07

Quantum measurement of the electromagnetic field amplitude and phase by optical homodyning

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The possibility of measuring the amplitude and phase of the electromagnetic field by optical light homodyning is shown. A new method for determining the relative phase of the field using the interference scheme of balanced homodyne detection is proposed. The method is based on direct measurement of the average complex amplitude of a quantum field mixed with a classical field using a passive symmetric beam splitter under conditions of a balanced homodyne measurement scheme. A „direct" measurement of the average field amplitude is a direct measurement of the real and imaginary parts of the average quantum mechanical value of the complex field amplitude by homodyning. By comparison with the quantum theory of the Hermitian phase operator of the electromagnetic field, the accuracy of this measurement is estimated for various quantum states of the microscopic electromagnetic field.

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1. Introduction

Classical theory uses the complex amplitude of the electromagnetic field as the basic and fundamental characteristic of light. The electromagnetic (EM) field amplitude in polar coordinates is written in the form $E = \sqrt{I}e^{i\varphi}$, where I — field intensity, φ — field phase. The description of the EM field within quantum mechanical theory involves replacing the c-numerical field parameters $\mathbf{E}, \mathbf{I}, \boldsymbol{\varphi}$ with their corresponding $\hat{a}, \hat{n}, \hat{\varphi}$ operators. The field creation (annihilation) operators $\hat{a}^+(\hat{a})$ are defined in quantum theory by replacing $E \to \hat{a} = \sqrt{\hat{n}+1}e^{i\widehat{\varphi}}$, $E^* \to \hat{a}^+ = e^{-i\widehat{\varphi}}\sqrt{\hat{n}+1}$, taking into account the commutation properties of the field operators.

Phase measurements of the φ field in classical theory are made using interferometry circuits in which the field to be measured is mixed with another EM field of a given phase and the phase difference of the two fields is measured. Thus, interferometric methods measure the relative phase of the field (phase difference of two fields) [1–7]. In order to measure the phase difference unambiguously, the values of the trigonometric functions sine and cosine of the phase difference shall be measured together.

The directly measurable quantities characterizing the EM field under the conditions of measuring the field phase difference are the intensities of these fields I (or the quantum-mechanical mean values of the photon numbers $\langle\hat{n}\rangle$). Such measurable values serve to determine (calculate) within the framework of a theory the values of trigonometric functions of the field phase difference or quantum mechanical mean trigonometric operators of the phase difference of the interfering fields present in the quantum theory of light.

2. Measuring the quantum mechanical mean electromagnetic field

The EM field signal at the output of an optical interferometer is sensitive to the phase difference of the fields at its inputs. The simplest example of an optical interferometer is a passive beam splitter. Let us consider the quantum theory of the passive beam splitter. The two inputs of the beam splitter receive quantum fields characterized by the creation (annihilation) operators $\hat{a}_1^+(\hat{a}_1)$ and $\hat{a}_2^+(\hat{a}_2)$, and the photon number operators $\hat{n}_j = \hat{a}_j^+\hat{a}_j$, $j = 1, 2$. We denote the creation (annihilation) operators for the fields emerging from the beam splitter as $\hat{b}_1^+(\hat{b}_1)$ and $\hat{b}_2^+(\hat{b}_2)$, and the photon number operators: $\hat{N}_j = \hat{b}_j^+\hat{b}_j$, $j = 1, 2$. We will assume that the incoming and outgoing field operators satisfy the following commutative relations: $[\hat{a}_i, \hat{a}_j^+] = \delta_{i,j}$, $[\hat{b}_i, \hat{b}_j^+] = \delta_{i,j}$, $i, j = 1, 2$.

