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Interband one photon absorption of light and its linear-circular dichroism in crystals with coherent saturation effect included

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The polarization, spectral and temperature dependences of the single-photon absorption coefficient of polarized radiation are calculated, and its linear-circular dichroism in crystals of tetrahedral symmetry is researched. In this case, the contribution to the coefficients of one- and two-photon absorption of light from the effect of coherent saturation of optical transitions is taken into account.

Keywords: polarization, spectral and temperature dependences of the single-photon light absorption coefficient, linear-circular dichroism, crystal of tetrahedral symmetry, coherent saturation effect.

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1. Introduction

Nonlinear light absorption in a semiconductor with a degenerate valence band caused by direct optical transitions between sub-bands of heavy and light holes and dependent on radiation polarization condition, was examined in [1–8]. These studies consider that nonlinearity in the dependence of the single-photon absorption coefficient on intensity occurs due to resonance absorption saturation. This saturation is caused by photo-induced variation of light and heavy hole distribution functions in the pulse space near the surface $E_{hh}(\mathbf{k}) - E_{hl}(\mathbf{k}) - \hbar\omega = 0$ corresponding to the resonance condition. Here $E_{hh}(\mathbf{k})(E_{hl}(\mathbf{k}))$ is the energy range of heavy (light) holes, ω is the light frequency.

In [8], the multi-photon linear-circular dichroism (LCD) was examined in p-Ge in the developed nonlinearity mode, when n-photon processes with $n = (1 \div 5)$ make the comparable contribution to absorption. The four-photon processes in semiconductors caused by optical transitions between sub-bands of the valence band were examined in [9,10]. In [11], theory of light absorption and circular photocurrent in Weyl semimetals was developed for arbitrary large light intensities and the direct optical transition rate was shown to be saturated at high intensity with the saturation behavior depending on light polarization and relation of elastic and inelastic relaxation times. Study [12] reported nonlinear dependence on the intensity of polarization-dependent photocurrents induced by direct optical transitions between topological surface and bulk states in topological insulators, [13,14] investigated dynamic conductivity of graphene, and [15] calculated the dynamic interband optical conductivity of an arbitrary electronic system in the Rabi oscillation mode. Microscopic theory of nonlinear-intensity boundary photocurrent in graphene induced by terahertz radiation is developed in [16].

However, the interband single-photon linear-circular dichroism in crystals with tetrahedral symmetry considering the coherent saturation effect remained open and is discussed herein.

Single-photon linear-circular dichroism of polarized radiation absorption will be addressed herein taking into account the coherent saturation effect [3,4] in direct-band crystals caused by direct optical transitions between sub-bands of the valence band and conduction band. For calculations of the interband single-band light absorption, the photon energy will be considered to satisfy the condition $\hbar\omega \geq E_g$, $E_g + \Delta_{SO}$, where E_g is the band gap, Δ_{SO} is the spin-orbit splitting of the valence band.

Due to the smallness of the photon wave vector compared with the electron (hole) wave vector formed as a result of absorption, $|\mathbf{q}| \ll |\mathbf{k}|$ and $|\mathbf{q}| = 0$ may be assumed for the calculation, where $\mathbf{q}(\mathbf{k})$ is the photon (hole) wave vector.

2. Polarized dependences of single-photon interband linear-circular dichroisms

In case of $\hbar\omega \leq E_g$, $E_g + \Delta_{SO}$, two interband optical transitions are available, $E_g \geq \hbar\omega\langle E_q + \Delta_{SO}$ satisfies to the former, and $\hbar\omega \geq E_g + \Delta_{SO}$ satisfies to the latter. Therefore, optical transitions between the light and heavy hole sub-bands of the valence band and conduction band occur in the first case, and optical transitions occur between the spin-orbit splitting and conduction bands in the second case which will be reviewed separately.

a) Let the initial states are in the heavy hole sub-band of the valence band, then in the Luttinger–Kohn and Kane approximation [11,12], the matrix element of the single-phase optical transition from the heavy hole sub-band $|V, \pm 3/2\rangle$ to the conduction band $|c, \pm 1/2\rangle$, i.e. $M^{(1)}_{\mathrm{C},\pm 1/2;\pm \mathrm{V},3/2}$, which is schematically shown as $|V, \pm 3/2\rangle \to |c, \pm 1/2\rangle$, is defined by the relations:

$$M^{(1)}_{\mathrm{C},+1/2;\mathrm{V}+3/2} = \left(\frac{eA_0}{c\hbar}\right) p_{cV} e'_{+},$$

$$M^{(1)}_{\mathrm{C},-1/2;\mathrm{V}-3/2} = -i\left(\frac{eA_0}{c\hbar}\right) p_{cV} e'_{-},$$

and $|V, \pm 3/2\rangle \to |c, \mp 1/2\rangle$ type optical transition is prohibited, where $e'_{\pm} = e'_x \pm i e'_y$, $e'_\alpha (\alpha = x, y, z)$ are light polarization vector projections with respect to the coordinates whose axis Oz is directed along the wave vector of photo-excited current carriers ($\mathbf{k}$), A_0 is the electromagnetic wave potential vector amplitude, p_{cV} is the Kane parameter [11,12], the rest are generally known varibales. The law of conservation of energy of this transition is described by $\delta\big(E_c(\mathbf{k}) - E_{hh}(\mathbf{k}) - \hbar\omega\big)$ function, where $E_c(\mathbf{k})$ is the electron energy spectrum in the conduction band, $E_L(\mathbf{k}) = \frac{\hbar^2 k^2}{2m_L}$ is the hole energy spectrum in the light hole sub-band $(L = lh)$ and heavy hole sub-band $(L = hh)$, $m_c(m_L)$ are effective masss of current carriers in the conduction band and valence band.

