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Measurement of the refractive index using a goniometric system

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The article proposes a method for measuring the refractive index using a goniometric system that does not require measuring the prism angle, which simplifies the measurement process compared to the widely used methods of minimum deviation and autocollimation. This method can be used to measure the refractive index of samples in the form of triangular prisms in the visible, ultraviolet and infrared ranges. To implement the method, a goniometric system was used. To obtain the reflection of the refracted beam, a fixed mirror was used, and the refractive index of the prism material was calculated from the solution of a system of equations. The results of an experimental study of a triangular prism made of optical glass N-BK7 using the proposed method and their comparison with the data obtained on the State Primary Standard of the Refractive Index Unit GET 138-2021 are presented.

Keywords: goniometer, refractive index, prism method.

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Introduction

Measurement of the refractive index of substances is the most important type of optical-physical measurements. Since the refractive index depends on many parameters: the state of aggregation and the chemical composition of the substance, temperature, radiation wavelength, etc., its value makes it possible to obtain information about the various properties of the samples under study. Accurate and reliable measurements of the refractive index are necessary in the optical industry for quality control of parts, in the chemical industry for the production of raw materials, for determining the quality of petroleum products, in the pharmaceutical industry and medicine for monitoring the composition and quality of medicines, in the food industry for the production of juices, oils, drinks etc.

To measure the refractive index, refractometric methods are used based on measuring the angle of refraction of light by a substance, interference methods based on measuring the phase delay of a light wave, laser spectroscopy methods, etc. When implementing these methods, the laws of optics are used, on the basis of which the parameters of light are calculated during its reflection, refraction and absorption by the substance. At the same time, goniometric methods based on measuring the angles of deviation of light passing through a sample are widely used due to their versatility and ease of implementation. These methods can be applied to both solids and liquids. As a samples, as a rule, triangular prisms are used, made of the studied material or filled with the studied substance [1].

Accurate measurements of the angles necessary to determine the refractive index are made on special instruments the goniometers. The goniometer has a fixed collimator, a rotary object table and is used to measure the deviation angles of rays refracted or reflected by the prism under study [2].

Prism Methods

Prism methods have become very popular for determining the refractive index of substances due to the simplicity and high accuracy of measurements. In addition, prism methods allow measurements in the invisible regions of the spectrum: ultraviolet (UV) and infrared (IR).

The essence of any of the prism methods is to measure the angles of incidence and refraction of the beam on the faces, from which the reflection index of the prism material is calculated.

A light beam incident on the face of a triangular prism with an angle α between the working faces at an angle of φ_1 is refracted first at the input face and then at the output face to the exit angle φ_2 and deviates by some angle ε from the original direction (Fig. 1), which can be calculated using the formula [3]:

$$\varepsilon = \varphi_1 - \alpha + \arcsin[n \sin(\alpha - \arcsin(\sin\varphi_1/n))], \qquad (1)$$

where n is the reflection index of the prism material.

The refractive index n can be calculated from the results of measurements of three angles : α, φ_1 and φ_2 using the

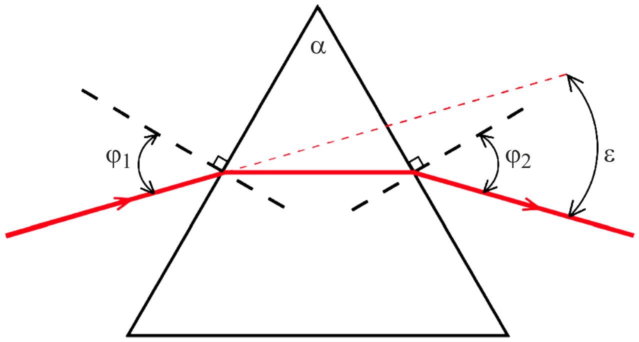

Figure 1. Refraction of a beam when passing through a triangular prism.

formula [4]:

$$n = \sqrt{\sin^2 \varphi_1 + \frac{(\sin \varphi_2 + \cos \alpha \sin \varphi_1)^2}{\sin^2 \alpha}}. \quad (2)$$

Usually, in practice, only two angles are measured, imposing some condition on the value of the third angle. Depending on the specific implementation of such a condition, there are various methods for measuring the refractive index of triangular prisms: the Fraunhofer method of minimum deviation [5], the Littrow-Abbe autocollimation method, and the constant deviation method.