For the photon numbers (intensities) of the incoming and outgoing fields of the beam splitter, the following relation is fulfilled (photon number conservation): $\hat{N}_1 + \hat{N}_2 = \hat{n}_1 + \hat{n}_2$. Outgoing photon numbers operators can be expressed using incoming field operators using beam splitter transmittance τ and reflectance $\rho = 1 - \tau$, as well as transmission and reflection phase shifts if the switching conditions are satisfied and the photon numbers are conserved. If the phase shifts of $\phi_\tau = \phi_\rho$ are equal, the photon number operators at the output of the beam splitter can be written in quantum theory as

$$\hat{N}_1 = \tau\hat{n}_1 + (1-\tau)\hat{n}_2 + \sqrt{\tau(1-\tau)}(\hat{a}_1^+\hat{a}_2 + \hat{a}_2^+\hat{a}_1), \quad (1a)$$

$$\hat{N}_2 = (1-\tau)\hat{n}_1 + \tau\hat{n}_2 - \sqrt{\tau(1-\tau)}(\hat{a}_1^+\hat{a}_2 + \hat{a}_2^+\hat{a}_1). \quad (1b)$$

If $\phi_\tau = \phi_\rho + \pi/2$, the operators of the numbers of photons of transmitted and reflected light are

$$\hat{N}_1' = \tau\hat{n}_1 + (1-\tau)\hat{n}_2 - i\sqrt{\tau(1-\tau)}(\hat{a}_1^+\hat{a}_2 - \hat{a}_2^+\hat{a}_1), \quad (2a)$$

$$\hat{N}_2' = (1-\tau)\hat{n}_1 + \tau\hat{n}_2 + i\sqrt{\tau(1-\tau)}(\hat{a}_1^+\hat{a}_2 - \hat{a}_2^+\hat{a}_1). \quad (2b)$$

The $\phi_\tau = \phi_\rho + \pi/2$ condition for a symmetrical beam splitter is effectively achieved by placing an $\lambda/4$ — plate at the input of the local oscillator.

Let us further assume that the $\hat{a}_2$ field is a strong classical field with a precisely defined complex amplitude a_{LO}: $\hat{a}_2 \to a_{LO}$, the field in the coherent state $|\alpha_{LO}\rangle$, $|\alpha_{LO}|^2 \gg 1$ can be used as such a local oscillator (LO) field. Under these conditions, we have $\langle\hat{a}_1^+\hat{a}_2\rangle = \alpha_{LO}\langle\hat{a}_1^+\rangle$, $\alpha_{LO} = |\alpha_{LO}|e^{i\varphi_{LO}}$. Thus, we will consider the optical homodyning scheme of a quantum field $\hat{a}_1$ while mixing it with a classical signal α_{LO}. We will hereafter assume that the beam splitter is symmetrical: $\tau = \rho = 1/2$. Adding equations (1a) and (2a) and using the photon number conservation condition $\hat{N}_1' + \hat{N}_2' = \hat{n}_1 + \hat{n}_2 = \hat{N}_1 + \hat{N}_2$, we find for the average quantum-mechanical creation operator for any quantum state of the measured field, the following expression:

$$\langle\hat{a}_1^+\rangle = \frac{\langle\hat{N}_-\rangle + i\langle\hat{N}_-'\rangle}{2\alpha_{LO}}, \quad (3a)$$

where $\langle\hat{N}\rangle \equiv \langle\hat{N}_1\rangle - \langle\hat{N}_2\rangle$, $\langle\hat{N}'\rangle \equiv \langle\hat{N}_1'\rangle - \langle\hat{N}_2'\rangle$. Similarly, using the difference of equations (1a) and (2a) and substituting the ratio $\langle\hat{n}_1\rangle = \langle\hat{N}_1'\rangle - \langle\hat{N}_2'\rangle$, we find

$$\langle\hat{a}_1\rangle = \frac{\langle\hat{N}_-\rangle - i\langle\hat{N}_-'\rangle}{2\alpha_{LO}^*}. \quad (3b)$$

The $\hat{a}_1^+(\hat{a}_1)$ field creation/annihilation operators can be written using the $\hat{n}_1 \equiv \hat{a}_1^+\hat{a}_1$ photon number operators and the $e^{i\hat{\varphi}}$ phase operator exponent operator in the following form:

$$\hat{a}_1 = \sqrt{\hat{n}_1 + 1}e^{i\hat{\varphi}_1} = e^{i\hat{\varphi}_1}\sqrt{\hat{n}_i},$$

$$\hat{a}_1^+ = e^{-i\hat{\varphi}_1}\sqrt{\hat{n}_1 + 1} = \sqrt{\hat{n}_1}e^{-i\hat{\varphi}_1}, \quad (4)$$

