The Effect of phonon focusing on the mutual drag of electrons and phonons and the electrical resistance of potassium

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The effect influence of elastic energy anisotropy on the mutual drag of electrons and phonons and the electrical resistance of potassium crystals at low temperatures have investigated. We have analyzed the momentum exchange between the electron and three phonon flows corresponding to three branches of the vibrational spectrum in the hydrodynamic approximation. The actual mechanisms of phonon momentum relaxation have taken into account: scattering at sample boundaries, dislocations, and in the processes of phonon-phonon transfer. It have shown that in the limiting case of strong mutual drag of electrons and phonons, the electrical resistance will be much lower than that given by the Bloch–Grüneisen theory, and the phonon and electron drift velocities are close and they are determined by the total phonon relaxation rate in resistive scattering processes. In the opposite case, when resistive scattering processes dominate for phonons and the phonon system remains in equilibrium, then the electrical resistance follows the Bloch–Grüneisen theory. In this case, the drift velocities of all modes are different and much less than the electron drift velocity.

Keywords: electrical resistance, elastic anisotropy, electron-phonon relaxation.

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1. Introduction

This paper analyzed influence of elastic anisotropy on electron-phonon relaxation, mutual drag of electrons and phonons and electric resistance of potassium crystals. Previously, in research of these effects, for phonons, a model of isotropic medium is used (see [1–4]). In this model only longitudinal phonons may participate in electron-phonon relaxation. It was used in Bloch–Grüneisen theory to calculate electric resistance of metals [5–12]. In papers [13–15] it was demonstrated that this approximation turned out to be incorrect for explanation of experimental data of thermo-emf of drag in potassium crystals [16]. In elastic anisotropic crystals, quasi-longitudinal and quasi-transverse phonons are distributed (see [17]). They have non-zero longitudinal component [17], therefore within the standard theory of potential deformations may interact with electrons. It turned out that at low temperatures the contribution of slow quasitransverse modes in thermo-emf of potassium crystal drag, which has not been previously taken into account (see [1–12]), by order of magnitude exceeded contribution of longitudinal phonons. Such approximation turned out to be insufficient to explain experimental data [16]. Therefore, in [18] the impact of shear waves at electron-phonon relaxation is taken into account, and constant of electron-phonon interaction was determined for shear components of oscillatory modes $E_{0t} = 0.11$ eV. Ziman in [9,19] mentioned the need to take into account impact of shear deformations at energy of conduction electrons in alkaline metals.

Paper [20] analyzed impact of elastic energy anisotropy at electron-phonon relaxation and electric resistance of potassium crystals in approximation of Bloch–Grüneisen [6–12]. Accounting of elastic energy anisotropy per phonon system, and also contribution of shear waves to electron-phonon relaxation made it possible to approve results of electric resistance calculation results with data [21,22] at temperatures above 40 K. It was demonstrated that at temperatures that are much lower than Debye temperature ($T \ll \Theta_D$), where resistance followed dependence $\rho_{e-ph} \approx B_1 T^5$, contribution of quasi-transverse phonons to electric resistance, which was not taken into account previously, 11.5 times exceeded contribution of longitudinal phonons, and relaxation of electrons on shear waves 4 times exceeded contribution of longitudinal phonons. However, at high temperatures ($T \gg \Theta_D$), where $\rho_{e-ph} \approx B_2 T$, contribution of longitudinal phonons was 4.5 times higher than total contribution of electron relaxation in fast and slow transverse modes.

In Bloch–Grüneisen theory [5–12] it was assumed that the system of phonons is balanced: the entire momentum transmitted by electrons to phonons through normal processes of electron-phonon scattering is not transmitted back to electrons, but relaxes inside phonon subsystem. In contrast to [19], in the present paper we have taken into account the most relevant phonon relaxation processes: scattering at sample boundaries, dislocations and in phonon-phonon umklapp processes, and analyzed mutual drag of electrons and phonons in potassium crystals with different dislocation concentrations. To solve this problem, we used the hydrodynamic approximation [23–25]: we considered

relaxation and momentum exchange between electron and three phonon fluxes corresponding to the three branches of the phonon spectrum. Hydrodynamic approximation allows inelasticity of electron-phonon relaxation to be correctly taken into account when calculating electrical resistance (see [19]). In the present paper it is shown that in the limiting case of strong mutual drag of electrons and phonons, electrical resistance will be much less than that given by Bloch–Grüneisen theory, and phonon drift velocities of all polarizations are equal and close to electron drift velocity. In the opposite limiting case, when resistive scattering processes dominate for phonons and the phonon system remains in equilibrium, we have the limiting case of Bloch–Grüneisen theory [7–9,20]. Electrical resistance is determined by normal electron-phonon relaxation processes, and drift velocities of all modes are different and much smaller than electron drift velocity.

2. Electron-phonon relaxation in elastically anisotropic metals

At temperatures much lower than Debye temperature, the main contribution to electron relaxation in metals will be made by long-wave phonons with wave vector $q \ll q_D$ (q_D — Debye wave vector) [26,27]. Therefore, for phonons, we will use the anisotropic continuum model [26,27]. In this model, phonon spectrum $\omega_q^\lambda = S^\lambda(\theta, \varphi)q$ and the phase velocity $S^\lambda(\theta, \varphi)$ for potassium crystals are defined in [13,18] and for polarization vectors, according to [17], we have

$$e_j^\lambda = \frac{1}{A_\lambda}\left\{\frac{h_j}{\psi_j^\lambda}\right\}, \quad A_\lambda = \pm\sqrt{\sum_j \frac{n_j^2}{(\psi_j^\lambda)^2}}, \quad (\mathbf{e}^\lambda\mathbf{n}) = \frac{1}{A_\lambda}\sum_j \frac{n_j^2}{\psi_j^\lambda},$$

$$\psi_j^\lambda = \frac{1}{3} + z_\lambda + (k-1)n_j^2, \quad k = (c_{12}+c_{44})/(c_{11}-c_{44}). \tag{1}$$

where c_{ij} — second-order elastic moduli, $\mathbf{n} = \mathbf{q}/q = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$ — unit phonon wave vector, z_λ — roots of Christoffel equation, determining the spectrum and polarization vectors (see [17] for details). The values of the second-order elastic moduli at $T = 4.2$ K are taken from [28]. Polarization index L corresponds to longitudinal phonons, while t_1 and t_2 — respectively, „fast" and „slow" quasi-transverse oscillatory modes. Anisotropy parameters $k-1$ in alkali crystals are much larger than those for Si (see Table). Therefore, phonon focusing and electron-phonon relaxation in potassium crystals is significantly different from semiconductor crystals (see [13–15] for more detail). Maximum value of the longitudinal component of the slow quasi-transverse mode t_2 reaches 28%, twice that of silicon crystals, and the average value of $\langle(\mathbf{e}^t\mathbf{n})^2\rangle$ included in the electron-phonon interaction constant increases four times when passing from Si crystals to potassium (see Table). Increase in the mean values of the longitudinal components $\langle(\mathbf{e}^\lambda\mathbf{n})^2\rangle$ leads to