Most often, the minimum deviation method (MDM) is used, which is based on determining the minimum possible beam deviation angle $\varepsilon_{\min}$. In this case, only the angles $\varepsilon_{\min}$ and α can be measured, and the equation for calculating the refractive index becomes much simpler [6]:

$$n = \sin\big((\alpha + \varepsilon_{\min})/2\big) / \sin(\alpha/2). \quad (3)$$

The widespread use of MDM is due to the high accuracy of refractive index measurements using this method [7].

In the autocollimation method, the directions of the incident beam and the one reflected back from the exit face are achieved, and in this case the refractive index is determined by the formula [5]

$$n = \sin \varphi_1 / \sin \alpha. \quad (4)$$

In some cases, the constant deviation method is used, when the value of ε is given some constant value. Then, in addition to the prism angle α, it is necessary to measure the exit angle φ_2, and the refractive index is calculated according to formula (2) taking into account the fact that $\varphi_1 = \varepsilon - \varphi_2 + \alpha$.

Thus, in order to calculate the refractive index by the considered methods, the apex angle of the prism must be preliminarily measured with high accuracy. Therefore, the development of new methods that make it possible to measure the refractive index without a priori information about the prism is an urgent task.

In the work [8] a method was proposed for measuring the refractive index of a triangular prism using a goniometer and a mirror. In this case, a mirror is installed on the path of the beam emerging through the face of the prism and the angles of incidence are measured at which autocollimation from the mirror occurs, as well as the apex angle of the prism, from which the refractive index is calculated. This method can be modified in such a way as to get rid of the need to measure α by measuring the angles of incidence when autocollimation is achieved from a fixed mirror on all three faces of the prism and determining the value n from the solution of the system of equations.

Thus, there is no need to measure the apex angle of the prism. In addition, the proposed method can be used to measure the refractive index in the UV and IR ranges. In this case, a photodetector should be installed instead of a fixed mirror, which eliminates the need to use an autocollimator with a beam-splitting cube [9], which limits the spectral range of radiation.

Goniometric system

To implement the proposed method in the work, a goniometric system manufactured by „Inertekh" OOO (Russia) [10], designed to measure the angles formed by flat surfaces of objects (Fig. 2), was used. The main technical and metrological characteristics of this system are given in Table 1.

Before starting measurements, the prism under study *1* must be installed on a rotating object table *2*, the platform of which is adjusted so that the normals to the prism faces are in the plane of the incident beam, and one of the reflecting faces is in the field of view of the autocollimator *3*. By moving the height adjustment screw, the sighting axis of the autocollimator is aligned with the geometric center of the entrance face of the prism. The object table can be rotated around a vertical axis using a motor or a manual rotation handle *6*. On the path of the refracted beam *4*, a fixed

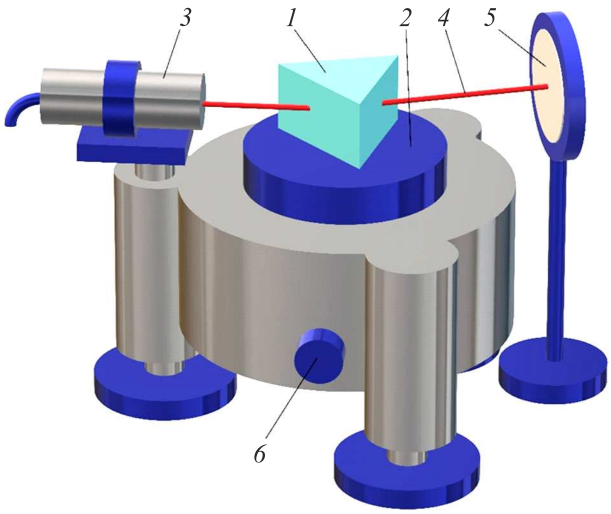

Figure 2. Goniometric system. *1* — prism, *2* — rotary object table, *3* — autocollimator, *4* — radiation beam, *5* — fixed mirror, *6* — table rotation knob.