Let us further assume that the quantum mechanical averages of the field exponent operator satisfy the approximate relation

$$\langle\hat{a}\rangle \approx \langle\sqrt{\hat{n}+1}\rangle\langle e^{i\hat{\varphi}}\rangle, \ \langle\hat{a}^+\rangle \approx \langle\sqrt{\hat{n}+1}\rangle\langle e^{-i\hat{\varphi}}\rangle, \quad (5)$$

where the lower index in the creation/destruction operator entry $\hat{a}_1^+/\hat{a}_1$, $\hat{n}_1$ and $\hat{\varphi}_1$ is omitted, which we will continue to do. Approximation (5) assumes that the $e^{i\hat{\varphi}}$ and $(\hat{n}+1)^{1/2}$ operators are weakly correlated. We will investigate the validity of this approximation for different quantum field states.

Equation (3b) in approximation (5) means that the exponent operator of the field phase operator is

$$e^{i\hat{\varphi}} \approx \frac{1}{2a_{LO}^*\sqrt{\hat{n}+1}}(\hat{N}_- - i\hat{N}_-'),$$

and the relative field phase exponent operator $\hat{\varphi} - \varphi_{LO}$ is defined as

$$e^{i(\hat{\varphi}-\varphi_{LO})} \approx \frac{1}{2\sqrt{n_{LO}(\hat{n}+1)}}(\hat{N}_- - i\hat{N}_-'). \quad (6)$$

It follows directly from (5) that the mean trigonometric operators of the relative field phase in this approximation satisfy the relations

$$\langle\cos(\hat{\varphi} - \varphi_{LO})\rangle \approx \frac{\langle\hat{N}_-\rangle}{2n_{LO}\langle(\hat{n}+1)^{1/2}\rangle},$$

$$\langle\sin(\hat{\varphi} - \varphi_{LO})\rangle \approx -\frac{\langle\hat{N}_-'\rangle}{2n_{LO}\langle(\hat{n}+1)^{1/2}\rangle}. \quad (7)$$

Our second assumption is that it is possible to replace the $(\hat{n}+1)^{1/2}$ operator with $(\langle\hat{n}\rangle+1)^{1/2}$ in expressions (6) and (7). Within the framework of this approximation,

$$(\hat{n}+1)^{1/2} \approx (\langle\hat{n}\rangle + 1)^{1/2}, \quad (8)$$

from which, we get the following:

$$\langle\cos(\hat{\varphi} - \varphi_{LO})\rangle \approx \frac{\langle\hat{N}_-\rangle}{2n_{LO}(\langle\hat{n}\rangle+1)^{1/2}},$$

$$\langle\sin(\hat{\varphi} - \varphi_{LO})\rangle \approx -\frac{\langle\hat{N}_-'\rangle}{2n_{LO}\langle(\hat{n}\rangle+1)^{1/2}}. \quad (9)$$

3. Comparing measured averages with the Hermite field phase operator theory

According to the Hermite theory of the field phase operator $\hat{\varphi}$ [8–14], the quantum-mechanical mean values of field operators for any quantum field state have the form

$$\langle\hat{a}\rangle_x = \sum_{n=0}^{\infty}\sqrt{n+1}\langle x|n\rangle\langle n+1|x\rangle,$$

$$\langle\hat{a}^+\rangle_x = \sum_{n=0}^{\infty}\sqrt{n+1}\langle x|n+1\rangle\langle n|x\rangle. \quad (10)$$

Using the Hermite theory of the Pegg-Barnett phase operator [8–14] for the mean values of the $e^{i\hat{\varphi}}$ and $\sqrt{\hat{n}+1}$ operators for an arbitrary field state we find

$$\langle e^{i\hat{\varphi}}\rangle_x = \sum_{n=0}^{\infty}\langle x|n\rangle\langle n+1|x\rangle, \ \langle\sqrt{\hat{n}+1}\rangle_x = \sum_{n=0}^{\infty}\sqrt{n+1}|\langle n|x\rangle|^2. \quad (11)$$