a significant increase in contribution of quasi-transverse modes to electron-phonon relaxation. As already noted in [9,19], spectrum of conduction electrons with Fermi energy in potassium crystals becomes anisotropic, and they get an opportunity to interact with shear waves (i.e., with the transverse component of quasi-transverse modes). Since all oscillatory modes in elastically anisotropic crystals have longitudinal and transverse components, then phonon polarization vectors $\mathbf{e}^\lambda(\mathbf{q})$ can be decomposed into the longitudinal $\mathbf{e}_\Uparrow^\lambda = \mathbf{n}(\mathbf{e}^\lambda\mathbf{n})$ (caused by compressive and tensile strain) and transverse component $\mathbf{e}_\perp^\lambda = [\mathbf{e}^\lambda\mathbf{n}]$ (due to shear strain of the lattice) (see [29]). In paper [18] it is shown that Fourier component of matrix element in electron-phonon interaction may be presented as

$$\left(C_0^\lambda(\theta, \varphi)\right)^2 \cong (E_{eff}^\lambda)^2\hbar/\left(S^\lambda(\theta, \varphi)\rho\right),$$

$$(E_{eff}^\lambda)^2 = \left(E_{0L}^2(\mathbf{e}^\lambda\mathbf{n})^2 + E_{0t}^2([\mathbf{e}^\lambda\mathbf{n}]^2)\right). \tag{2}$$

In addition to the constant $E_{0L} = 1.41$ eV characterizing the interaction of electrons with longitudinal components, we have introduced the constant E_{0t} characterizing the interaction of electrons with transverse components of oscillatory modes. It was determined in [18] from comparison of drag thermo-emf calculation results with data of [23] and turned out to be lower than E_{0L}; $E_{0t} = 0.11$ eV by order of magnitude. This is not surprising, since, according to estimates of [19], deviation of Fermi surface from the spherical one in potassium crystals is 7%. This ratio of the constants E_{0L} and E_{0t} is significantly different from semiconductor crystals, where, due to the much greater anisotropy of the current carrier spectrum, constant E_{0t} is two orders of magnitude greater, and usually exceeds value E_{0L} [29,30]. It should be noted that in contrast to the isotropic medium model, effective bonding constant $\left(E_{eff}^\lambda(\theta, \varphi)\right)^2$ is a function of θ and φ angles, whose dependences are determined by squares of longitudinal and transverse polarization vector components (see [18] Fig. 2). For longitudinal phonons, deviation of function $\left(E_{eff}^L(\theta, \varphi)\right)^2$ from isotropic distribution does not exceed 10%. However, for slow transverse mode, it changes quite dramatically for both cross sections (see [18] Fig. 2).

3. Influence of focusing on mutual drag of electrons and phonons in potassium crystals

Let us calculate charge flux in metal due to the action of a constant electric field. We proceed from a system of kinetic equations for non-equilibrium electron $f(\mathbf{k}, \mathbf{r})$ and phonon $N^\lambda(\mathbf{q}, \mathbf{r})$ distribution functions (see [2–4,13]):

$$\frac{e}{\hbar}\mathbf{E}_0\frac{\partial f_\mathbf{k}}{\partial\mathbf{k}} + (\mathbf{v}_k\nabla_r)f_\mathbf{k} = I_{ei}(f_\mathbf{k}) + I_{eph}(f_\mathbf{k}, N_q^\lambda), \tag{3}$$

$$\mathbf{v}_q^\lambda\nabla_r N_q^\lambda = -\left(N_q^\lambda - N_{q\lambda}^{(0)}\right)\nu_{ph}^{(1)\lambda} + I_{phe}(N_q^\lambda, f_\mathbf{k}). \tag{4}$$

Elastic modules of the second order c_{ij} (10^{12} dyne/cm^2), density ρ (g/cm^3), anisotropy parameter $k-1$ for crystals HgSe:Fe and Si, K, Li, Na

Compound	c_{11}	c_{12}	c_{44}	ρ	$k-1$	$\langle(\mathbf{e}^L\mathbf{n})^2\rangle$	$\langle(\mathbf{e}^{t1}\mathbf{n})^2\rangle$	$\langle(\mathbf{e}^{t2}\mathbf{n})^2\rangle$
HgSe:Fe	0.69	0.51	0.23	8.26	0.61	0.99	$6.7\cdot10^{-4}$	$7.0\cdot10^{-3}$
Si	1.677	0.65	0.804	2.33	0.67	0.99	$7.5\cdot10^{-4}$	$7.9\cdot10^{-3}$
K	0.0457	0.0374	0.0263	0.91	2.284	0.965	0.0028	0.0323
Li	0.148	0.125	0.108	0.55	4.825	0.942	0.0044	0.0536
Na	0.0615	0.0469	0.0592	1.01	45.13	0.902	0.0069	0.0908

Here $\mathbf{v}_k = \partial\varepsilon_k/\hbar\partial\mathbf{k}$, $\mathbf{v}_q^\lambda = \partial\omega_q^\lambda/\partial\mathbf{q}$ — group velocities of electrons and phonons with polarization λ, $N_{q\lambda}^0$ — Planck function, total velocity of phonon relaxation $v_{ph}^{(1)\lambda} = v_{phB}^\lambda(\theta,\varphi) + v_{phd}^\lambda + v_{phU}^\lambda(q)$ includes all non-electron resistive relaxation velocities, caused by scattering of phonons on the boundaries of sample $v_{phB}^\lambda(\theta,\varphi)$, dislocations v_{phd}^λ and in processes of phonon-phonon umklapp $v_{phU}^\lambda(q)$. Integrals of electron collisions with impurities I_{ei}, phonons I_{eph} and phonons with electrons I_{phe} are determined in [2,14].

$$I_{eph}(f_\mathbf{k}, N_\mathbf{q}) = \frac{2\pi}{\hbar}\frac{1}{V}\sum_{\mathbf{q},\lambda}|C_q^\lambda|^2\bigg\{\Big[f_{\mathbf{k+q}}(1-f_\mathbf{q})(N_\mathbf{q}^\lambda+1)$$

$$- f_\mathbf{k}(1-f_{\mathbf{k+q}})N_\mathbf{q}^\lambda\Big]\delta(\varepsilon_{\mathbf{k+q}}-\varepsilon_\mathbf{k}-\hbar\omega_q^\lambda) - \Big[f_\mathbf{k}(1-f_{\mathbf{k-q}})$$

$$\times(N_\mathbf{q}^\lambda+1) - f_{\mathbf{k-q}}(1-f_\mathbf{k})N_\mathbf{q}^\lambda\Big]\delta(\varepsilon_{\mathbf{k-q}}-\varepsilon_\mathbf{k}+\hbar\omega_q^\lambda)\bigg\},$$

$$\tag{5}$$

$$I_{phe} = \frac{4\pi}{\hbar}\frac{1}{V}\sum_\mathbf{k}|C_q^\lambda|^2\Big[f_{\mathbf{k+q}}(1-f_\mathbf{k})(N_\mathbf{q}^\lambda+1)$$

$$- f_\mathbf{k}(1-f_{\mathbf{k+q}})N_\mathbf{q}^\lambda\Big]\delta(\varepsilon_{\mathbf{k+q}}-\varepsilon_\mathbf{k}-\hbar\omega_q^\lambda), \tag{6}$$

where $|C_q^\lambda|^2 = (C_0^\lambda(\theta,\varphi))^2 q \cong (E_{eff}^\lambda)^2\hbar/(S^\lambda(\theta,\varphi)\rho)q$, and E_{eff}^λ is determined by equations (2). Spectrum of conduction electrons in potassium crystals is assumed to be isotropic, and anisotropic continuum model [13–15,20] is used for phonons.