Using the latter relations, polarization dependence of the probabilities of the considered optical transitions can be derived by equation

$$W^{(1)} = \frac{2\pi}{\hbar} \sum_{k,l';l} \left(f^{(0)}_{lk} - f^{(0)}_{l''k}\right) \times \frac{\left|M^{(1)}_{l'\mathbf{k};l\mathbf{k}}(\mathbf{k})\right|^2}{\sqrt{1 + 4\frac{\alpha_\omega}{\hbar^2\omega^2}\left|M^{(1)}_{l'\mathbf{k};l\mathbf{k}}(\mathbf{k})\right|^2}} \delta\big(E_{i'k} - E_{ik} - \hbar\omega\big), \tag{1}$$

where

$$\alpha_\omega = 6\omega^2 T^{(1)}_1 T^{(1)}_2 \frac{I}{I_0}, \quad I_o = \frac{c n_\omega \hbar^3 \omega^3}{2\pi |B|}.$$

For example, for p-GaAs and $I_0 = 13420\,\frac{\mathrm{kWt}}{\mathrm{cm}^2}$ at $\hbar\omega = 17$ meV, $m_2 = 0.045 m_0$. $M^{(1)}_{l'\mathbf{k};i\mathbf{k}}(\mathbf{k})$ is the matrix element of the single-photon optical transition from state $|l\mathbf{k}\rangle$ to state $|l'\mathbf{k}\rangle$, $\mathbf{k}$ is the wave vector of holes, $I = \frac{n_\omega \omega^2 A_0^2}{2\pi c}(A_0)$ is the light intensity (potential vector amplitude), $E_{l\mathbf{k}}$ is the current carrier energy spectrum in the sub-band l, $f^{(0)}_{lk}$ are their equilibrium distribution function, n_ω is the refraction index at frequency ω, $\hbar\omega$ is the photon energy. The rest variables are commonly known. Here, states with angular momentum projection $m = \pm 3/2$ on direction $\mathbf{k}$ correspond to the electrons in sub-band $l = 1$ (heavy holes), and states with $m = \pm 1/2$ correspond to the electrons in the sub-band $l = 2$ (light holes).

In particular, for $|\mathrm{V}, \pm 3/2\rangle \to |\mathrm{C}, \pm 1/2\rangle$ type optical transitions, the polarization dependence of the probability of this transition on the coherent saturation effect parameter

$$\xi_\omega = 4\frac{\alpha_\omega}{\hbar^2\omega^2}\left(\frac{eA_0}{c\hbar}\right)^2 p^2_{cV},$$

defined by the polarization dependence

$$\left|M^{(1)}_{\mathrm{C},\pm 1/2;\mathrm{V},\pm 3/2}\right|^2 = \left(\frac{eA_0}{c\hbar}\right)^2 p^2 |e'_{\pm}|^2$$

and calculated by equation (1)[1] is of the oscillatory type with respect to the angle between the polarization vector and current carrier wave vector for the fixed parameter value ξ_ω both for the linear and circular polarization. In this case, the interband linear-circular dichroism coefficient defined as the relation of probabilities of the optical transitions in the linear and circular polarization is equal to 1, i.e. the linear-circular dichroism is not observed, and is almost independent on ξ_ω.

b) If the initial states are in the light hole subband of the valence band, then the matrix element of the single-photon optical transition from the light hole sub-band $|V, m\rangle (m \pm 1/2)$ to the conduction band $|c, m'\rangle \big(m' = \pm 1/2\big)$, i.e. $M^{(1)}_{\mathrm{C,m';V,m}}$, which is schematically shown as $|V, m\rangle \to |c, m'\rangle$, is defined by the relations

$$M^{(1)}_{c,+1/2;\mathrm{V},+1/2} = \left(\frac{eA_0}{c\hbar}\right) \frac{1}{\sqrt{3}} p_{cV} e'_{-},$$

$$M^{(1)}_{c,-1/2;\mathrm{V},-1/2} = \left(\frac{eA_0}{c\hbar}\right) \frac{-i}{\sqrt{3}} e'_{+} p_{cV},$$

$$M^{(1)}_{c,+1/2;\mathrm{V},-1/2} = \left(\frac{eA_0}{c\hbar}\right) \frac{1}{\sqrt{3}} e'_z p_{cV},$$

$$M^{(1)}_{c,+1/2;\mathrm{V},-1/2} = \left(\frac{eA_0}{c\hbar}\right) i\sqrt{\frac{1}{3}}\, e'_z p_{cV}.$$