It follows from (10) and (11), that the trigonometric phase operators are

$$\langle e^{i\hat{\varphi}}\rangle_x = \langle\cos\hat{\varphi}\rangle_x + i\langle\sin\hat{\varphi}\rangle_x,$$

$$\langle\cos\hat{\varphi}\rangle_x = \mathrm{Re}\sum_{n=0}^{\infty}\langle x|n\rangle\langle n+1|x\rangle,$$

$$\langle\sin\hat{\varphi}\rangle_x = \mathrm{Im}\sum_{n=0}^{\infty}\langle x|n\rangle\langle n+1|x\rangle. \qquad (12)$$

Taking further the value $\varphi_{LO} = 0$ for the phase of the local oscillator (classical field) and using formulas (7), we will compare the results of the quantum Pegg-Barnett (PB) theory with the results of the proposed scheme for measuring the average values of trigonometric field phase operators carried out by the balancing optical homodyning method. To do this, consider a coherent state as the quantum state of the field to be measured

$$|\alpha\rangle = e^{-n_\alpha/2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle,\ \alpha = \sqrt{n_\alpha}e^{i\varphi_\alpha},\ n_\alpha \equiv |\alpha|^2.$$

Let us investigate the validity of the assumptions (5) and (8) we have made, serving to determine the measured quantum mean values of the trigonometric operators of the relative phase (9), for the coherent field state $|x\rangle = |\alpha\rangle$. Fig. 1, *a* shows the ratio of $\langle\sqrt{\hat{n}+1}\rangle_\alpha\langle e^{i\hat{\varphi}}\rangle_\alpha$ to $\langle\hat{a}\rangle_\alpha = \langle\sqrt{\hat{n}+1}e^{i\hat{\varphi}}\rangle_\alpha$ as a function of the average number of coherent state photons: $0 \le n_\alpha \le 10$ for any coherent phase angle value φ_α. The Figure shows that the correlation $\langle\sqrt{\hat{n}+1}e^{i\hat{\varphi}}\rangle_\alpha$ differs markedly from the product of the average $\langle\sqrt{\hat{n}+1}\rangle_\alpha\langle e^{i\hat{\varphi}}\rangle_\alpha$ only for small values n_α near 2, and the difference between the two values does not exceed 7%. At $n_\alpha > 5$, the weak correlation for these operators is assumed almost exactly for a coherent field state and the difference between the product of averages and the correlation is less than 1%.

The $\sqrt{\langle\hat{n}\rangle_\alpha + 1}/\langle\sqrt{\hat{n}+1}\rangle_\alpha$ relation for the same values of the coherent field state parameters is given in Fig. 1, *b*. As can be seen from the figure, the approximation (8) that we use is highly accurate and the error does not exceed 3% for all values of n_α. The maximum error is observed at $n_\alpha \approx 1$.

Thus, our calculations have shown that the assumptions and approximations we have made are valid for the case of a coherent measured field state with high accuracy. This means that measurements of mean values of trigonometric operators of relative field phases agree well with the quantum theory of the Hermite phase operator for microscopic coherent quantum fields. Fig. 1, *c* shows the dependence of the ratio of the theoretical values of the phase cosine operator to the measured mean values of $r(n_\alpha) \equiv \langle\cos\hat{\varphi}\rangle_{\alpha,T}/\langle\cos\hat{\varphi}\rangle_{\alpha,M}$ on the mean number of photons of the coherent field state. The difference between the theoretical and measured values does not exceed 10%, the maximum difference being achieved at $n_\alpha \approx 2.5$.