A scheme describing momentum relaxation of quasiparticles in non-equilibrium electron-phonon system is given in Fig. 1. Non-equilibrium momentum of electrons received from electric field relaxes on impurities and defects, and in normal electron-phonon scattering processes part of this momentum is transferred to phonons and provides drift motion of phonons. If all the momentum transferred to phonons is completely relaxed in resistive phonon scattering processes: in scattering on sample boundaries, defects, dislocations and in phonon-phonon umklapp processes, then the phonon system remains in equilibrium, as assumed in the Bloch-Grüneisen [5–8] theory. In this case, normal electron-phonon scattering processes ensure relaxation of the electronic system to a drift locally equilibrium state
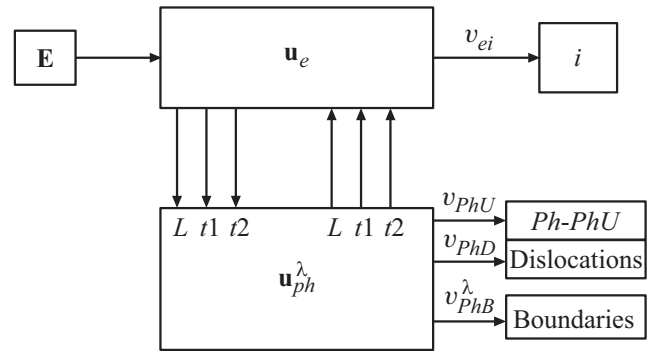

Figure 1. Scheme illustrating momentum relaxation gained by electrons from electric field in non-equilibrium electron-phonon system

and, consequently, together with impurities and defects at $\gg\Theta_D$ condition provide a linear temperature dependence of metal electrical resistance. Since in perfect potassium crystals at sufficiently low temperatures phonon-phonon umklapp processes are frozen out, only a small part of non-equilibrium momentum obtained by phonons relaxes on impurities, defects and dislocations, while most of it returns back to the electron subsystem. In contrast to Bloch-Grüneisen theory, we have taken into account influence of elastic energy anisotropy on relaxation of phonons and analyzed contributions of all oscillatory modes to electrical resistance of potassium crystals. As shown in [13–15,20], the main role in this relaxation is played by slow quasi-transverse phonons. Let us restrict ourselves to linear approximation by external perturbations and represent the electron and phonon distribution functions as [1–4]:

$$f_\mathbf{k} = f_0(\varepsilon_k) + \delta f_\mathbf{k}, \quad N_\mathbf{q}^\lambda = N_{q\lambda}^0 + g_\lambda(\mathbf{q}), \tag{7}$$

where $f_0(\varepsilon_k)$ and $N_{q\lambda}^0$ — equilibrium distribution functions for electrons and phonons, and δf_k and $g_\lambda(\mathbf{q})$ — non-equilibrium additives to them. Let us linearize collision integrals (3), (4) by these additives. Let us consider the momentum balance in a system of electrons interacting with impurities and phonons under isothermal conditions. The electric field is assumed to be sufficiently weak that we can restrict ourselves to a linear approximation. We

assume that a drift-local equilibrium state is realized for electron and phonon subsystems, which can be described in a hydrodynamic approximation [23–25]:

$$f(\mathbf{k}, \mathbf{u}) = \left(\exp\left(\frac{\varepsilon_k - \xi - \hbar\mathbf{k}\mathbf{u}_e}{k_B T}\right) + 1\right)^{-1}$$

$$= f_0(\varepsilon_k) + \delta f_{\mathbf{k}}, \quad \delta f_{\mathbf{k}} = (\hbar\mathbf{k}\mathbf{u}_e)\left(\frac{\partial f_0}{\partial \varepsilon_k}\right). \quad (8)$$

$$N(\mathbf{q}, \mathbf{u}_\lambda) = N_{q\lambda}^0 + g_\lambda(\mathbf{q}), \quad g_\lambda(\mathbf{q}) = \left\{\frac{\hbar\mathbf{q}\mathbf{u}_\lambda}{k_B T_0}\right\} N_{q\lambda}^0 (N_{q\lambda}^0 + 1). \quad (9)$$

Here $f_0(\varepsilon_k)$ and $N_{q\lambda}^0$ — Fermi and Planck functions, $\mathbf{u}_e$ and $\mathbf{u}_\lambda$ — drift velocities of electrons and phonons, which are determined from the bound system of quasiparticle momentum balance equations in the non-equilibrium electron-phonon system. So, in the hydrodynamic description of the electron transfer in elastically anisotropic metals, the electron flux interacts with three phonon ones, corresponding to the three branches of the phonon spectrum. To obtain balance equations, we multiply equations (5) and (6) by $\hbar\mathbf{k}$ and $\hbar\mathbf{q}$ and summarize, respectively, over the electron and phonon momenta

$$\frac{1}{V}\sum_{k,\sigma}\hbar\mathbf{k}\left\{I_{eph}(\delta f_{\mathbf{k}}, N_{q\lambda}^0) + I_{eph}\left(f_0(\varepsilon_k), g_\lambda(\mathbf{q})\right)\right\}$$

$$= m_F n_e \sum_\lambda \left\{u_e^\alpha \nu_{eph(\delta f)}^\lambda(k_F) - \mathbf{u}_\lambda^\alpha \nu_{eph(g)}^{\lambda(B)}(k_F)\right\}. \quad (10)$$

$$\frac{1}{V}\sum_{\mathbf{q}}\hbar\mathbf{q}^\alpha\left\{I_{phe}(\delta f_{\mathbf{k}}, N_{q\lambda}^0) + I_{phe}^\lambda\left(f_0(\varepsilon_k), g_\lambda(\mathbf{q})\right)\right\}$$

$$= m_F n_e \left\{\mathbf{u}_e^\alpha \nu_{phe(\delta f)}^\lambda(k_F) - u_\lambda^\alpha \nu_{phe(g)}^{\lambda(B)}(k_F)\right\}. \quad (11)$$

