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# Optimization of vertical acceptance of a magnet mirror 

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The mirror type magnetic mass-analyzer axial aberration, caused by ions travelling through the mirror outside the median plane is estimated. A technique for the mass-analyzer acceptance optimization is given.

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A magnetic mirror (MM) is used in various analytical instruments and devices. Dempster was the first to apply an MM in mass spectrometry [1], paving the way for significant discoveries in isotope chemistry. An ion beam in Dempsters mass spectrometers was introduced orthogonally to an MM. It has been demonstrated in later studies that the introduction of an ion beam at an oblique angle to an MM allows one to focus the beam tighter, reduce the thirdorder spherical aberration coefficient, and shift the focus line (FL) out the magnetic field $[2,3]$. The contribution of ions travelling outside the median MM plane to the beam width was neglected. At the same time, it was found in the analysis of this contribution (let us call it „axial aberration") in sector magnetic mass analyzers [4] that the indicated aberration may be significant. In practice, it is suppressed to achieve the needed resolving power by collimating an ion beam with the exit slit of an ion source and the receiving slit of a detector. This has a marked negative effect on the sensitivity of a mass spectrometer.

In the present study, we analyze axial aberration of a magnetic mirror with a uniform field under the conditions of second-order focusing and show how the collimating MM system may be optimized with the aim of minimizing the ion current loss in the process of collimation that is performed to achieve the needed resolving power.

Various aspects of focusing of ions in the median plane of an MM with a uniform field were discussed in sufficient detail in [3]. There is a unique correspondence between angles of rotation $\varphi$ of axial orbits of single-mass components of an ion beam introduced into the magnetic field, which become separated in the MM field, and angle $\varepsilon$ of the beam introduction into an MM:

$$
\begin{equation*}
\varphi+2 \varepsilon \equiv \pi . \tag{1}
\end{equation*}
$$

Therefore, with the exit slit of an ion source located at distance $l_{1}$ from an MM measured along the optical axis, first-order focusing of single-mass components, which have axial orbits shaped as arcs of a circle with radius $r$ in the MM field, is achieved after they leave the MM at distance
$l_{2}(r)$ (formula (6) in [5]):

$$
\begin{equation*}
l_{2}(r)=r \sin (\varphi)-l_{1} . \tag{2}
\end{equation*}
$$

Distance $l_{2}(r)$ is measured along the corresponding continuation of an axial orbit (i.e., the exit arm of the optical axis of a single-mass component; see Fig. 1). Calculations of broadening $\delta(\alpha)$ of the cross section of single-mass components on the FL, which is induced by angular spread $\alpha$ of escape directions of ions relative to the optical axis in the median MM plane, in the third-order approximation in $\alpha$ yield the following result [3]:

$$
\begin{gather*}
\delta(\alpha)=\delta_{\alpha \alpha}+\delta_{\alpha \alpha \alpha},  \tag{3}\\
\delta_{\alpha \alpha}=\left[\left(2 \tan ^{2} \varepsilon-1\right) r\right] \alpha^{2},  \tag{4}\\
\delta_{\alpha \alpha \alpha}(r)=2\left[\sin (2 \varepsilon)\left(\tan ^{2} \varepsilon+1 / 2\right) r\right] \alpha^{3} . \tag{5}
\end{gather*}
$$

With the angle of beam introduction into an MM being

$$
\begin{equation*}
\varepsilon=\arctan (1 / \sqrt{2}) \tag{6}
\end{equation*}
$$

the first term in (3) - second-order aberration contribution - vanishes (second-order focusing), and the size of the cross section of single-mass components on the FL under conditions (6) depends cubically on $\alpha$ :

$$
\begin{equation*}
\delta_{\alpha \alpha \alpha}(r)=2\left(l_{1}+l_{2}(r)\right) \alpha^{3}=\frac{4 \sqrt{2}}{3} r \alpha^{3} . \tag{7}
\end{equation*}
$$

With aberration (5) taken into account, the coefficient of dispersion in mass number $\left(X_{m}\right)$ and the resolving power of a mass analyzer ( $R s$ ) are

$$
\begin{equation*}
X_{m}=\frac{2}{3} r, \quad R s=1 /\left(\frac{3}{2} \frac{S_{0}}{r}+\sqrt{8} \alpha^{3}\right) \tag{8}
\end{equation*}
$$

where $S_{0}$ is the width of the exit slit of an ion source. Note that the angle of rotation of single-mass components and the exit arm of their focusing under conditions (6) are

$$
\begin{gather*}
\varphi=2 \arctan (\sqrt{2}) .  \tag{9}\\
l_{2}(r)=(\sqrt{8} / 3) r-l_{1} \tag{10}
\end{gather*}
$$



Figure 1. Geometry of a magnetic mass analyzer of the magnetic mirror type. 1 - Axial orbits of single-mass components of an ion beam; $\psi$ - tilt angle of the focus line.