Therefore, the squared absolute value of the matrix elements of the optical transitions of interest is written as

$$\left|M^{(1)}_{c,\pm 1/2;\mathrm{V},\pm 1/2}\right|^2 = \left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{3} p^2_{cV} \left|e'_{\mp}\right|^2,$$

$$\left|M^{(1)}_{c,\mp 1/2;\mathrm{V},\pm 1/2}\right|^2 = \left(\frac{eA_0}{c\hbar}\right)^2 \frac{2}{3} p^2_{cV} \left|e'_z\right|^2.$$

The law of conservation of energy of these transitions is described by $\delta\big(E_c(\mathbf{k}) - E_{lh}(\mathbf{k}) - \hbar\omega\big)$ function. Then the wave vector of photo-excited current carriers is defined by the relation:

$$k^{(1\omega)}_{c,lh} = \sqrt{\frac{2\mu^{(c,lh)}_{+}}{\hbar^2}(\hbar\omega - E_g)},$$

where

$$\mu^{c,lh}_{+} = \frac{m_c m_{lh}}{m_c + m_{lh}}$$

is the reduced effective mass with respect to the effective mass of electrons in the conduction band and light hole band.

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[1] For simplicity in the calculations of the probabilities of optical transitions, it was assumed in paragraphs a), b) and c) that the initial state of the current carriers is fully filled, and the final state is empty (according to „the golden rule of quantum mechanics").

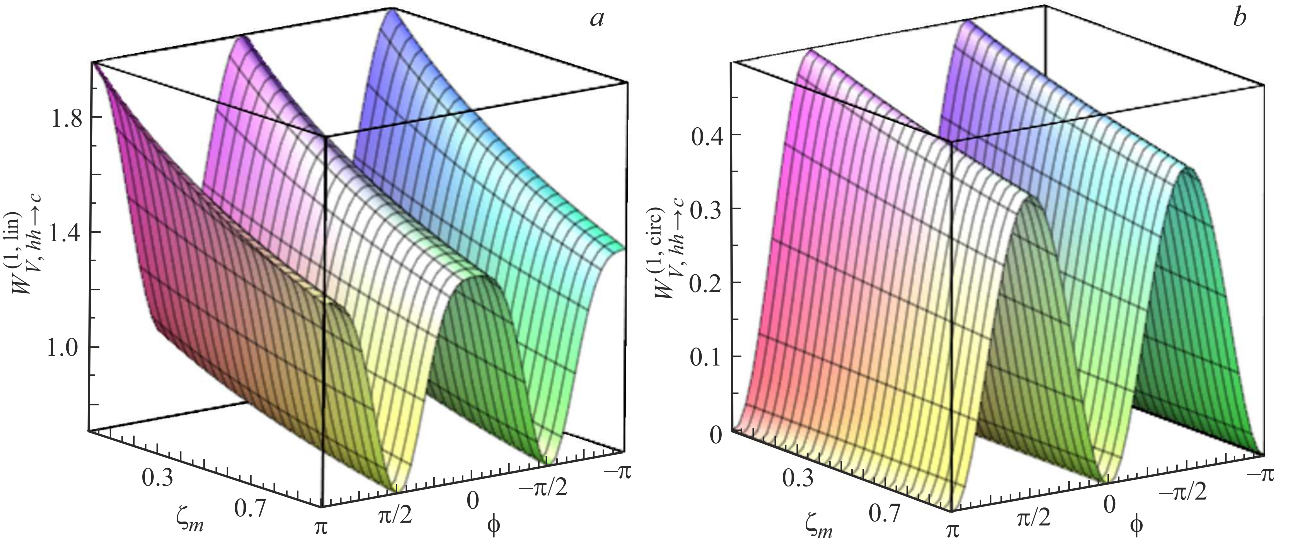

Figure 1. The polarization dependence of the probability for $|V, hh\rangle \to |C\rangle$ type optical transitions for linearly-polarized (*a*) and circularly-polarized (*b*) light depending on the coherent saturation effect parameter.

Considering the polarization dependence of matrix elements $M^{(1)}_{c,\pm1/2;\mathrm{V},\pm1/2}$ and $M^{(1)}_{c,\mp1/2;\mathrm{V},\pm1/2}$ for $|\mathrm{V}, \pm1/2\rangle \to |\mathrm{C}, \pm1/2\rangle$ and $|\mathrm{V}, \pm1/2\rangle \to |\mathrm{C}, \mp1/2\rangle$ type optical transitions, the polarization dependence of the probability of this transition calculated by (1) can be determined depending on ξ_ω, which is shown in Figure 1, *a*. Figure 1, *b* shows that the polarization dependence of the probability of the considered optical transition both for linear and circular polarization is of the oscillatory type with respect to the angle between the polarization vectors and current carrier wave vector, but with growing ξ_ω , the oscillation amplitude decreases: by 20% for the linear polarization and by 15% for the circular polarization.

