are dashed).

$\exp(-i\omega t)$, the real part sign, and a single vector indicating the direction of the linear polarization of radiation, let us record the solutions to the wave equation (Helmholtz equation)

$$\frac{d^2E}{dz^2} + k^2(z)E = 0 \quad (4)$$

for distribution of the electric field strength in the following form

$$z < 0: \; E = A\exp(ik_0z) + B\exp(-ik_0z),$$

$$0 < z < L: \; E = C\exp(ikz) + D\exp(-ikz),$$

$$z > L: \; E = F\exp[ik_0(z - L)]. \quad (5)$$

Here $k(z) = k_0 = \omega/c$ at $z < 0$ and $z > L$ and $k(z) = k = k_0\sqrt{\varepsilon}$ at $0 < z < L$ (the root branch is chosen based on the requirement of decay of the amplitude of forward wave C upon increase of z). The conditions of continuity E and dE/dz at the interfaces result in the ratios

$$\frac{C}{A} = \frac{2\frac{k_0}{k}(1 + \frac{k_0}{k})}{(1 + \frac{k_0}{k})^2 - (1 - \frac{k_0}{k})^2\exp(2ikL)},$$

$$\frac{D}{A} = \frac{2\frac{k_0}{k}(1 - \frac{k_0}{k})}{(1 + \frac{k_0}{k})^2\exp(-2ikL) - (1 - \frac{k_0}{k})^2},$$

$$\frac{F}{A} = C\exp(ikL) + D\exp(-ikL),$$

$$\frac{B}{A} = -1 + \frac{C}{A} + \frac{D}{A}. \quad (6)$$

Electric area

In the limit $\omega \to 0$ the coefficients A, B and F change into the electric area of accordingly the forward and backward transmitted pulses; with a certain convention one can also assume that C and D are the electric areas of forward and backward transmitted pulses within the layer. Due to linearity of the task, without the loss of generality, one may assume that $A = 1$. Then B represents the amplitude coefficient of reflection, and F — the amplitude coefficient of transmission.

For a dielectric the dielectric permittivity at zero frequency is equal to its (finite) static value ε_0. From (6) at $\omega \to 0$ we get

$$C = \frac{1}{2}\left(1 + \frac{1}{\sqrt{\varepsilon_0}}\right),$$

$$D = \frac{1}{2}\left(1 - \frac{1}{\sqrt{\varepsilon_0}}\right), \; B = 0, \; F = 1. \quad (7)$$

Therefore, the electric area everywhere coincides with the area of the incident pulse. For the electric area the layer is completely transmissive, there is no reflection. This is compliant with the more general conclusion [11].

In case of the medium with the electric conductivity we will use the Drude model

$$\varepsilon(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}, \quad (8)$$

where ω_p — plasma frequency and γ — decay index. In the limit of $\omega \to 0$ the dielectric permittivity (8) has singularity: $\varepsilon(\omega) \approx \frac{i\omega_p^2}{\gamma\omega}$. But the wave number in such medium is $k = k_0\sqrt{\varepsilon} \approx \frac{q}{c}\sqrt{\omega}$, where $q = \sqrt{i\frac{\omega_p^2}{\gamma}}$. In this case using (6) we find

$$C = D = \frac{1}{2 + \frac{\omega_p^2 L}{c\gamma}},$$

$$F = \frac{1}{1 + \frac{\omega_p^2 L}{2c\gamma}},$$

$$B = -\left(1 - \frac{1}{1 + \frac{\omega_p^2 L}{2c\gamma}}\right). \quad (9)$$

Therefore, if there are free charges, the electric area differs from the area of the incident pulse. In the definition (1) it is constant everywhere in accordance with the general rule of its conservation, without changes within the layer. Besides, inside the layer the areas of the forward and backward transmitted waves determined as specified above do not depend on the longitudinal coordinate z and are equal to each other.

Role of transition layers

The model of the stepwise variation of dielectric permittivity at the boundaries of the layer is idealization. In the more accurate model such boundaries are replaced with additional layers with smooth variation of dielectric permittivity between its boundaries values (Fig., *b*). The impact of such layers may be taken into account in the perturbation theory [12]; there is a known series of dependences $\varepsilon(z)$, for which the analytic solutions are available for equation (4), including the Rayleigh layer, $\varepsilon(z) = a(\omega)/z^2$ [13].