The first terms in equations (10) and (11) account for momentum transfer from non-equilibrium electrons to phonons, and the second terms — from non-equilibrium phonons to electrons. Note that relaxation rates in electron-phonon and phonon-electron scattering processes are equal: $\nu_{eph(\delta f)}^\lambda(k_F) = \nu_{phe(\delta F)}^\lambda(k_F)$, $\nu_{eph(g)}^{\lambda(B)} = \nu_{phe(g)}^{\lambda(B)}$. They can be represented as

$$\nu_{eph(\delta f)}^\lambda(k_F) = \nu_{phe(\delta f)}^\lambda(k_F) = \frac{m_F}{24\pi^4\hbar^3 n_e}\int d\Omega |C_0^\lambda|^2 (q_{T\lambda})^5$$

$$\times \int dZ_q^\lambda (Z_q^\lambda)^4 N_{q\lambda}^0 \int dy_{\mathbf{k}}\left[\left(f_0(y_{\mathbf{k}})\right)\left(1 - f_0(y_{\mathbf{k}} + Z_q^\lambda)\right)\right]$$

$$= \frac{m_F}{24\pi^4\hbar^3 n_e}\frac{1}{\rho}\left(\frac{k_B T}{\hbar}\right)^5 \Phi_\Omega^\lambda \cdot J_{eph(\delta f)}(Z_D^\lambda),$$

$$\Phi_\Omega^\lambda = \left\langle \frac{(E_{eff}^\lambda)^2}{[S^\lambda(\theta, \varphi)]^5} \right\rangle = \int_0^\pi \sin(\theta)d\theta \int_0^{2\pi} d\varphi \frac{(E_{eff}^\lambda)^2}{[S^\lambda(\theta, \varphi)]^5}, \quad (12)$$

$$q_T^\lambda = (k_B T)/\left(\hbar S^\lambda(\theta, \varphi)\right),$$

$$J_{eph(\delta f)}^\lambda = J_{phe(\delta f)}(Z_D^\lambda) = \int_0^{Z_D^\lambda} dZ_q^\lambda (Z_q^\lambda)^4 N_{q\lambda}^0$$

$$\times \int dy_{\mathbf{k}}\left[\left(f_0(y_{\mathbf{k}})\right)\left(1 - f_0(y_{\mathbf{k}} + Z_q^\lambda)\right)\right]$$

$$\nu_{eph(g)}^{\lambda(B)} = \nu_{phe(g)}^{\lambda(B)} = \frac{m_F}{24 n_e \pi^4 \hbar^3}\int d\Omega_q (q_T^\lambda)^5 |C_0^\lambda|$$

$$\times \int_0^{Z_{qD}^\lambda} dZ_q^\lambda Z_q^{\lambda 4}\left\{N_{q\lambda}^0(N_{q\lambda}^0 + 1)\int dy_{\mathbf{k}}\left[f_0(y_{\mathbf{k}}) - f_0(y_{\mathbf{k}} + Z_q^\lambda)\right]\right\}$$

$$= \frac{m_F}{24 n_e \pi^4 \hbar^3}\frac{1}{\rho}\left(\frac{k_B T}{\hbar}\right)^5 \Phi_\Omega^\lambda \cdot J_{phe(g)}(Z_D^\lambda),$$

$$J_{phe(g)}(Z_D^\lambda) = \int_0^{Z_{qD}^\lambda} dZ_q^\lambda Z_q^{\lambda 4}$$

$$\times \left\{N_{q\lambda}^0(N_{q\lambda}^0 + 1)\int dy_{\mathbf{k}}\left[f_0 y_{\mathbf{k}} - f_0(y_{\mathbf{k}} + Z_q^\lambda)\right]\right\} \quad (13)$$

Hydrodynamic approximation allows us to introduce distribution functions of the most effective phonons in electron-phonon and phonon-electron scattering processes:

$$\Phi_{eph(\delta f)}(Z_q^\lambda) = (Z_q^\lambda)^4 N_{q\lambda}^0 \int dy_{\mathbf{k}}\left[\left(f_0(y_{\mathbf{k}})\left(1 - f_0(y_{\mathbf{k}} + Z_q^\lambda)\right)\right)\right],$$

$$\Phi_{phe(g)}(Z_q^\lambda) = (Z_q^\lambda)^4 N_{q\lambda}^0(N_{q\lambda}^0 + 1)$$

$$\times \int dy_{\mathbf{k}}\left[f_0(y_{\mathbf{k}}) - f_0(y_{\mathbf{k}} + Z_q^\lambda)\right]. \quad (14)$$

It is not difficult to see from equation (14) that the functions of the most effective scattering processes for electron-phonon $\Phi_{eph(\delta f)}(Z_q^\lambda)$ and phonon-electron $\Phi_{phe(g)}(Z_q^\lambda)$ processes coincide. Therefore, momentum relaxation rates in the electron-phonon and phonon-electron scattering processes are equal: $\nu_{eph(\delta f)}^\lambda(k_F) = \nu_{phe(g)}^{\lambda(B)}$. In this case integral parts of balance equations (10) and (11) will be equal and proportional to the difference of drift velocities of electrons and phonons:

$$m_F n_t \sum_\lambda \nu_{eph(\delta f)}^\lambda(k_F)\left\{\mathbf{u}_e^\alpha - \mathbf{u}_\lambda^\alpha\right\}. \quad (15)$$

Therefore, if electron and phonon drift velocities coincide, electron-phonon interaction makes no contribution to electron and phonon momentum relaxation.

Let us consider the role of inelasticity of electron-phonon relaxation in metals. In monographs [24,25] they believed that inelasticity of electron-phonon scattering in metals may be neglected, since Fermi energy is much higher than phonon energy $\hbar\omega_q^\lambda$ and $k_B T$. In Ziman monographs [14,16]
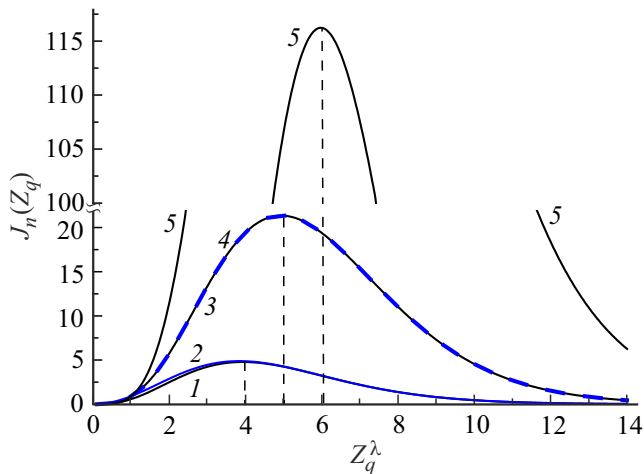
Figure 2. Distribution functions of most effective phonons from parameter Z_q^λ: curve *1* — $\Phi_{eph(el)}(Z_q^\lambda) = (Z_q^\lambda)^4 N_q^\lambda$ for elastic scattering of electrons, curve *2* — $\Phi_4(Z_q^\lambda) = (Z_q^\lambda)^4 N_q^\lambda (N_q^\lambda + 1)$ (for heat capacity), curve *3* — $\Phi_{eph(\delta f)}(Z_q^\lambda)$ for inelastic scattering of electrons, dotted curve *4* — $\Phi_5(Z_q^\lambda) = (Z_q^\lambda)^5 N_q^\lambda (N_q^\lambda + 1)$ determines distribution of phonons when solving the problem by variation method [8-12], curve *5* — for $\Phi_6(Z_q^\lambda) = (Z_q^\lambda)^6 N_q^\lambda (N_q^\lambda + 1)$ phonon-phonon umklapp processes.