Calculations performed using equation (1) show that the polarization dependence of the single-photon linear-circular dichroism coefficient for $|\mathrm{V}, hh\rangle \to |\mathrm{C}\rangle$ type optical transitions is also of the oscillatory type with respect to the angle between the polarization vectors and current carrier wave vector, whose amplitude value is almost independent of the coherent saturation effect parameter ξ_ω. The optical transition probability during linearly-polarized light absorption is approximately five times higher than the optical transition probability during circular-polarized light absorption. The latter is explained by the dependence of the selection rule of the optical transition of interest on the light polarization degree.

c) If the initial states are in the spin-split band, then the matrix elements of the single-photon optical transitions $M^{(1)}_{\mathrm{C,m';}SO,\mathrm{m}}$, which are written as $|SO, m\rangle \to |c, m'\rangle$, are determined by the relations

$$M^{(1)}_{\mathrm{C},+1/2;\mathrm{SO},+1/2} = \left(\frac{eA_0}{c\hbar}\right)\frac{1}{\sqrt{3}}\, p_{cV} e'_z,$$

$$M^{(1)}_{\mathrm{C},-1/2;\mathrm{SO},+1/2} = \left(\frac{eA_0}{c\hbar}\right)\frac{1}{\sqrt{3}}\, p_{cV} e'_-,$$

$$M^{(1)}_{\mathrm{C},+1/2;V,-1/2} = \left(\frac{eA_0}{c\hbar}\right)\frac{1}{\sqrt{3}}\, e'_z p_{cV},$$

$$M^{(1)}_{\mathrm{C},-1/2;\mathrm{SO},-1/2} = \left(\frac{eA_0}{c\hbar}\right)\frac{i}{\sqrt{3}}\, p_{cV} e'_z.$$

The law of conservation of energy for these transitions is described by $\delta\big(E_c(\mathbf{k}) - E_{SO}(\mathbf{k}) - \hbar\omega\big)$ function, where $E_{SO}(\mathbf{k}) = \frac{\hbar^2 k^2}{2m_c} + \Delta_{SO}$ is the hole energy spectrum in the spin-orbit splitting band, Δ_{SO} is the spin-orbit splitting energy. Whence we have

$$\left|M^{(1)}_{\mathrm{C},\pm1/2;\mathrm{SO},\pm1/2}\right|^2 = \left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{3}\, p^2_{cV} e'^2_z,$$

$$\left|M^{(1)}_{\mathrm{C},\mp1/2;\mathrm{SO},\pm1/2}\right|^2 = \left(\frac{eA_0}{c\hbar}\right)^2 \frac{1}{3}\, p^2_{cV} e'^2_\pm.$$

In this case, the wave vector of photo-excited current carriers is defined as

$$k^{(1\omega)}_{c,SO} = \sqrt{\frac{2\mu^{(c,SO)}_+}{\hbar^2}\big(\hbar\omega - E_g - \Delta_{SO}\big)},$$

$\mu^{(c,SO)}_+$ is the reduced effective mass with respect to the current carriers in the conduction band and spin-orbit splitting band.

Considering the polarization dependences of the squared absolute values of the matrix elements $\left|M^{(1)}_{\mathrm{C},\pm1/2;\mathrm{SO},\pm1/2}\right|^2$ and $\left|M^{(1)}_{\mathrm{C},\mp1/2;\mathrm{SO},\pm1/2}\right|^2$ for $|\mathrm{V}, \pm1/2\rangle \to |\mathrm{C}, \pm1/2\rangle$ and $|\mathrm{V}, \pm1/2\rangle \to |\mathrm{C}, \mp1/2\rangle$ type optical transitions, polarization dependence of the probability of this transition may be defined taking into account the coherent saturation effect (see Figure 2). Figure 2 shows that the polarization dependence of the probabilities of optical transitions is of the oscillatory type with respect to the angle between

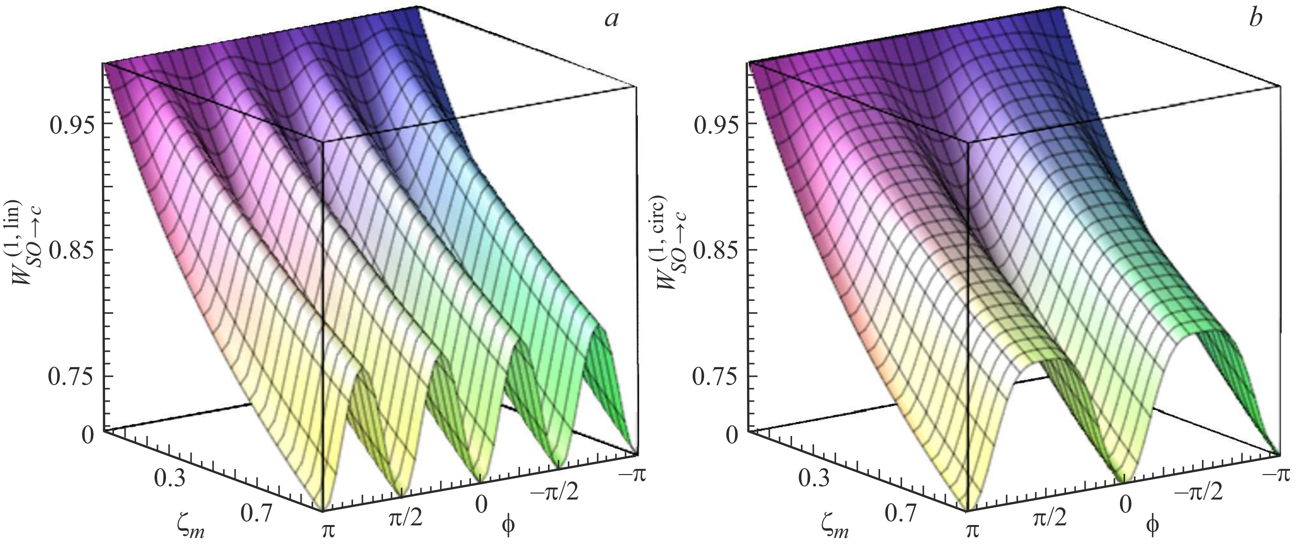

Figure 2. polarization dependence of the probability of $|SO\rangle \to |C\rangle$ optical transitions for the linear light polarization (*a*) and circular light polarization (*b*).