Table 1. Main characteristics of the goniometric system

Characteristics	Value
Angle measurement range in the horizontal plane	$0-360^\circ$
Angle measurement range in the vertical plane	$\pm 15'$
Limits of allowable absolute error	$\pm 0.25''$
Turntable diameter	100 mm
Dimensions	$650 \times 380 \times 370$ mm
Weight	47 kg
Radiation wavelength*	650 nm

Note. * Wavelength of the radiation source of autocollimator is not specified in the documentation, so its spectrum was previously studied using a mini-spectrometer Hamamatsu C10083CA [13].

mirror *5* is installed at an angle ε_1 relative to the initial direction of the beam, with $\varepsilon_1 > \varepsilon_{\min}$. Thus, when changing the angular position of the prism, it is possible to obtain an image of an autocollimation mark twice for each working face (Fig. 3). At the same time, angles φ_{11} (Fig. 3, a) and φ_{12} (Fig. 3, b),corresponding to the moments of reflection of the beam from the mirror when the object table is rotated for each working face of the prism and apex angles α, β and γ, are measured.

Since the initial angular position of the prism is unknown, the value of φ_{11} can be used as the origin, then $\varphi_{12} = \varphi_{11} + \Delta\varphi$, where the angle $\Delta\varphi$ needs to be measured. Next, we compose a system of equations (5) based on formula (1) and the properties of the sum of the angles of a triangle:

$$\begin{cases} \varepsilon_1 = \varphi_{11\alpha} - \alpha + \arcsin\left[n \sin\left(\alpha - \arcsin\left(\frac{\sin\varphi_{11\alpha}}{n}\right)\right)\right], \\ \varepsilon_1 = \varphi_{11\alpha} + \Delta\varphi_\alpha \\ -\alpha + \arcsin\left[n \sin\left(\alpha - \arcsin\left(\frac{\sin(\varphi_{11\alpha}+\Delta\varphi_\alpha)}{n}\right)\right)\right], \\ \varepsilon_1 = \varphi_{11\beta} - \beta + \arcsin\left[n \sin\left(\beta - \arcsin\left(\frac{\sin\varphi_{11\beta}}{n}\right)\right)\right], \\ \varepsilon_1 = \varphi_{11\beta} + \Delta\varphi_\beta \\ -\beta + \arcsin\left[n \sin\left(\beta - \arcsin\left(\frac{\sin(\varphi_{11\beta}+\Delta\varphi_\beta)}{n}\right)\right)\right], \\ \varepsilon_1 = \varphi_{11\gamma} - \gamma + \arcsin\left[n \sin\left(\gamma - \arcsin\left(\frac{\sin\varphi_{11\gamma}}{n}\right)\right)\right], \\ \varepsilon_1 = \varphi_{11\gamma} + \Delta\varphi_\gamma - \gamma \\ + \arcsin\left[n \sin\left(\gamma - \arcsin\left(\frac{\sin(\varphi_{11\gamma}+\Delta\varphi_\gamma)}{n}\right)\right)\right], \\ \alpha + \beta + \gamma = \pi. \end{cases} \quad (5)$$

In a system of 7 equations, 7 unknowns are obtained: n, α, β, γ, $\varphi_{11\alpha}$, $\varphi_{11\beta}$, $\varphi_{11\gamma}$ and there is a unique solution that can be found using math software packages. From the solution of this system of equations, the values of the apex angles are found, as well as the relative refractive index of the prism material n.

Table 2. The results of measurements of the angle for deviation and the difference between the angles of reaching autocollimation for three faces of a prism made of N-BK 7 glass using a goniometric system

Parameter	Measured Value	MSD
ε_1, rad	0.769094	$1.7 \cdot 10^{-5}$
$\Delta\varphi_\alpha$, rad	0.912044	$5.8 \cdot 10^{-5}$
$\Delta\varphi_\beta$, rad	0.054550	$1.3 \cdot 10^{-4}$
$\Delta\varphi_\gamma$, rad	0.632308	$5.2 \cdot 10^{-5}$

Table 3. The results of calculating the absolute refractive index and refractive angles for a prism made of N-BK 7 glass

Parameter	Calculated value	Reference value	Absolute error
n_{abs}	1.515147	1.515082	$6.5 \cdot 10^{-5}$
α, rad	0.958908	0.959008	$-1.0 \cdot 10^{-4}$
β, rad	1.135440	1.135397	$4.2 \cdot 10^{-5}$
γ, rad	1.047245	1.047186	$5.7 \cdot 10^{-5}$

Since the prism is in the air during measurements, the value of the absolute refractive index n_{abs} is calculated using the formula [11]:

$$n_{\text{abs}} = n n_{\text{air}}, \quad (6)$$

where n_{air} is refractive index of air, $n_{\text{air}} \approx 1.00027$.