it was argued that since incident and scattered phonon energies lie within thermal blur of Fermi energy, they differ by order of magnitude $k_B T$, which is much smaller than ε_F, so their difference can be neglected. Naturally, Fermi energies ε_F in metals at helium temperatures are $4-5$ orders of magnitude greater than thermally excited phonon energy $\hbar\omega_q^\lambda$ and $k_B T$, but these considerations have nothing to do with the criterion of considering or neglecting inelasticity of electron-phonon scattering. The fact is that the integrals of electron-phonon collisions include Fermi functions $f_0(\varepsilon_k \pm \hbar\omega_q^\lambda)$. Inelasticity parameter $Z_q^\lambda = \hbar\omega_q^\lambda/k_B T$ should not be compared with the ratio $\varepsilon_F/(k_B T)$ but with the value $y_k = (\varepsilon_k - \varepsilon_F)/(k_B T)$. For electrons at Fermi level $y_k = 0$, and within the Fermi level thermal blur is $|k_k| \leq 1$. Electron momentum relaxation is provided by all thermally activated phonons, and their distribution is determined by distribution functions that are most efficient for electron-phonon relaxation $\Phi_{eph(\delta f)}(Z_q^\lambda) = \Phi_{phe(g)}(Z_q^\lambda)$ (see (14)). As seen in Fig. 2, this function (curve *1*) reaches maximum at $Z_q^\lambda = 5$ and decreases rapidly at $Z_q^\lambda > 12$ due to Planck distribution function. It should be noted that it practically coincides with dashed curve *4*, which determines equation for the distribution function of the most effective phonons $\Phi_5(Z_q^\lambda)$ when solving the problem by the variation method (see [5–10], and also Fig. 2, curves *3, 4*). Neglecting inelasticity of electron-phonon scattering $Z_q^\lambda \to 0$, we obtain $\Phi_{eph(el)}(Z_q^\lambda) = (Z_q^\lambda)^4 N_q^\lambda$ (see Fig. 2, curve *2*). Calculation of the integrals over Z_q^λ at low temperatures gives $J_{eph(\delta f)}^\lambda = 124.4$, and in elastic approximation $J_{eph(el)}^\lambda = 24.9$. Thus, taking into account inelasticity of electron-phonon scattering leads to 5 fold increase in electri-

cal resistance compared to elastic approximation. It should be noted that for heat capacity, the distribution function of thermally activated phonons, according to [14–16], is determined by equation $\Phi_{tepl}(Z_q^\lambda) = (Z_q^\lambda)^4 N_q^\lambda (N_q^\lambda + 1)$. It reaches maximum at $Z_q^\lambda = 4$ and is nonzero in the interval $1 < Z_q^\lambda < 9$ (see Fig. 2, curve *2*). So, at low temperatures, phonons that are most relevant to electrical resistance are distributed in interval $1 < Z_q^\lambda < 12$ with maximum at $Z_q^\lambda \approx 5$. For them, inequality $Z_q^\lambda \ll 1$ is not complied with. Moreover, for relevant phonons in Fermi level thermal blur region $|y_k| \leq 1$, the opposite inequality is complied with: $y_k = (\varepsilon_k - \varepsilon_F)/(k_B T) \ll Z_q^\lambda$. Therefore, the main contribution to electric resistance at low temperatures is made not by „vertical" transitions $\hbar\omega_q^\lambda \approx k_B T$ or „horizontal" $\hbar\omega_q^\lambda \gg k_B T$ transitions by Ziman terminology [14,16], but „oblique" transitions, for which $k_B T \leq \hbar\omega_q^\lambda \leq 12 k_B T$ (see Fig. 3). So, hydrodynamic approximation allows inelasticity of electron-phonon scattering in metal to be correctly taken into account.

For electron scattering on impurities, we have:

$$\frac{1}{V}\sum_{\mathbf{k},\sigma} \hbar\mathbf{k} I_{ei}(f_\mathbf{k}) = -\frac{1}{V}\sum_{\mathbf{k},\sigma} \hbar\mathbf{k}(\hbar\mathbf{k}\mathbf{u})\nu_{ei}(k)\left(\frac{\partial f_0}{\partial \varepsilon_k}\right)$$

$$= -\mathbf{u}^\alpha m_F n_e \cdot \nu_{ei}(k_F). \qquad (16)$$

From equations (7) and (8) for full momentum balance equations under isothermal conditions $\nabla_\alpha T(r) = 0$ we obtain a system of four algebraic equations for drift velocities:

$$e\mathbf{E}_\alpha n_e = m_F n_e \left[\nu_{ei}(k_F)\mathbf{u}_e^\alpha + \sum_\lambda \nu_{eph(\delta f)}^\lambda(k_F)\{\mathbf{u}_e^\alpha - \mathbf{u}_\lambda^\alpha\}\right]$$
$$- \mathbf{u}_\lambda^\alpha R_\lambda + m_F n_e (\mathbf{u}_e^\alpha - \mathbf{u}_\lambda^\alpha)\nu_{phe(\delta f)}^\lambda()k_F = 0. \qquad (17)$$

From (17) we express phonon drift velocity through drift velocity of electrons:

$$\mathbf{u}_\lambda^\alpha = \mathbf{u}_e^\alpha \frac{m_F n_e^\alpha \nu_{phe(\delta f)}^{\lambda(2)}(k_F)}{\{R_\lambda + m_F n_e \nu_{phe(g)}^{\lambda(B)}(k_F)\}}$$
$$= \mathbf{u}_e^\alpha \frac{1}{\{1 + R_\lambda/m_F n_e \nu_{phe(g)}^{\lambda(B)}(k_F)\}}. \qquad (18)$$

As it should be: direction of electron flux coincides with direction of phonon flux and is opposite to direction of electric current. It is obvious that in performance of inequality $\lfloor R_\lambda/(m_F n_e \nu_{phe(g)}^{\lambda(B)})\rfloor \ll 1$ drift velocity of mode λ will be close to electron drift velocity $\mathbf{u}_\lambda^\alpha \cong \mathbf{u}_e^\alpha$. Value R_λ includes all non-electron mechanisms of phonon momentum relaxation: scattering on boundaries, dislocations and in

phonon-phonon umklapp processes:

$$R_\lambda = \frac{(k_B T)^4}{3(2\pi\hbar)^3} J_5(Z_q^\lambda) \left\langle \frac{\nu_{phB}^\lambda(\theta,\varphi)}{(S^\lambda(\theta,\varphi))^5} \right\rangle$$
$$+ \frac{\nu_{phd}^*(k_B T)^5}{\hbar 3(2\pi\hbar)^3} J_5(Z_q^\lambda) \left\langle \frac{1}{(S^\lambda(\theta,\varphi))^5} \right\rangle$$
$$+ \frac{(k_B T)^7}{\hbar^3 3(2\pi)^3} A_U^\lambda \cdot \exp\left(-\frac{C_U^\lambda}{T}\right) \cdot J_6^\lambda(Z_D^\lambda) \left\langle \frac{1}{(S^\lambda(\theta,\varphi))^5} \right\rangle. \tag{19}$$

Phonon relaxation rates $\nu_{phB}^\lambda(\theta,\varphi)$ in diffuse scattering at boundaries of circular, square and rectangular samples are defined in [17], value ν_{phd}^λ — in [24]. For umklapp processes, coefficient C_U^λ is determined by Debye temperature for each mode: $C_U^\lambda \cong T_D^\lambda/\delta_\lambda$, where adjustable parameter $\delta_\lambda \approx 2-3$, we use the method [17,31] to estimate coefficient A_U^λ.