the polarization vector and current carrier wave vector, but oscillation for the linear polarization is approximately twice as high as for the circular polarization. For both polarizations, the oscillation amplitude drops with the growing coherent saturation effect parameter.

Figure 3 shows a nonmonotonic polarization dependence of the single-photon linear-circular dichroism coefficient for $|SO\rangle \to |C\rangle$ type optical transitions, it occurs due to the consideration of the contribution of the coherent saturation effect to the optical transition probabilities, i.e. by the squared composite matrix element of the optical transition of interest, which is under the radical (see, for example, [3–5]).

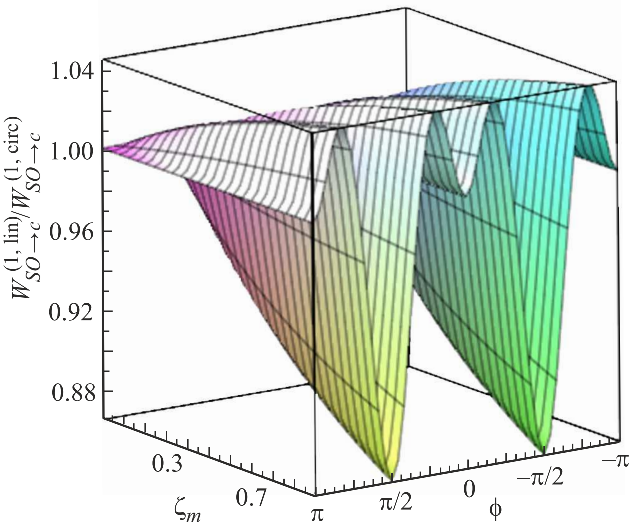

Figure 3. polarization dependence and dependence on the Rabi parameter of the single-photon linear-circular dichroism for $|SO\rangle \to |C\rangle$ type optical transitions.

3. Interband single-photon absorption of the polarized light taking into account the coherent saturation effect

Next, various options of the single-photon interband absorption of the polarized light will be investigated taking into account the contribution of the coherent saturation effect [3–5] to the light absorption coefficient. In this case, calculate the spectral-temperature dependence of the single-photon light absorption coefficient $K^{(1)}$ by equation [3–5]:

$$K^{(1)}_{\mathrm{C},\pm1/2;\mathrm{V},\pm3/2} = \frac{2\pi}{\hbar}\,\hbar\omega\,\frac{1}{I}\,\rho(\hbar\omega)F(\beta,\omega) \times \left(\left\langle \frac{|M^{(1)}_{\mathrm{C},\pm1/2;\mathrm{V},\pm3/2}(\mathbf{k})|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}|M^{(1)}_{\mathrm{C},\pm1/2;\mathrm{V},\pm3/2}(\mathbf{k})|^2}} \right\rangle + \left\langle \frac{|M^{(1)}_{\mathrm{C},\pm1/2;\mathrm{V},\mp3/2}(\mathbf{k})|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}|M^{(1)}_{\mathrm{C},\pm1/2;\mathrm{V},\mp3/2}(\mathbf{k})|^2}} \right\rangle \right), \quad (2)$$

where $I(\omega)$ is the light intensity (frequency), $\rho(\hbar\omega)$ is the density of current carrier states involved in the optical transitions where the law of conservation of energy is considered, $F(\beta,\omega)$ is the current carrier distribution function in the initial state, $\beta^{-1} = k_BT$, k_B is the Boltzmann constant, T is the sample temperature:

$$F(\beta,\omega) = \left[1-\exp(\beta\hbar\omega)\right]\exp\left[\beta\left(\mu - E_{L=hh}(k^{(\omega)}_{c,L=hh})\right)\right],$$

$$E_{L=hh}\left(k^{(\omega)}_{c,L=hh}\right) = \frac{m_c}{m_c+m_{hh}}\left(\hbar\omega - E_g\right),$$

$$\rho(\hbar\omega) = \mu^* k_\omega/(\pi^2\hbar^2),$$

μ^* — reduced effective mass of the current carriers, which is written depending on the type of optical transitions.