Experimental Studies

To confirm the applicability of the proposed method, an experimental study of a triangular prism by Schott AG (Germany) from N-BK7 glass was carried out. This prism was used in international comparisons COOMET.PR-S3 [12] and the value of its refractive index at different wavelengths is known with very high accuracy.

To obtain the results of measuring angles, the goniometric system is connected to a personal computer with specialized GonioScan software installed, designed to set up operating modes, display an autocollimation mark and display measurement data.

Table 2 shows the values obtained as a result of multiple measurements of the difference between the angles of reaching autocollimation for three prism faces. The value of the beam deviation angle ε_1 was also preliminarily measured using a vertical mirror mounted on the turntable of the goniometric system.

Note that due to the dispersion properties of the prism material, the image of the autocollimation mark is blurred, which complicates the automation of measurements and leads to an increase in the mean square deviation (MSD) of the measurement results. Therefore, to reduce the random error, it is desirable to use laser radiation sources with a discrete wavelength.

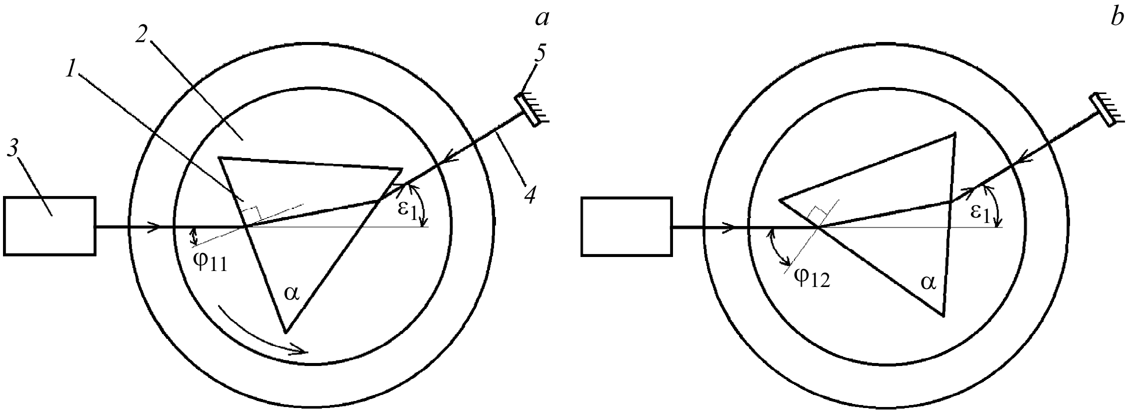

Figure 3. Measurement scheme. *1* — prism, *2* — rotary object table, *3* — autocollimator, *4* — radiation beam, *5* — stationary mirror.

Table 3 shows the values obtained as a result of solving the system (5) using the „Solver" add-in of Microsoft Excel.

To estimate the absolute measurement error, the nominal value of the refractive index of the studied prism at the wavelength of the radiation source of the autocollimator of the goniometric system 650 nm was calculated. The Sellmeier formula [13] was used for the calculation:

$$n^2(\lambda) - 1 = \frac{A_1\lambda^2}{\lambda^2 - B_1} + \frac{A_2\lambda^2}{\lambda^2 - B_2} + \frac{A_3\lambda^2}{\lambda^2 - B_3}, \qquad (7)$$

where $A_1, A_2, A_3, B_1, B_2, B_3$ are experimentally determined Sellmeier coefficients for a given prism, λ is wavelength in μm.

Thus, the experimental error in measuring the refractive index does not exceed $6.5 \cdot 10^{-5}$, which confirms the possibility of using the proposed method for high-precision measurements of the refractive index in cases where the prism angles are unknown. An important advantage of the proposed method is that it can be used to measure the refractive index in the UV and IR ranges. In this case, instead of a fixed mirror, it is necessary to install a radiation receiver, and a source install instead of an autocollimator, which eliminates the need to use beam-splitting elements that limit the spectral range.

Conclusions

The method proposed in this paper makes it possible to determine the refractive index and apex angles of a prism from the results of measuring the difference in the angles of reaching autocollimation of a beam from a fixed mirror for three faces. This method can be applied to triangular prisms made of optically transparent materials, as well as to liquid optically transparent substances placed in a hollow triangular prism.

Conflict of interest

The authors declare that they have no conflict of interest.

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