4. Phonon focusing and electrical resistance of potassium crystals

From the system of equations (17), we find drift velocities of electrons and phonons:

$$\mathbf{u}_e^\alpha = \frac{e\mathbf{E}_\alpha m_F}{\tilde{\nu}_{eR}(k_F)},$$

$$\tilde{\nu}_{eR}(k_F) = \left\{ \nu_{ei}(k_F) + \sum_\lambda \nu_{eph(\delta f)}^{(\lambda)}(k_F)\left[1 - \frac{1}{1+K_U^\lambda}\right]\right\}, \tag{20}$$

$$\mathbf{u}_\lambda^\alpha = \frac{e\mathbf{E}_\alpha m_F}{\tilde{\nu}_{eR}(k_F)} \frac{m_F n_e^\alpha \nu_{phe(\delta f)}^{\lambda(2)}(k_F)}{\left\{R_\lambda + m_F n_e \nu_{phe(g)}^{\lambda(B)}(k_f)\right\}}$$

$$= \frac{e\mathbf{E}_\alpha m_F}{\tilde{\nu}_{eR}(k_F)} \frac{1}{\left\{1+K_U^\lambda\right\}},$$

$$K_U^\lambda = \left[R_\lambda/(m_F n_e \nu_{phe(g)}^{\lambda(B)}).\right] \tag{21}$$

Let us calculate conduction current and determine electrical resistance of potassium crystals:

$$J = -|e|\mathbf{u}^\alpha n_e = -\frac{e^2 n_e}{\tilde{\nu}_{eR}(k_F)\cdot m_F}\mathbf{E}_\alpha,$$

$$\rho_{xx} = \frac{\tilde{\nu}_{eR}(k_F)\cdot m_F}{e^2 n_e}$$

$$= \left(\frac{m_F}{e^2 n_e}\right)\left[\sum_\lambda \frac{\nu_{eph(\delta f)}^{(\lambda)}(k_F)K_U^\lambda}{1+K_U^\lambda}\right]. \tag{22}$$

As can be seen from (22), the entire effect of the partial transfer of non-equilibrium momentum from phonons to electrons is determined by factor K_U^λ and is reduced to renormalizing the effective electron relaxation rate on phonons due to resistive phonon relaxation mechanisms.

Scattering on impurities ensures electric resistance output to a constant value. We will not consider this effect but will analyze the effect of phonon relaxation in resistive processes on the momentum exchange within the electron-phonon system and electrical resistance of potassium crystals. Obviously, in the case of weak phonon scattering in non-electron relaxation mechanisms $K_U^\lambda \ll 1$ electrical resistance of potassium crystals will be much smaller than the Bloch–Grüneisen theory gives:

$$\rho_{xx} \cong \left(\frac{m_F}{e^2 n_e}\right)\left[\sum_\lambda \nu_{eph(\delta f)}^{(\lambda)}(k_F)K_U^\lambda\right] \ll \rho_{xx}^{BG}$$

$$= \left(\frac{m_F}{e^2 n_e}\right)\left[\sum_\lambda \nu_{eph(\delta f)}^{(\lambda)}(k_F)\right]. \tag{23}$$

Within this limit, normal electron-phonon relaxation processes dominate for both electrons and phonons. In this case most of the momentum of the non-equilibrium electrons, transferred to phonons, returns back to the electron system. As a result we obtain a rather curious result: under conditions of strong mutual drag of electrons and phonons, electrical resistance no longer depends on electron-phonon relaxation, but will be completely determined by resistive mechanisms of phonon relaxation: scattering on boundaries, dislocations and in phonon-phonon umklapp processes:

$$\rho_{xx} \cong \left(\frac{m_F}{e^2 n_e}\right)\sum_\lambda \nu_{eph(\delta f)}^{(\lambda)}(k_F)\left[R_\lambda/(m_F n_e \nu_{phe(g)}^{\lambda(B)})\right]$$

$$= \left(\frac{1}{e^2}\right)\left[\sum_\lambda R_\lambda\right]. \tag{24}$$

To this end, phonon drift velocities of all polarizations are equal and coincide with the electron drift velocity. They are determined by the phonon momentum relaxation rate in resistive scattering processes:

$$u_e^\alpha \approx u_\lambda^\alpha \approx \frac{eE_\alpha m_F}{\tilde{\nu}_{eR}(k_F)} \approx eE_\alpha m_F / \sum_\lambda R_\lambda. \tag{25}$$

In the opposite limiting case $K_U^\lambda \gg 1$ most of the drift momentum from electrons is relaxed in resistive scattering processes inside the phonon system. This case is realized at increasing temperature, when phonon-phonon umklapp processes dominate. The phonon system remains in equilibrium, and we move to the Bloch-Grüneisen approximation [5–8]:

$$\rho_{xx} \cong \left(\frac{m_F}{e^2 n_e}\right)\left[\sum_\lambda \nu_{eph(\delta f)}^{(\lambda)}(k_F)\left(1 - (K_U^\lambda)^{-1}\right)\right]$$

$$\approx \left(\frac{m_F}{e^2 n_e}\right)\left[\sum_\lambda \nu_{eph(\delta f)}^{(\lambda)}(k_F)\right] = \rho_{xx}^{BG}. \tag{26}$$

In this case drift velocities of all modes are different and much smaller than the electron drift velocity:

$$\mathbf{u}_e^\alpha = \frac{e\mathbf{E}_\alpha m_F}{\tilde{\nu}_{eR}(k_F)} \cong \frac{e\mathbf{E}_\alpha m_F}{\sum_\lambda \nu_{eph(\delta f)}^{(\lambda)}(k_F)},$$

$$\mathbf{u}_\lambda^\alpha = \frac{e\mathbf{E}_\alpha m_F}{\tilde{\nu}_{eR}(k_F)} \frac{1}{\{1 + K_U^\lambda\}} \approx \frac{e\mathbf{E}_\alpha m_F}{K_U^\lambda \sum_\lambda \nu_{eph(\delta f)}^{(\lambda)}(k_F)},$$