Equation (2) shows that angular averaging of the squared composite matrix elements are required with respect to the solid angles of the current carrier wave vector, i. e.

$$\left\langle \frac{\left|M^{(1)}_{\mathrm{C},\pm1/2;\mathrm{V},\pm3/2}(\mathbf{k})\right|^2}{\sqrt{1+4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left|M^{(1)}_{\mathrm{C},\pm1/2;\mathrm{V},\pm3/2}(\mathbf{k})\right|^2}}\right\rangle = \left(\frac{eA_0}{c\hbar}\right)^2 p_{cV}^2\left[\langle\Re_1(I)\rangle+\langle\Re_2(I)\rangle\right], \qquad (3)$$

type integration shall be performed where

$$\Re_1(\mathrm{I})=\frac{|e'_\pm|^2}{\sqrt{1+\xi_\omega|e'_\pm|^2}},\ \Re_2(\mathrm{I})=\frac{|e'_z|^2}{\sqrt{1+\xi_\omega|e'_z|^2}},$$

$\langle|M^{(N)}_{n'\mathbf{k}',n\mathbf{k}}|^2\rangle$ is the squared absolute value of the matrix element $M^{(N)}_{n'\mathbf{k}',n\mathbf{k}}$ averaged with respect to the solid angles of vector $\mathbf{k}$, $\xi_\omega=4\,\frac{\alpha_\omega}{\hbar^2\omega^2}\left(\frac{eA_0}{c\hbar}\right)^2p_{cV}^2$, wave vector k_ω is defined from the law of conservation of energy. In particular, for the optical transition described above

$$k_\omega=k_{c,L}=\sqrt{\frac{2\mu_+^{c,L}}{\hbar^2}\,(\hbar\omega-E_g)},\quad \mu_+^{c,L}=\frac{m_cm_L}{m_c+m_L}.$$

Single-photon absorption of the polarized light associated with the optical transition from the light and heavy hole sub-band to the conduction band shall be calculated by equation [6–10]

$$K^{(1)}=\frac{4\pi e^2}{c\omega m_0^2n_\omega}\times\sum_{nn'k}|\mathbf{e}\mathbf{p}_{nn'}(\mathbf{k})|^2(f_{n\mathbf{k}}-f_{n'\mathbf{k}})\delta(E_{n'}(\mathbf{k})E_n(\mathbf{k})-\hbar\omega), \qquad (4)$$

whence, in the Luttinger–Kohn approximation and in Kane's three-band model [17,18], the spectral-temperature dependence of the interband single-photon light absorption coefficient is written as

$$K^{(1)}_{c,V}=\frac13\,p_{cV}^2\,\frac{e^2p_{cV}^2}{c\hbar^3n_\omega}\Big\{\Big(f_{hh,k^{(1\omega)}_{c,hh}}-f_{c,k^{(1\omega)}_{c,hh}}\Big)\mu_+^{(c,hh)}k^{(1\omega)}_{c,hh}+\Big(f_{lh,k^{(1\omega)}_{c,lh}}-f_{c,k^{(1\omega)}_{c,lh}}\Big)\mu_+^{(c,lh)}k^{(1\omega)}_{c,lh}\Big\}, \qquad (5)$$

where the distribution functions of the photo-excited light and heavy holes are determined as

$$f_{lh,k^{(1\omega)}_{c,lh}}=\exp\Big[\frac{E_F}{k_BT}\Big]\cdot\exp\Big[-\frac{1}{k_BT}\frac{\mu_+^{(c,lh)}}{m_{lh}}(\hbar\omega-E_g)\Big], \qquad (6)$$

$$f_{hh,k^{(1\omega)}_{c,hh}}=\exp\Big[\frac{E_F}{k_BT}\Big]\exp\Big[-\frac{E_{hh}(k^{(1\omega)}_{c,hh})}{k_BT}\Big]=\exp\Big[\frac{E_F}{k_BT}\Big]\cdot\exp\Big[-\frac{1}{k_BT}\frac{\mu_+^{(c,hh)}}{m_{hh}}\,(\hbar\omega-E_g)\Big]. \qquad (7)$$

Note here that if the temperature (e.g. experimental) dependence of the hole concentration is known, then in the Maxwell distribution, the temperature dependence of the Fermi energy is determined by the relation

$$e^{\frac{E_F}{k_BT}}=\frac12\,p\Big(\frac{k_BT}{2\pi\hbar^2}\Big)^{-3/2}\times\big(m_{hh}^{3/2}+m_{lh}^{3/2}+m_{SO}^{3/2}(T)\cdot\exp(-\Delta_{SO}/k_BT)\big)^{-1}, \qquad (8)$$

where p is the hole concentration.

The Fermi energy is known to be determined on the assumption that the semiconductor is neutral. For example, if the electrons in the conduction band and holes in the valence band sub-bands obey the traditional statistics and if the electron and hole energies are counted from the upper edge of the valence band, then the temperature dependence of the Fermi energy is defined by the relation

$$E_F=\frac12\left[E_g(T)+k_BT\times\ln\left(\frac{m_{hh}^{3/2}+m_{lh}^{3/2}+m_{SO}^{3/2}(T)\cdot\exp(-\Delta_{SO}/k_BT)}{m_c^{3/2}(T)}\right)\right], \qquad (9)$$

where [19]

$$E_g(T)=E_g(T=0)-\gamma_TT^2/(T+T_c), \qquad (10)$$

is the temperature dependence of the band gap,

$$m_{SO}(T)=m_0\left\{\gamma_1-\frac{E_p\Delta_{SO}}{3E_g(T)(E_g(T)+\Delta_{SO})}\right\}^{-1}, \qquad (11)$$

$$m_c(T)=m_0\left\{1+F+\frac{E_p(E_g(T)+2\Delta_{SO}/3)}{E_g(T)(E_g(T)+\Delta_{SO})}\right\}^{-1}. \qquad (12)$$