Consider the case where the measured field is in the Fock state $|n\rangle$. It is not difficult to see that for this state, the average values of (10) are $\langle\hat{a}\rangle_{n,T} = \langle\hat{a}^+\rangle_{n,T} = 0$. It follows that the measured values of the mean trigono-

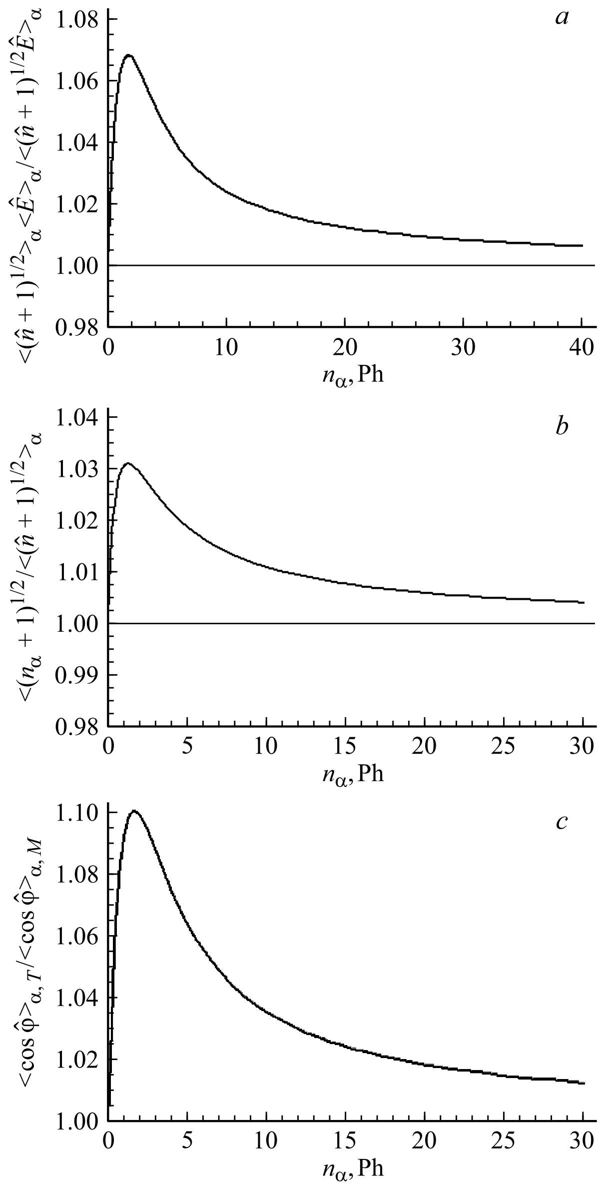

Figure 1. (*a*) Dependence of the ratio $\langle\sqrt{\hat{n}+1}e^{i\hat{\varphi}}\rangle_\alpha/\langle\sqrt{\hat{n}+1}\rangle_\alpha\langle e^{i\hat{\varphi}}\rangle_\alpha$ for fields in coherent states on the average values of the number of photons n_α for any value of the phase angle φ_α of the coherent state $|\alpha\rangle$. (*b*) The ratio of mean values for $\langle\sqrt{\hat{n}+1}\rangle_\alpha/\sqrt{\langle\hat{n}\rangle_\alpha+1}$ field in coherent states as a function of mean value of photon number n_α for any value of phase angle φ_α of coherent state $|\alpha\rangle$. (*c*) The dependence of the ratio of the average value of the phase difference cosine operator of PB theory to the average value of the measured phase operator $r(n_\alpha) \equiv \langle\cos\hat{\varphi}\rangle_{\alpha,T}/\langle\cos\hat{\varphi}\rangle_{\alpha,M}$ for the coherent field state $|\alpha\rangle$ on the average values of the photon number n_α for any value of the phase angle φ_α of the coherent state and $\varphi_{LO} = 0$. The mean values of the measured trigonometric phase operators according to (9) and (3) are $\langle\cos(\hat{\varphi}-\varphi_{LO})\rangle_{\alpha,M} \equiv \frac{\mathrm{Re}\langle\hat{a}\rangle_\alpha}{(\langle\hat{n}\rangle_\alpha+1)^{1/2}}$, $\langle\sin(\hat{\varphi}-\varphi_{LO})\rangle_{\alpha,M} \equiv \frac{\mathrm{Im}\langle\hat{a}\rangle_\alpha}{(\langle\hat{n}\rangle_\alpha+1)^{1/2}}$.

metric operators of the relative phase (9) are also zero: $\langle\cos(\hat{\varphi}-\varphi_{LO})\rangle_{n,M}=\langle\sin(\hat{\varphi}-\varphi_{LO})\rangle_{n,M}=0$, which corresponds to a uniform distribution of random field phase values from 0 to 2π.