$$u_e^\alpha / K_U^\lambda \ll u_e^\alpha. \tag{28}$$

Let us analyze temperature dependences of the renormalization coefficient of the effective electron relaxation rate on phonons K_U^λ. As can be seen from Fig. 3, phonon scattering at sample boundaries cannot ensure equilibrium of the phonon system: only a small part of the non-equilibrium momentum obtained by phonons relaxes at the boundaries, while most of it returns back to the electron subsystem. For potassium crystals without dislocations the limiting case $K_U^\lambda \ll 1$ is realized for slow t_2-mode in temperature range $1\,\mathrm{K} < T < 5\,\mathrm{K}$, for L-phonons in range $1\,\mathrm{K} < T < 15\,\mathrm{K}$ (see Fig. 3, curves $2, 2a$). Since at low temperatures the slow t_2-mode dominates electrical resistance, for potassium crystals the limiting case of strong mutual drag can be realized only in the interval $1\,\mathrm{K} < T < 5\,\mathrm{K}$. As can be seen from Fig. 3, slow t_2-phonons scatter much stronger on dislocations than L-phonons. For potassium crystals with minimum dislocation concentration of $N_d = 0.03$ at $T < 15\,\mathrm{K}$ the K_U^L factor is much less than unity (see Fig. 3, curves 3). Therefore, for L-phonons the case of strong mutual drag can be realized. In the model that we adopted, we calculated temperature dependences of electrical resistance of potassium crystals with different dislocation concentrations and compared the calculation results with experimental data (see Fig. 4, curves $1, 2, 3$). Concentration values were taken from papers [16,20], where thermal-emf but not electrical resistance was measured. As can be seen from the figure, for potassium crystals with maximum concentration of dislocations $N_d = 0.55$ temperature dependences of the electrical resistance are close to those calculated in Bloch–Grüneisen approximation. Resistive processes of phonon scattering dominate for it: at low temperatures due to dislocations, and at high — due to umklapp processes (see Fig. 3, curves $4, 4a$). For t_2-mode, coefficient is $K_U^{t2} \gg 1$. Virtually the entire momentum coming from the electrons to t_2-mode is relaxed on dislocations at low temperatures, and at high — in umklapp processes. For such crystals the ratio of contributions to electric resistance is: $\rho_{e-ph}^{t2} : \rho_{e-ph}^L$; $\rho_{e-ph}^{t1} = 0.94 : 0.035 : 0.025$. Resistive processes of phonon scattering dominate for them: at low temperatures due to dislocations, and at high — due to umklapp processes (see Fig. 3, curves $4a$). For this mode the coefficient is $K_U^{t2} \gg 1$. Therefore t_2-mode is in equilibrium, and Bloch–Grüneisen regime is realized for it. For potassium crystals with minimum concentration of dislocations $N_d = 0.03$ (with deformation $\varepsilon = 0.027$) contribution of slow t_2-mode to electrical resistance increases to 95%, and ratio of contributions is: $\rho_{e-ph}^{t2} : \rho_{e-ph}^L$; $\rho_{e-ph}^{t1} = 0.95 : 0.01 : 0.04$. At the same time the contribution of shear component of t_2-mode increases from 32% in Bloch–Grüneisen model to 56%. For such crystals $K_u^L \approx 0.5$, and relaxation rate of L-phonons in non-electron scattering mechanisms is comparable to relaxation rate on electrons. Therefore, the

effect of mutual drag is mild. It provides minor effect at full electrical resistance (see Fig. 4, curves $1, 2$).

However, as the temperature increases, the contribution of L-phonons to electrical resistance increases rapidly and at $T > 30\,\mathrm{K}$ becomes greater than the contribution of the
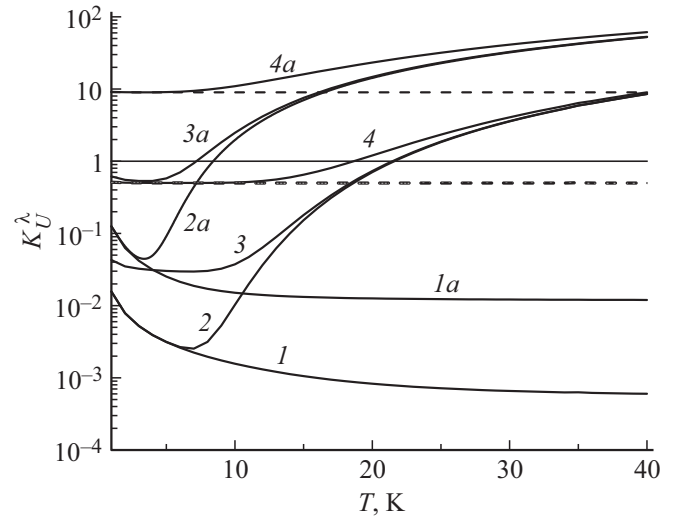

Figure 3. Temperature dependence of coefficient $K_U^\lambda = \left[R_\lambda / (m_F n_e \nu_{phe-g}^{\lambda(B)}) \right]$ in potassium crystals for longitudinal (curves $1, 2, 3, 4$) and slow t_2-phonons (curves $1a, 2a, 3a, 4a$). Curves $1, 1a$ — for boundary scattering, curves $2, 2a$ — for boundary scattering and umklapp processes, curves $3, 3a$ — for scattering on boundaries, dislocations with minimum concentration $N_d = 0.03$ and in umklapp processes, curves $4, 4a$ — for scattering on boundaries, dislocations with maximum concentration $N_d = 0.55$ and in umklapp processes.
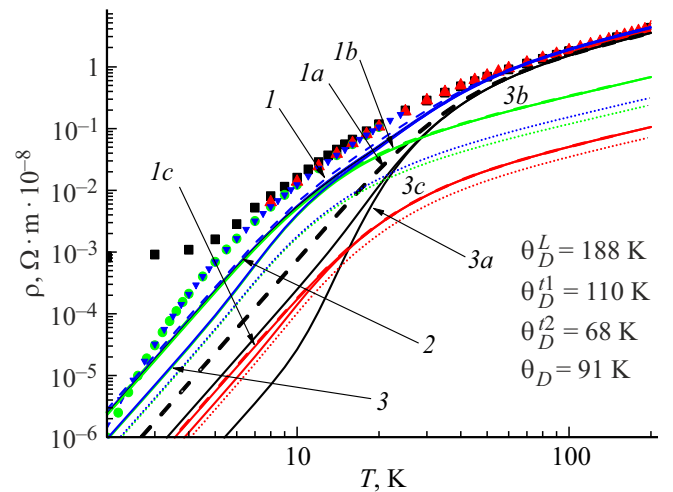

Figure 4. Temperature dependences of electrical resistance of potassium crystals: curves $1, 1a, 1b, 1c$ — results for Bloch–Grüneisen theory; curves $2, 2a, 2b, 2c$ — for crystals with maximum concentration of dislocations $N_d = 0.55$; curves $3, 3a, 3b, 3c$ — for crystals with minimum concentration of dislocations $N_d = 0.03$; curves $1a, 2a, 3a$ — contributions of L-phonons, curves $1b, 2b, 3b$ — contributions of slow transverse phonons, curves $1c, 2c, 3c$ — contributions of fast quasitransverse phonons. Symbols — experimental data from papers [21,22,34,35].

slow t_2-mode. Moreover, the coefficient becomes much greater than unity at $T > 40\,\mathrm{K}$ (see Figs. 3 and 5). It should be noted that with increasing temperature, phonon-phonon umklapp processes begin to play a major role, and at $T > 40\,\mathrm{K}$ the results of calculations of electrical resistance of potassium crystals with different dislocation concentration agree well with both Bloch–Grüneisen theory and results of measurements for pure samples [21,22,32,33]. In the Bloch–Grüneisen model it was shown that at low temperatures $T < 10\,\mathrm{K}$ the contribution of the slow t_2-mode to electrical resistance was an order of magnitude greater than that of the longitudinal phonons, ratio of contributions being as follows: $\rho_{e-ph}^{t2} : \rho_{e-ph}^{L}; \rho_{e-ph}^{t1} = 0.9 : 0.08 : 0.02$ (see [20]). The dominant role of the slow transverse mode in electrical resistance of potassium crystals at low temperatures is due to the fact that it has the minimum phase velocity and therefore the maximum wave vector at a fixed phonon energy (see [20], Fig. 1) and as such it makes the maximum contribution to electrical resistance. As can be seen from equation (12), ratio of the coefficients $\Phi_\Omega^\lambda(T)$ for the slow t_2-mode and L-phonons looks like:

$$\Phi_\Omega^{t2}/\Phi_\Omega^L = \left\langle \left(E_{eff}^{t2}(\theta,\varphi)\right)^2\right\rangle / \left\{S^{t2}(\theta,\varphi)\right\}^6 /$$
$$\left\langle \left(E_{eff}^L(\theta,\varphi)\right)^2 / \left\{S^L(\theta,\varphi)\right\}^6\right\rangle. \qquad (29)$$