In particular, for GaAs: $\gamma_T=0.46\,\mathrm{meV/K}$, $T_c=204\,\mathrm{K}$, $F=-1.94$ is the Kane constant $E_g(T=0)=1.519\,\mathrm{eV}$, $\gamma_1=6.98$ is the Luttinger parameter, $E_p=28.8\,\mathrm{eV}$ is the Kane energy, $\Delta_{SO}=0.341\,\mathrm{eV}$, and for InSb: $\gamma_T=0.32\,\mathrm{meV/K}$, $T_c=170\,\mathrm{K}$, $F=-0.23$, $E_g(T=0)=$ $=0.235\,\mathrm{eV}$, $\gamma_1=34.8$, $E_p=23.3\,\mathrm{eV}$, $\Delta_{SO}=0.81\,\mathrm{eV}$ [19]. The calculations show that the relative variation of the effective masses of electrons in the conduction band and of holes in the light and heavy hole sub-bands for GaAs (InSb) is negligible with temperature variation from 0 to 300 K, but for the effective mass of holes in the spin-orbit splitting sub-band, it accounts for 5% (25%), and for the temperature dependence of the band gap — it accounts for 10 (34%). Hence it follows that in the calculations of the temperature dependence of the parameter of interest, including also $E_F(T)$, for the narrow-band semiconductors, not only $E_g(T)$, but also $m_{SO}(T)$ shall be considered, which are considered further.

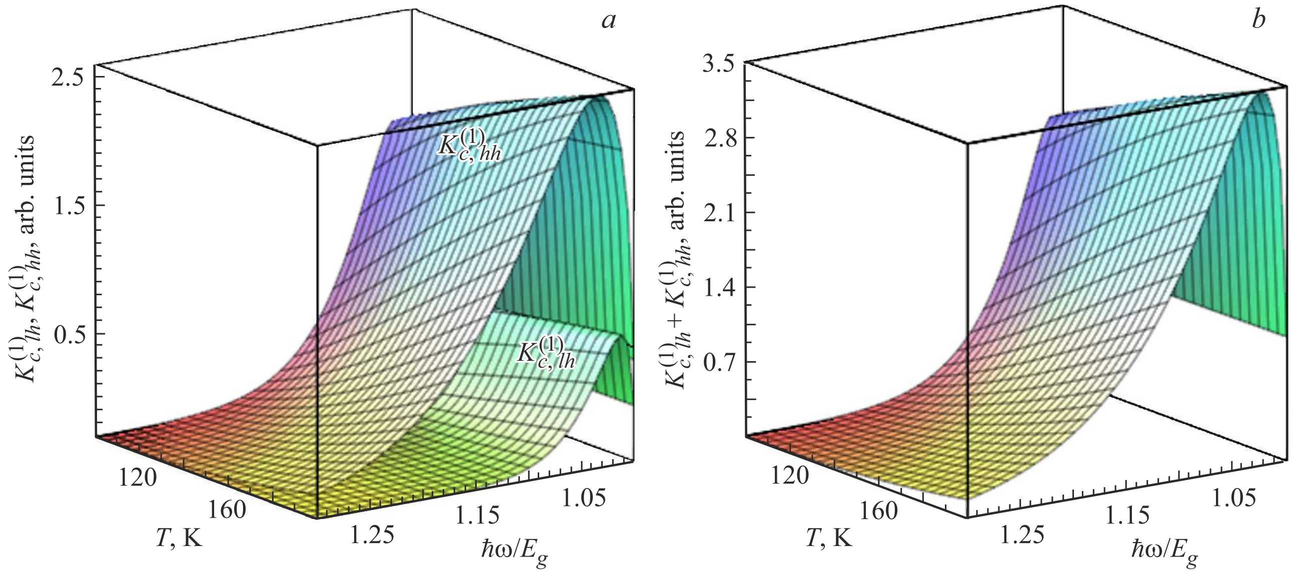

Figure 4. Spectral–temperature dependence of the single-photon absorption coefficient of the polarized light in *GaAs* associated with the optical transitions between light hole sub-bands ($K^{(1)}_{c,lh}$) and heavy hole sub-bands ($K^{(1)}_{c,hh}$) and conduction band (a) and sum thereof (b).