On the other hand, the PB theory of the phase operator, from which the expressions for the trigonometric operator mean (12) follow, also indicates equality 0 of these quantum-mechanical means: $\langle\cos(\hat{\varphi}-\varphi_{LO})\rangle_{n,T}=\langle\sin(\hat{\varphi}-\varphi_{LO})\rangle_{n,T}=0$.

Under conditions where the measured field is in the „states of the Schrödinger cat" (SC) $|\psi_{SC,\pm}\rangle=N_{SC,\pm}(|\alpha\rangle\pm|-\alpha\rangle)$, it can be shown that $\langle\hat{a}\rangle_{SC\pm}=\langle\hat{a}^{+}\rangle_{SC\pm}=0$ and the measured values of the average cosine and sine phase operators are zero: $\langle\cos(\hat{\varphi}-\varphi_{LO})\rangle_{SC\pm,M}=\langle\sin(\hat{\varphi}-\varphi_{LO})\rangle_{SC\pm,M}=0$. The results of PB quantum theory in the case under consideration are in exact agreement with the theory of measurable trigonometric operators, since it follows from formulae (12) that $\langle\cos(\hat{\varphi}-\varphi_{LO})\rangle_{SC\pm,T}=\langle\sin(\hat{\varphi}-\varphi_{LO})\rangle_{SC\pm,T}=0$.

Thus, it is shown that the measurement of mean values of trigonometric phase operators carried out within the light homodyning balancing scheme we consider is in exact agreement with the theory of PB Hermitian phase operator in the case of Fock states as well as „states of the Schrödinger cat" EM field.

4. Measuring field phase uncertainties

The measure of quantum phase uncertainty (dispersion of the phase operator), proposed in [4] and used in [1–3,5,6] to interpret experimental data, is of the form

$$\langle(\delta\hat{\varphi})^2\rangle=1-|\langle e^{i\hat{\varphi}}\rangle|^2=1-\langle\cos\hat{\varphi}\rangle^2-\langle\sin\hat{\varphi}\rangle^2 \quad (13)$$

takes a value equal to 1 in cases of total phase uncertainty in the quantum field state in question and equal to 0 for quantum field states with exactly defined phase or in the classical limit of close to 0 fluctuations (dispersion) of the field phase. It is not difficult to see, that the value $\langle(\delta\hat{\varphi})^2\rangle$ can serve to estimate absolute phase uncertainty as well as relative phase uncertainty, since it does not depend on the phase of the local oscillator φ_{LO}: $\langle(\delta(\hat{\varphi}-\varphi_{LO}))^2\rangle=\langle(\delta\hat{\varphi})^2\rangle$.

The results of the calculation of the field phase dispersion for the coherent field state $\langle(\Delta\hat{\varphi})^2\rangle$, obtained within the PB [8–10] theory, are compared by us with the phase uncertainty measure $\langle(\delta\hat{\varphi})^2\rangle$ (13) obtained using formulas for the average values of trigonometric phase operators included in (13) and following from quantum PB and Susskind-Glogover theories [11]. The comparison of $\langle(\delta\hat{\varphi})^2\rangle$ with the theoretical $\langle(\Delta\hat{\varphi})^2\rangle$ following from the above theories is carried out by us, in turn, also using the corresponding approximations for the average measured operators of the form (7). Fig. 2 shows the dependences of the field phase uncertainty measure $\langle(\delta\hat{\varphi})^2\rangle_\alpha$ and the variance of the PB phase operator $\langle(\Delta\hat{\varphi})^2\rangle_\alpha$ for the EM field in the coherent state on the average photon number n_α for any value of the phase angle φ_α of the coherent state $|\alpha\rangle$