The analysis shows that the transition layers do not change the results of the model of stepwise variation of dielectric permittivity, if their thickness l is essentially smaller than the corresponding length of the radiation mode. Since the electric area of the pulse complies with the zero frequency and therefore with the infinitely large wave length $\lambda = \infty$, the presence of the transition layers will not in any way impact the coefficients of reflection and transmission given in the previous section. This may easily be confirmed for the Rayleigh layer model. The criterion here would be the unitless value [13]

$$p = \left(\frac{2\pi l}{\lambda}\right)^2 \left|\frac{\sqrt{\varepsilon}}{\sqrt{\varepsilon}-1}\right|^2. \qquad (10)$$

In the case of our interest $p = 0$, which justifies the model of stepwise variation of dielectric permittivity.

Conclusion

In this paper within the plane wave (unidimensional) approximation the electric area is found for the pulses of radiation that was reflected and transmitted through the layer of the homogeneous medium with electric conductivity if there are the transition near boundary layers of the medium available. The conclusions are essentially different from the predictions of the unidirectional propagation approximation. Besides, the plane wave approximation may be justified in respect to propagation of pulses in coaxial waveguides [14]. Other options for justification of the results applicability, such as replacement of the boundaries with inclined or scattering surface, requires going beyond the one-dimensional geometry and are not considered here.

The importance of the analytical nature of the conclusions is emphasized by the fact that the numerical calculations of the maximally short pulses are complicated with the potential availability of their hard-to-account for fronts. Therefore, the numerical simulation, for example in [15], may mean the existence of a pronounced unipolar pulse accompanied with the extended front of opposite polarity and small amplitude. Such conclusion is compliant with the result of [11] and at the same time is not lessening the importance of calculations of [15], since such front will not impact the effectiveness of the pulse impact at the microobjects [9].

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Conflict of interest

The authors declare that they have no conflict of interest.

References

[1] M.B. Vinogradova, O.V. Rudenko, A.P. Sukhorukov. *Teoriya voln* (Nauka, M., 1979). (in Russian)
[2] M. Kolesik, J.V. Moloney. Phys. Rev. E, **70**, 036604 (2004).
[3] A.V. Bogatskaya, A.M. Popov. Pisma v ZhETF, **118**, 291–296 (2023). (in Russian)
[4] N.N. Rosanov, M.V. Arkhipov, R.M. Arkhipov, A.V. Pakhomov. JETP Lett., **118**, 608–609 (2023).
[5] A.V. Pakhomov, N.N. Rosanov, M.V. Arkhipov, R.M. Arkhipov, JETP Lett., **119**, 94–103 (2024).
[6] N.N. Rozanov. Opt. Spectrosc., **107**, 721 (2009).
[7] J.D. Jackson. *Classical Electrodynamics* (J. Wiley, New York, 1962).
[8] N.N. Rosanov, R.M. Arkhipov, M.V. Arkhipov. Phys. Usp., **61**, 1227 (2018).
[9] R.M. Arkhipov, M.V. Arkhipov, N.N. Rosanov. Quant. Electron., **50**, 801 (2020).
[10] N.N. Rosanov. Physics — Uspekhi, **66** (10), 1059–1064 (2023).
[11] M.V. Arkhipov, R.M. Arkhipov, A.V. Pakhomov, I.V. Babushkin, A. Demircan, U. Morgner, N.N. Rosanov. Opt. Lett., **42**, 2189 (2017).
[12] N.N. Rozanov, V.M. Zolotarev. Opt. Spectrosc., **49**, 506 (1980).
[13] S.G. Rautian. *Vvedenie v fizicheskuyu optiku* (Knizhny dom „LIBROKOM“, M., 2009).
[14] N.N. Rosanov. Opt. Spectrosc., **127**, 1050 (2019).
[15] V.V. Kozlov, N.N. Rosanov, C. De Angelis, S. Wabnitz. Phys. Rev. A, **84**, 023818 (2011).

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