Square of the effective bonding constant of longitudinal phonons to electrons $(E_{eff}^L)^2$ is 25 times larger than for the slow transverse mode. However, in directions such as [110], the phase velocity of the t_2-mode is 4 fold less than that for the longitudinal phonons, and the ratio of the averaged values gives $\langle S^L\rangle/\langle S^{t2}\rangle = 2.54$. The sixth degree of this ratio gives a value that is greater by two orders of magnitude. As a result of calculating angular averages in equation (29) we get: $\Phi_\Omega^{t2}/\Phi_\Omega^L \cong 11.5$. For thermally excited phonons with the same energy, the wave vector of the t_2-mode is 2.5 times larger than for longitudinal phonons, so their contribution to electron momentum relaxation is an order of magnitude greater than that of longitudinal phonons. It is necessary to note a significant role of electron relaxation on shear component of the t_2-mode, which provides 32% of electrical resistance of potassium crystals and is 4 fold greater than contribution of longitudinal phonons. However, as temperature increases, contribution of longitudinal phonons increases much faster than that of the slow t_2-mode, and at temperatures above $30\,\mathrm{K}$ it already dominates (see Fig. 5). With further increase in temperature at $T \geq T_D^L$, contribution of longitudinal phonons dominates, and at room temperature it reaches 82%, whereas the contribution of the slow t_2-mode decreases to 15%, and contribution of the fast transverse mode is only 3% (see Fig. 5).

Comparison of the calculation results with experimental data [21,22,32,33] shows that at temperatures above $40\,\mathrm{K}$ they agree well (see Fig. 4). This can be explained by the fact that the entire momentum obtained by phonons in normal electron-phonon scattering processes relaxes within the phonon system mainly due to phonon-phonon umklapp
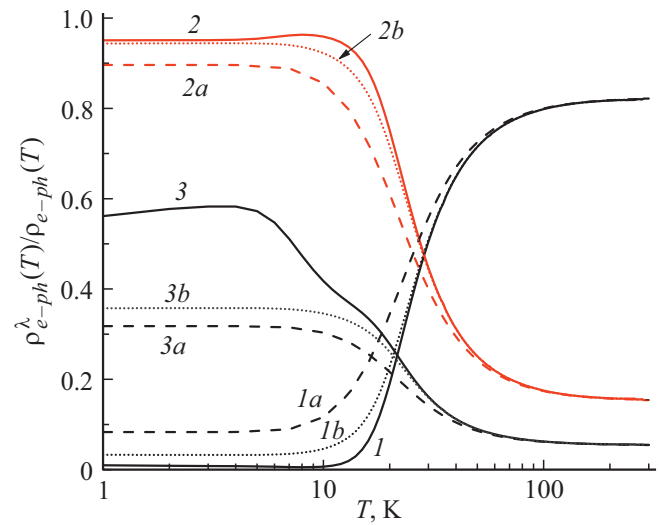

Figure 5. Temperature dependences of relative contributions to electrical resistance of potassium crystals $\rho_{e-ph}^\lambda(T)/\rho_{e-ph}(T)$ for slow quasitransverse phonons — curves 2, 2a, 2b, for L-phonons — 1, 1a, 1b, for shear component of t_2-mode — curves 3, 3a, 3b; for crystals with minimum concentration of dislocations $N_d = 0.03$ — curves 1, 2, 3, for crystals with maximum concentration of dislocations $N_d = 0.55$ — curves 1b, 2b, 3b, calculation according to Bloch–Grüneisen theory — curves 1a, 2a, 3a.

processes. These processes are activated at temperatures of the order of θ_D^λ/γ, where $\gamma \sim (2-3)$. This result indicates that accounting of elastic energy anisotropy per phonon system, and also contribution of shear waves to electron-phonon relaxation make it possible to approve results of electric resistance calculation results with data [21,22,32,33] without using adjustable parameters. However, as the temperature drops below $40\,\mathrm{K}$, discrepancy between experimental data [21,22,32,33] and the calculation results for potassium samples with different dislocation concentration increases and reaches a maximum at $T \sim 7-8\,\mathrm{K}$. This is due to the fact that the phonon-phonon umklapp processes are frozen out, but the role of electron-phonon umklapp processes, which are not considered in the present paper, increases. The activation temperature of these processes is $10-20\,\mathrm{K}$, and they play a maximum role in the $5-10\,\mathrm{K}$ [34,35] interval. According to the thermo-emf data [18] as well as the analysis [35,36] the electron-phonon scattering processes are frozen out at temperatures $T < 4-5\,\mathrm{K}$, and at $T < 2\,\mathrm{K}$ the normal electron-phonon scattering processes already dominate. So, at temperatures below $30\,\mathrm{K}$, taking into account the mutual drag of electrons and phonons only leads to an increase in discrepancy between the calculation results and the experimental data. For their agreement it is necessary to take into account the electron-phonon umklapp processes. Obviously, analysis of the role of electron-phonon umklapp processes taking into account the anisotropy of the phonon spectrum presents an independent problem that requires a separate consideration.

5. Conclusion

Influence of phonons focusing on mutual drag of electrons and phonons in potassium crystals was studied. Its main results can be formulated as follows.

1. In the limiting case of strong mutual drag of electrons and phonons, when normal electron-phonon relaxation processes dominate, both for electrons and phonons the electrical resistance will be much lower than Bloch–Grüneisen theory gives. It is determined by resistive mechanisms of phonon scattering: on boundaries, dislocations and in phonon-phonon umklapp processes. To this end (1) phonon drift velocities of all polarizations are equal and close to electron drift velocity; (2) they are determined by total phonon relaxation rate in resistive scattering processes.

2. In the opposite limiting case, when resistive scattering processes dominate for phonons and the phonon system remains in equilibrium. We have the limiting case of Bloch–Grüneisen theory [5–9]: electrical resistance is determined by normal electron-phonon relaxation processes. In this case phonon drift velocities for all modes are different and much smaller than the electron drift velocity:

3. The role of inelasticity of electron-phonon relaxation under conditions of mutual drag of electrons and phonons is investigated. The distribution functions of the most relevant mechanisms for phonon momentum relaxation are determined: in the electron-phonon relaxation processes, as well as for phonon scattering on the boundaries, dislocations, and in the phonon-phonon umklapp processes. It was shown that relaxation rates in electron-phonon and phonon-electron scattering processes coincide.

4. Role of shear waves in the electrical resistance of potassium crystals is analyzed. It is shown that relaxation of electrons on the shear component of the slow t_2-mode in Bloch–Grüneisen approximation at temperatures of $T \ll \Theta_D$ provides 32% of the total electrical resistance, which is 4 fold the contribution of L-phonons. In the mode of strong mutual drag of electrons and phonons for potassium crystals with a minimum dislocation concentration of $N_d = 0.03$ the shear component of the t_2-mode contributes up to 56% to the electrical resistance.

It is obvious that the theory of electron transfer in metals needs a substantial revision related to the effect of elastic energy anisotropy on the dynamic characteristics of phonons.

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Conflict of interest

The authors declare that they have no conflict of interest.

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