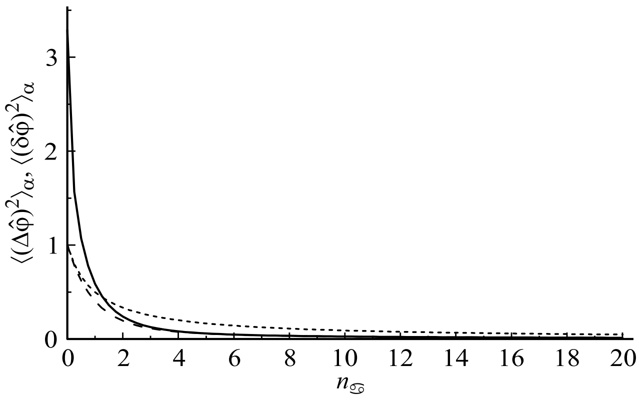

Figure 2. Dependence of the field phase uncertainty measure $\langle(\delta\hat{\varphi})^2\rangle_\alpha$ and the variance of the PB phase operator $\langle(\Delta\hat{\varphi})^2\rangle_\alpha$ for the EM field in the coherent state on the average photon number n_α for any value of the phase angle φ_α of the coherent state $|\alpha\rangle$ and $\varphi_{LO}=0$. Dashed curve —in formula (13), the average of the measured sine and cosine operators (14) are used to calculate $\langle(\delta\hat{\varphi})^2\rangle_\alpha$;Dashed curve — in formula (13) for calculating $\langle(\delta\hat{\varphi})^2\rangle_\alpha$, the average values of trigonometric operators of PB sine and cosine theory (12) are used; solid curve — results of PB phase operator variance calculation for field in coherent state $|\alpha\rangle$.

and $\varphi_{LO}=0$. The figure shows that the phase dispersion in PB [15,16] theory is quantitatively different from the uncertainty measure $\langle(\delta\hat{\varphi})^2\rangle_\alpha$ [4] for small $n_\alpha\sim 1$ for both the mean trigonometric phase operators (12) included in (11) and the mean measured trigonometric operators

$$\langle\cos\hat{\varphi}\rangle_{\alpha,M}\equiv\frac{\mathrm{Re}\langle\hat{a}\rangle_\alpha}{(\langle\hat{n}\rangle_\alpha+1)^{1/2}},\ \langle\sin\hat{\varphi}\rangle_{\alpha,M}\equiv\frac{\mathrm{Im}\langle\hat{a}\rangle_\alpha}{(\langle\hat{n}\rangle_\alpha+1)^{1/2}}. \quad (14)$$

Note that at $n_\alpha\gg 1$, the fluctuations (dispersion) and the phase uncertainty measure tend to 0. For EM field states „of the Schrödinger cat" the field phase uncertainty measure $\langle(\delta\hat{\varphi})^2\rangle_{SC}$ takes its maximum value (as in the case of a Fokowski field state), equal to 1 (11), for any values of the mean photon number. At the same time, as shown in [15], the phase dispersion of the field in the „state of the Schrödinger cat" decreases with increasing average photon number n_α of the coherent state $|\alpha\rangle$ and decreases markedly from the value $\langle(\Delta\hat{\varphi})^2\rangle_{SC}=\frac{\pi^2}{3}$ at $n_\alpha=0$ to $\langle(\Delta\hat{\varphi})^2\rangle_{SC}=\frac{\pi^2}{4}$ at $n_\alpha\gg 1$. Thus, the uncertainty measure $\langle(\delta\hat{\varphi})^2\rangle_{SC}$ can only serve as a qualitative estimate of the EM field phase fluctuations found in quantum states „of the Schrödinger cat" $|\psi_{SC,\pm}\rangle=N_{SC,\pm}(|\alpha\rangle\pm|-\alpha\rangle)$.

5. Conclusion

A procedure for measuring the mean value of the complex amplitude of a quantum EM field is described. The quantitative measurement of average quantum-mechanical values of trigonometric field phase operators using a balanced optical homodyning scheme, based on the measurement of the average EM field amplitude, is shown to be possible in the approximations made. The applicability limits of the proposed method for microscopic fields in different quantum states are investigated. It is shown that the quantum mechanical averages of the measured trigonometric phase operators agree with high accuracy with the theory of the Hermite field phase PB operator for a coherent field state and exactly match the results of the phase operator theory for Fock states and „states of the Schrödinger cat".

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