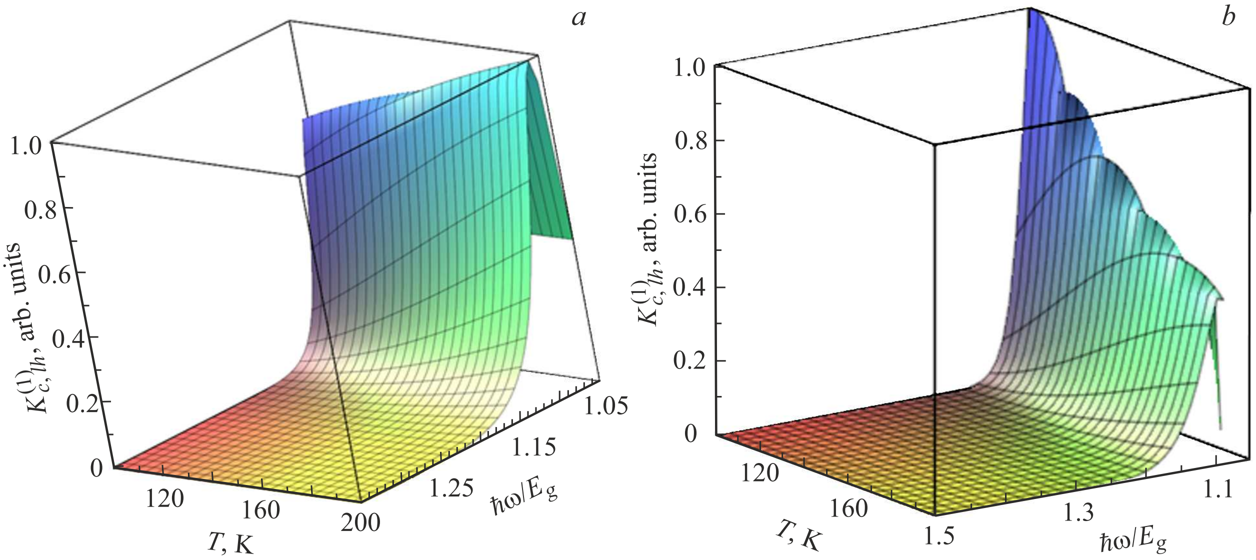

Figure 5. Spectral-temperature dependence of the single-photon absorption coefficient of the polarized light in *GaAs* associated with the optical transition between the light hole sub-bands and conduction band ($K^{(1)}_{c,lh}$) without (a) and with (b) temperature dependence of the band gap on temperature.

Figure 4 shows spectral-temperature dependences of the single-photon absorption coefficient of the polarized light in GaAs associated with the optical transitions between the light hole sub-bands ($K^{(1)}_{c,lh}$) and heavy hole sub-bands ($K^{(1)}_{c,hh}$) and the conduction band (Figure 4, a), as well as the resulting single-photon light absorption (Figure 4, b), where the contribution of the coherent saturation effect to the single-photon light absorption coefficient is not taken into account. In the numerical calculations, the maximum value of $K^{(1)}_{c,lh}$ was assumed as equal to 1. Figure 4, a, b shows that the spectral (temperature) dependence of the single-photon light absorption coefficient in GaAs associated with the optical transitions between the valence band sub-bands and conduction band, at first increases with growing frequency (temperature) and then decreases when crossing the maximum. This is explained by the fact that the spectral dependence of the single-photon light absorption coefficient is determined by the product of the density of states (with the growing frequency, which increases as the power function of frequency) and by the current carrier distribution function in the initial state (with the growing frequency which decreases exponentially). The product of these values gives the curve shown in Figure 4. Note that the temperature dependence of the band gap is not considered here, and its consideration will result in significant variation of spectral and temperature dependence of the light absorption coefficient which is shown in Figure 5 for GaAs[2]. Figure 5 shows that when the temperature dependence of band gap with the growing temperature is considered in the low frequency region, the amplitude value $K^{(1)}_{c,lh}+K^{(1)}_{c,hh}$ decreases in the oscillatory manner and remains almost unchanged in the large frequency region.

4. Conclusions

Equations (1) and (3) show that the polarization dependence of the probability of the optical transition of interest, both for the linear and circular polarization, is determined by the relations

$$\Re_1(\mathrm{I})=\frac{|e'_\pm|^2}{\sqrt{1+\xi_\omega|e'_\pm|^2}},\quad \Re_2(\mathrm{I})=\frac{|e'_z|^2}{\sqrt{1+\xi_\omega|e'_z|^2}}$$

that are used to obtain the results shown in Figure 1–3. Note here that at low values of ξ_ω (with $I \ll I_0$), the numerical calculations may be performed approximately, i. e. expanding into a series of the radical in $\Re_1$ and $\Re_2$ as $(1+x)^n \approx 1+nx$ (with $x \ll 1$). Then, according to the calculations, in the region of values $0.01 < \xi\omega < 0.4$, the approximately calculated polarization dependences of the transition probabilities do not differ from the results shown in Figure 1–3.

Thus, we have determined the following:

1. Polarization dependence of the squared absolute values of the matrix elements for the interband optical transitions both for the linear and circular polarization is of the oscillatory type with respect to the angle between the polarization vector and current carrier wave vector.

2. For the single-photon optical transition between the spin-orbit splitting band and conduction band, the number of oscillations for the linear polarization is approx. twice as high as for the circular polarization. For both polarizations, the oscillation amplitude drops with the growing coherent saturation effect parameter.

3. Oscillation in the spectral-angular dependence of the single-photon absorption of the polarized light in GaAs associated with the optical transitions between the valence band sub-bands and conduction band, and between the spin-orbit splitting band and conduction band is defined by the angular dependence of the matrix elements corresponding to them.

4. Figure 5 shows that when the temperature dependence of band gap with the growing temperature is considered in the low frequency region, the amplitude value $K^{(1)}_{c,lh}+K^{(1)}_{c,hh}$ decreases in the oscillatory manner and remains almost unchanged in the large frequency region.

Conflict of interest

The authors declare that they have no conflict of interest.

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[2] Numerical values of the band parameters of the crystal were selected from [19].