In a second-order approximation, the contribution of ions travelling through a mass analyzer outside of the median plane is characterized by quadratic trinomial [4]

$$
\begin{equation*}
X(b, y)=(X, b b) b^{2}+(X, b y) b y+(X, y y) y^{2}, \tag{11}
\end{equation*}
$$

where $(X, b b),(X, b y)$, and $(X, y y)$ are matrix aberration coefficients. Their dimension is specified by the following: function $X(b, y)$ has a dimension of $\mu \mathrm{m}, b$ are the escape angles of ions in mrad, and $y$ are the coordinates of their escape in mm in the direction perpendicular to the median MM plane (vertical direction). In the case of an MM with a uniform field, all the calculated matrix coefficients in (11) are of the same sign, and discriminant $D$ is positive. This implies that levels $X(b, y)=S$ take the form of ellipses

$$
\begin{equation*}
(X, b b) b^{2}+(X, b y) b y+(X, y y) y^{2}=S \tag{12}
\end{equation*}
$$

It is known from the transport theory [6] that any slitshaped aperture blocking the path of ions is represented by an oblique band in the $(b, y)$ coordinate plane. The tilt angle of this band is specified by the position of the aperture, and the band width depends on the slit size. Therefore, a set of $N$ slit-shaped apertures collimating an ion beam is rendered as a $2 N$-gon, which represents the vertical acceptance of a mass analyzer, in the ( $b, y$ ) plane (Fig. 2). Thus, in geometric terms, the problem of maximizing the transmission of a mass analyzer with a fixed magnitude of axial aberration $S$ boils down to finding this 2 N -gon of the maximum area under condition

$$
\begin{equation*}
(X, b b) b^{2}+(X, b y) b y+(X, y y) y^{2} \leqslant S \tag{13}
\end{equation*}
$$

This problem was formulated and discussed in detail in [7]. Let us consider an example collimating system of four slitshaped apertures that are positioned at the output of an ion


Figure 2. Aberration ellipse and a phase octagon inscribed into it (shaded area).
source, at the entrance and exit MM boundaries, and on the FL. The passage of ions through aperture $k$ with slit height $2 h_{k}(k=1,2,3,4)$ is represented by inequality

$$
\begin{equation*}
\left|e_{k} b+f_{k} y\right| \leqslant h_{k}, \tag{14}
\end{equation*}
$$

where coefficients $e_{k}$ (in mm ) and $f_{k}$ (dimensionless quantity) are specified by the positioning of aperture $k$, while their ratio defines the tilt of the corresponding band with respect to axis $b$ in plane $(b, y)$. The vertices of an octagon corresponding to the chosen collimating system are defined by the set of $h_{k}(k=1-4)$ values.

To perform specific calculations, we set the angle of introduction of an ion beam into an MM to
$\varepsilon=\arctan (1 / \sqrt{2})=35.26^{\circ}$ to the normal and assume that $l_{1}=20 \mathrm{~mm}$ and $r=120 \mathrm{~mm}$. Axial aberration coefficients (11) calculated with these values are as follows:

$$
\begin{gather*}
(X, b b)=0.374 \mu \mathrm{~m} / \mathrm{mrad}^{2},(X, b y)=6.38 \mu \mathrm{~m} / \mathrm{mm} \cdot \mathrm{mrad} \\
(X, y y)=32.1 \mu \mathrm{~m} / \mathrm{mm}^{2} \tag{15}
\end{gather*}
$$

Let us estimate the beam emittance parameters that would correspond to the maximum acceptance of a mass analyzer with resolving power $R s=1000$. Since dispersion coefficient $X_{m}=80 \mathrm{~mm}$ at $r=120 \mathrm{~mm}$ (i.e., $X_{m} / R s=80 \mu \mathrm{~m}$ ), the ion-beam width at focus for the chosen $R s$ value should not exceed

$$
\begin{equation*}
S_{0}+\delta_{\alpha \alpha \alpha}+S=80 \mu \mathrm{~m} \tag{16}
\end{equation*}
$$

In view of the cubic dependence of $\delta_{\alpha \alpha \alpha}$ on the initial angular spread $\alpha$ of ions in the horizontal direction and the linear dependence of the vertical acceptance of a mass analyzer on $S$ [7], the value of product $\left[\alpha \cdot S_{0} \cdot S\right.$ ] under conditions (14) is, as is easy to demonstrate using the variational Lagrangian method, maximized at

$$
\begin{equation*}
2 \alpha=0.074\left(\approx 4^{\circ}\right), \quad S_{0}=S=35 \mu \mathrm{~m} \tag{17}
\end{equation*}
$$

Maximizing octagon area (14) under condition (13) with coefficients (15) and $S=35 \mu \mathrm{~m}$, we find the following resulting $\left\{h_{k}\right\}(k=1-4)$ values:

$$
\begin{equation*}
h_{1}=1.80 \mathrm{~mm}, h_{2}=1.64 \mathrm{~mm}, h_{3}=2.00 \mathrm{~mm}, h_{4}=3.85 \mathrm{~mm}, \tag{18}
\end{equation*}
$$

which yield a vertical acceptance of $58.1 \mathrm{~mm} \cdot \mathrm{mrad}$. This corresponds to $70 \%$ of the area of ellipse (12) at $S=35 \mu \mathrm{~m}$.

Let us compare the obtained acceptance value to the acceptance of a double-slit system that is commonly used in practice to suppress axial aberration and consists of an entrance slit of a source with half-height $h_{1}$ and a receiving slit of a detector with half-height $h_{4}$. The indicated parameters $h_{1}$ and $h_{4}$ are calculated in accordance with the same procedure. With the maximum admissible aberration $S=35 \mu \mathrm{~m}$, calculations yield the following values:

$$
\begin{equation*}
h_{1}=1.67 \mathrm{~mm}, \quad h_{4}=3.43 \mathrm{~mm} \tag{19}
\end{equation*}
$$

which correspond to a vertical acceptance of $43.1 \mathrm{~mm} \cdot \mathrm{mrad}(52 \%$ of the aberration ellipse area at $S=35 \mu \mathrm{~m})$.

Thus, compared to an optimum double-slit collimating system, an optimum four-slit collimating MM system with resolving power $R s=1000$ provides an opportunity to raise the transmission of the considered mass analyzer by $35 \%$. It is important to note that the discussed method for collimating system optimization may be used to estimate the maximum transmission of a mass analyzer, which may serve as an objective measure of efficiency of a collimating system at any given mass resolution.

## Conflict of interest

The authors declare that they have no conflict of interest.

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